ELEMENTS OF ELECTRICITY

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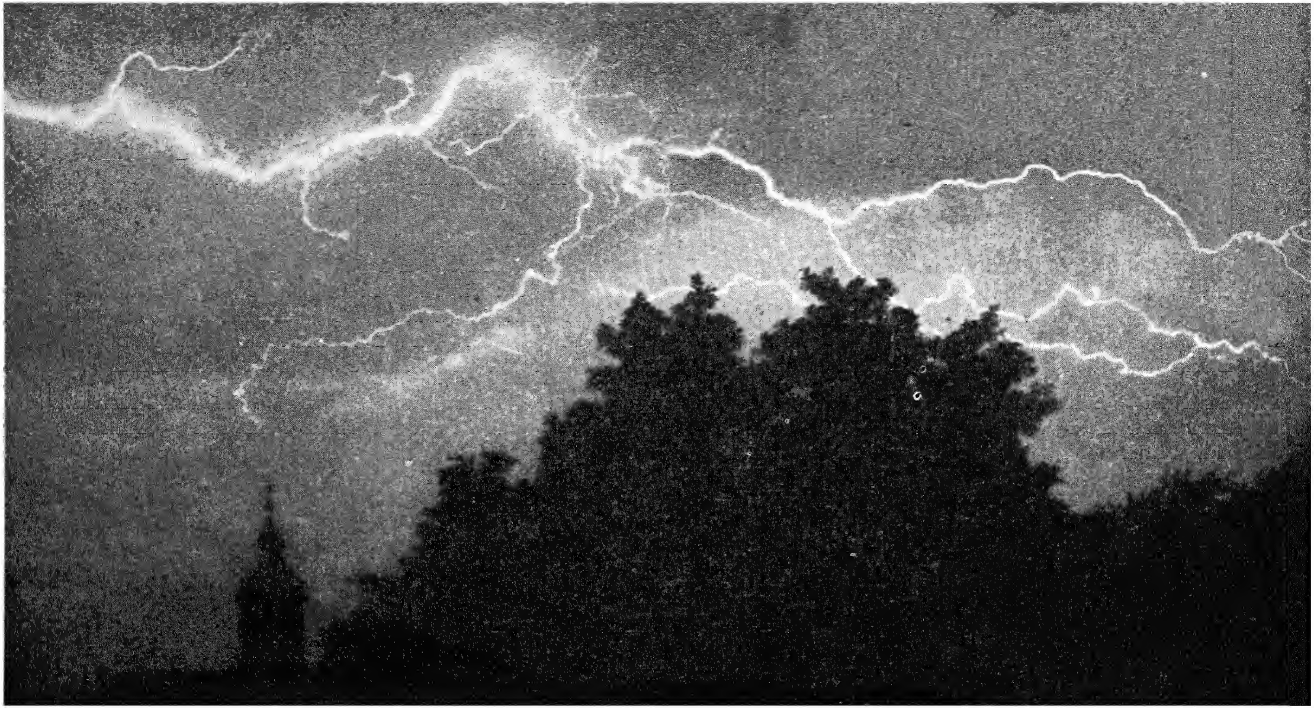

Lightning between clouds. (Photograph by Nicolas Befort, Wetzlar, Germany, 1928.)
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ELEMENTS
OF
ELECTRICITY

BY

ANTHONY ZELENY, PH.D.

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SECOND EDITION
FIFTH IMPRESSION

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PREFACE TO THE SECOND EDITION

The treatment of the electromotive force impressed by a moving magnetic field on a stationary conductor has been changed from that given in the first edition to one requiring the introduction of one more "basic phenomenon," which, however, is shown to be a consequence of relative motion. This treatment makes clear the distinction between the conservative and the nonconservative electric fields and thereby also clarifies the picturing of electromagnetic pulses.

The action of the thermocouple is treated at length and unusual clarity is believed to have been attained by representing diagrammatically the magnitudes of the contributing Thomson and Peltier effects. The chapter on Alternating Currents has been enlarged, and capacitive circuits are now treated as fully as the inductive. The reciprocal relationship between electric and magnetic fields is fully developed.

Practically every article is improved in some manner, the more modern topics are brought to date, and three appendixes are added.

The author wishes to acknowledge his indebtedness to those who pointed out errors in the first edition and suggested changes. They contributed much toward the making of a significant improvement in the text.

ANTHONY ZELENY.

MINNEAPOLIS, MINN.,
April, 1935

PREFACE TO THE FIRST EDITION

This text is the outgrowth of many years of teaching the subject of electricity to engineering, pre-medical, and arts students in the first course of college physics. The ever-increasing need in all fields for a better understanding of electricity and the continual encroachment of new subjects on the time of the student require that much be taught in the least possible time. The first course, being the only one in electricity for the great majority of the students, must be sufficiently comprehensive and contain enough practical information to meet, as far as possible, the needs common to all the groups. It must give the clearest possible picture of the interrelations between the various phenomena by explaining them in terms of the most basic facts and concepts. The student then acquires a foundation which enables him to formulate the quantitative relationships and to obtain, thereby, a training in analytical thinking and an ability to read intelligently the ordinary literature of the subject.

In attempting to meet these requirements, and in the hope of improving the exposition of the subject, the author has departed considerably from the usual method of presentation. All the major phenomena are explained in terms of physical concepts which are reducible to two basic phenomena. This explanation is made possible by taking the point of view that electric fields have inertia, interpenetrate freely, and are inseparable from their elemental charges. Superposed fields are then construed to exist as independent fields even though they neutralize one another's action on electric charges. It follows that an uncharged body is surrounded by both a negative electric field, which is the resultant of the superposed elemental fields of all its electrons, and an equal opposite positive field due to the fields of all its protons. It also follows that when electrons are in motion within or with a conductor, each electron is surrounded by a magnetic field which is inseparably associated with the moving elemental electric field. A magnetic field is therefore regarded as an aspect of a moving electric field. From this

point of view a large number of the most important phenomena, such as induction and the production of electromagnetic waves, which usually are only inadequately described, are readily explained in a satisfactory manner.

Since clearness of exposition is materially improved by referring to the direction of electron flow rather than to the conventional direction of the current, all explanations are made in terms of the flow, and arrows in the figures are drawn to represent the direction of the flow and not that of the current.

The basic principles of the subject are presented in the first twenty chapters so that it is possible to omit any or all of the last ten chapters when it is necessary to abridge the course.

The author wishes to express his gratitude to his colleagues in his own and other departments of the University for many helpful suggestions and criticisms. He is especially indebted to Associate Professor J. W. Buchta, who taught from the text in its preparatory mimeographed form.

The author also wishes to express his appreciation to the following corporations for furnishing cuts or photographs of apparatus: Central Scientific Company, Chicago; W. M. Welch Scientific Company, Chicago; Westinghouse Electric and Manufacturing Company, East Pittsburgh; General Electric Company, Schenectady; The Electric Storage Battery Company, Philadelphia; Edison Storage Battery Company, Orange, N. J.; E. Leitz, Inc., New York; and Science Service, Inc., Washington, D. C. He also wishes to thank Dr. W. H. Ude, Minneapolis, for the x-ray photographs and Dr. C. A. Coulomb, Philadelphia, for the portrait of his great-grandfather, Charles Augustin Coulomb. The cuts of apparatus manufactured by the Leeds and Northrup Company, Philadelphia, and the Weston Electrical Instrument Company, Newark, N. J., were kindly furnished by the Central Scientific Company.

The author further takes pleasure in expressing his gratitude to Lester F. Borchardt and George H. Shortley, advanced students of physics, for their faithful reading of the manuscript and of the proof and for their many helpful suggestions.

ANTHONY ZELENY.

UNIVERSITY OF MINNESOTA,
February, 1930.

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A NOTE TO THE STUDENT

Before undertaking the study of this text the student must have sufficient mathematical preparation to understand:

1. How to solve simple algebraic equations.
2. The simple rules of variation; *i e*,
   a. If $a \propto b$, then $a = K_1 b$, where K_1 is a constant. (A)
   b. If the magnitude of a depends on more than one variable so that, for example, $a \propto b$, $a \propto c$, and $a \propto 1/d$, the dependence of a on all the variables is expressed by the product of the foregoing second terms; *i.e.*

$$a \propto \frac{bc}{d}.$$

   From this it follows that

$$a = K_2 \frac{bc}{d}. \qquad (B)$$

   If in any special case c and d are constants, then

$$a = \frac{K_2 c}{d}\, b = K_3 b,$$

   which has the form of Eq. (A), where the value of a depends on only one variable.
   c. The value of the constant K in any expression is determined by finding or defining the magnitudes of the other quantities for a particular case. If in Eq. (B), $a = 1$ when b, c, and d are each equal to 1, as is frequently the case, it follows that $K_2 = 1$, and the equation for all values of b, c, and d becomes

$$a = \frac{bc}{d}. \qquad (C)$$

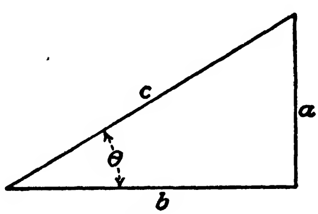


FIG. A.

3. The trigonometric functions of sine, cosine, and tangent, and that $\sin\theta/\cos\theta = \tan\theta$ In the right triangle, Fig. A,

$$a = c\sin\theta, \quad b = c\cos\theta, \quad a = b\tan\theta.$$

The student must see clearly that in any number of right triangles having in common the same acute angle θ, $a \propto c$, $b \propto c$, and $a \propto b$, from which it follows that the quantities $\sin\theta$, $\cos\theta$, and $\tan\theta$ are constants for that set of triangles. The values of these constants are tabulated in tables called tables of trigonometric functions (Appendix X).

4. How to use the symbols of calculus in a few cases. A knowledge of calculus, however, is not required. Imagine the value of a quantity to increase uniformly from Q_1 to Q_2 during the time interval t. Then the rate with which the quantity is increasing in magnitude is

$$I = \frac{Q_2 - Q_1}{t} = \frac{q}{t},$$

where q is the total amount of the change, *i e.*, $Q_2 - Q_1$

If the same rate of change continues for a shorter interval, $(1/n)t$, the change q will be smaller in the same proportion and therefore have a value of $(1/n)q$.

Then the rate of change is

$$I = \frac{(1/n)q}{(1/n)t} = \frac{q}{t}. \tag{D}$$

The ratio of the change in quantity to the time interval remains the same regardless of what the magnitude of the interval may be. The fraction $1/n$ then can be an infinitesimal.

Infinitesimal changes in quantity and in time are represented by dq and dt. These are read respectively as *differential of the quantity* q and *differential of the time* t. The rate of change then is represented by

$$I = \frac{dq}{dt},$$

which, when the rate of change is uniform, expresses the same thing as Eq. (D), *i e.*, the amount of change in the quantity per second. If the quantity does not change uniformly with time, dq/dt still represents the rate of change, but it is the rate at any particular instant under consideration.

Any one who is able to use with understanding the foregoing simple mathematical relationships and has a knowledge of the simple laws of mechanics, and preferably of heat also, can, with the proper amount of study, master the contents of this text.

It is needless to state that the student must learn the language of the science and acquire such a clear comprehension of the

concepts involved as to enable him to formulate the definitions and to derive the desired quantitative relationships. The test of his mastery of the subject is his ability to derive these relationships and to answer specific questions by the use of methods which he can develop for himself from basic concepts.

The power to analyze and to understand phenomena is not enhanced by memorizing the wording of any part of the text, not even the definitions. Read the ~~lessons,~~ before they are considered in the classroom, with care and understanding. A clear mental picture of a phenomenon is all that matters and not what the text may say about it.

A laboratory course should accompany the study of this text, because the text and lecture demonstrations alone cannot give the intimate knowledge for the proper appreciation and understanding of the subject.

A concise outline, often consisting largely of figures, should be made of each lesson. The preparation of each lesson should include a study of at least the three preceding outlines. Once a week the outlines of all previous lessons should be carefully reviewed. These reviews take comparatively little time, but the frequent repetition gives a mastery of the subject which probably cannot be obtained so effectively in any other way. Such a frequent repetition is as important as the associating of newly learned facts with those already known.

The facts and concepts of Chaps. I and V must be thoroughly mastered before attempting to read the remaining chapters. Special study should be given progressively to the topics noted in Appendix VIII, the tabulated definitions and laws in Appendixes III and IV, and the basic topics.

The student should acquaint himself with the appendixes so that he may refer to them when necessary.

The cross reference Art. V-3,4, for example, refers to Arts. 3 and 4 of Chap V; the reference Art. 3 refers to Art. 3 in the chapter being read. The lesson assignment Probs. XI-1 to 5 in Appendix XV refers to Probs. 1 to 5, inclusive, in Chap. XI.

The questions accompanying each chapter cover the whole subject matter in that chapter and should be answered by the individual to test his knowledge. In solving problems the student is expected to express the quantities involved in the form

of a general equation before making substitutions. The use of a slide rule or logarithm table in these solutions is desirable. The suggested list of experiments at the end of each chapter is principally for the convenience of the instructor.

The student takes greater interest in the subject if he holds continually in mind the following chief objectives of the course:

1. Learning the main facts about electricity and magnetism.
2. Relating these facts to one another and, directly or indirectly, to the three most basic observed facts.
3. Learning the language of the subject so that the ordinary literature on electricity and magnetism may be read with understanding.
4. Learning the chief scientific and practical applications.
5. Training in scientific and analytical thinking.
6. Obtaining a better comprehension of the physical world in the hope that each individual may in his own way be benefited thereby.

ELEMENTS OF ELECTRICITY

PART I

This part gives the basic facts and concepts and the technical language necessary for understanding the ordinary literature of the subject.

CHAPTER I

ELECTRIC CHARGES AND ELECTRIC FORCES (BASIC PHENOMENON 1)

1. Electric Charges.—A piece of sealing wax rubbed with cat's fur acquires an abnormal power to attract other bodies. The force of attraction is large enough to lift bits of paper against the force of gravity and to deflect a suspended pith ball.

If the pith ball is covered with some conducting material and is suspended by a nonconducting thread, such as silk, it is attracted as before but is repelled immediately after coming in contact with the sealing wax. This repellent force persists but disappears when the pith ball is touched by some conducting material through which it is connected to the earth.

That which is the cause of these phenomena is called *electricity*. The sealing wax becomes charged with it, and to this charge is attributed the original attraction. The pith ball on touching the sealing wax takes electricity from that part with which it comes in contact. The repulsion of the pith ball is then attributed to the two charges, one that remains on the sealing wax and the other that was transferred to the pith ball. When the pith ball is connected to the earth by means of a conducting material, the electricity "escapes" from it to the earth; when, however, the nonconducting sealing wax is grounded in the same manner, only that part of the electricity escapes which comes in direct contact with the conducting material.

A similar set of experiments may be performed by rubbing a glass rod with silk.

The charged pith ball which is repelled by the charged sealing wax, however, is attracted by the charged glass, and the charged pith ball which is repelled by the charged glass is attracted by the charged sealing wax. The charge on the sealing wax and that on the glass, therefore, produce reverse effects; and the two may have such magnitudes or be at such distances as to completely neutralize each other's action on the pith ball.

These experiments show that there are two kinds of electricity having in some respects similar and in other respects reverse characteristics. The electricity which appears on sealing wax

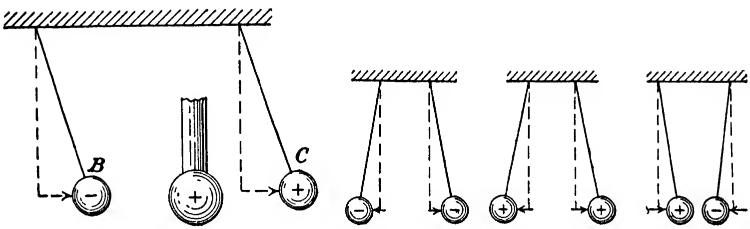


FIG. 1.—(*a*) Action of a charge on oppositely charged pith balls *B* and *C*. (*b*) Forces acting between charged pith balls

after it is rubbed with fur is arbitrarily called *negative electricity*, and that which appears on glass after being rubbed with silk is called *positive electricity*. These are usually represented by the signs − and +, respectively. The names and the signs call attention to the reverse characteristics of the two kinds of electricity, especially to the fact that one neutralizes the effect of the other.

Any two dissimilar materials brought in contact become charged, more or less, with equal amounts of the unlike kinds of electricity. The rubbing is only for the purpose of increasing the area of actual contact. The sealing wax and the glass are selected as examples because they are convenient materials which give comparatively large quantities of the unlike kinds of electricity when brought in contact with fur and silk, respectively, and which, because of their insulating properties, retain the charges at the points where they are formed.

The effect of a + charge on two pith balls charged oppositely is illustrated in Fig. 1(*a*), and the forces acting between charged pith balls in Fig. 1(*b*). The forces acting between such electric charges are called *electric forces.*

The results of these basic experiments are included in the following statement which will be referred to as *basic phenomenon* 1 (B.P.1):

There are two kinds of electricity of which charges of like kind repel one another, and charges of unlike kind attract.
(B.P.1) (1)

The attracted or repelled charges account for the attraction or the repulsion of the bodies of which they are a part.

A neutral body (also called an *uncharged body*) is believed to be composed of equal quantities of the unlike kinds of electricity. An external charge of either kind attracts the unlike kind of

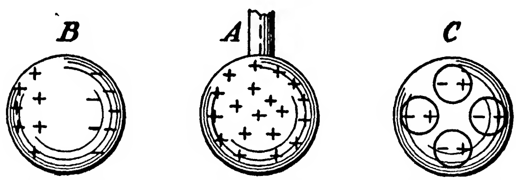


Fig. 2.—The charged body *A* produces a separation of the two kinds of electricity in the conductor *B* and in the atoms of the non-conductor *C*.

charge of this neutral body and repels the like charge. A separation of the two kinds of electricity occurs as illustrated in Fig. 2. The near unlike charges on the two bodies attract with a greater force than the more distant like charges repel. This results in the observed attraction between the charged and the neutral body. The separation occurs whether one or both kinds of electricity are mobile. In a solid conductor some of the negative charges only are believed to be mobile, which, moving to one side (Fig. 2*B*), leave the other side with an excess of positive electricity. In a perfect nonconductor there are no such free charges, but an equivalent displacement of the charges in individual atoms takes place (Fig. 2*C*), making each atom an *electric doublet.* The attraction of the nonconductor is then due to the attraction of these doublets and therefore, as in the case of conductors, to forces between electric charges. The gravitational

attraction between the masses is insignificant compared with these electric forces.

2. Electrons and Protons.—Each of the two kinds of electricity is shown (Chap. XXVI) to be composed of discrete particles or "atoms" of electricity each having a definite charge and a definite mass. The particle of the negative charge with the mass inseparably associated with it is called an *electron;* the corresponding particle of the positive charge with its associated

Sir Joseph John Thomson (1856–), Cavendish professor of experimental physics, Cambridge University, Cambridge, England; the leading contributor to the establishment of the theory of conduction by ions in gases and to the discovery of the electron (1897–1899).

mass is called a *proton.* Any electric charge, then, is composed of one or more electrons or protons. The electron and proton are therefore designated as the *elemental charges.* This designation does not mean that the electron and proton are the ultimate indivisible particles of electricity but has reference to their being the smallest particles of electricity with which we are now ordinarily concerned and which we can easily measure.

The negative charge of the electron is found experimentally to be equal to the positive charge of the proton. At the same distance, then, the two act with equal and oppositely directed

forces on other charges. The masses associated with these unlike kinds of charges, however, are not equal. The mass of the electron is 9×10^{-28} grams. The mass of the proton is 1,847 times that of the electron and is the same as the mass of the nucleus of a hydrogen atom. The proton and the hydrogen nucleus are therefore assumed to be identical.

Particles of positive electricity, called *positrons*, have been discovered recently (Art. XXVII–9) which apparently have the mass of an electron and the charge of the proton. The positron, therefore, may be a positively charged particle which is attached normally to a recently discovered neutral particle or *neutron*. The two together, on this supposition, form the proton. This assigned relationship is at this time only a conjecture.

The electric charge as such is not visible and its presence is detected only by the forces it exerts on material bodies through its action on the electric charges of which they are composed.

The atoms of all material bodies are believed to contain an equal number of electrons and protons (Arts. XXVII–12,13). Normally these electrons and protons therefore produce no action on electric charges at a distance. The body, then, is said to be "uncharged" or electrically neutral. A body is electrically charged when the number of its electrons is not equal to that of its protons. The excess of one or of the other constitutes the "electric charge on a body."

When two dissimilar solids are brought in contact, some of the free electrons (Art. II–4) flow across the boundary (Art. XIV–1) from one of the materials to the other; hence on separation the two materials are oppositely charged, as noted in Art. 1.

3. Electric Field—Electric Lines of Force.—In the study of electric phenomena it is found convenient to call the space surrounding an electric charge an *electric field*. Instead of attributing the observed force to one charge acting on another placed in its neighborhood, it is usually more convenient to consider each charge (with its field) to be acted on by the electric field of the other charge. *An electric field, then, is a space which exerts a force on an electric charge, placed in it, regardless of its state of motion or rest.* The direction in which a + charge is urged by the field is called the *direction of the electric field*. Lines drawn to represent these directions are called *electric lines of force*. The

direction of the electric field in the neighborhood of charges is represented by the lines of force shown in Fig. 3. The arrowheads in each case indicate, by convention, the direction of the field, *i.e.*, the direction in which a positive charge is urged by the field. A negative charge placed in the field, then, is urged in a direction opposite that in which the lines of force of the field point. The directions of the forces acting on charges are indicated by the short arrows attached to the oppositely charged bodies, A and B, shown in the electric fields of Fig. 3.

The electric fields about electrons and protons are called *elemental fields.* Since the elemental charge cannot be imagined

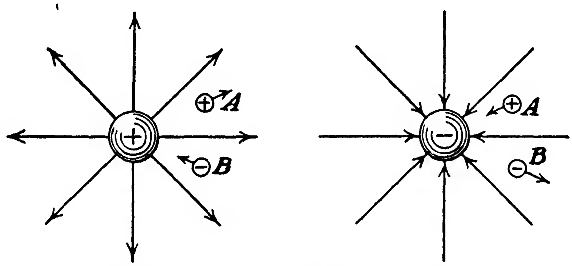


FIG. 3.—Electric fields about positive and negative electric charges (on the larger spheres) and their action on the oppositely charged bodies A and B.

apart from its elemental field, the elemental charge and its field are, in this text, considered as inseparable parts of one and the same physical entity. (Those who are unable to picture the elemental field as a physical entity can use the field concept as a convenient tool without committing themselves as to its nature provided they think of it as being inseparable from the elemental charge with which it is associated.) In order to explain the various electric and magnetic phenomena in terms of these elemental fields, it must be assumed that

Elemental electric fields extend indefinitely, interpenetrate one another freely, exert forces on electric charges placed in them, possess inertia, and, when in motion, their lines of force move parallel to themselves. (2)

The resultant electric field about a charged body is pictured as being composed of the superposed elemental fields of all the excess elemental charges, each of these elemental fields retaining its physical identity unaltered, as represented in Fig. 4. For pur-

poses of simplifying analysis, the electric field about a charged conductor is represented by the resultant of the elemental fields, as illustrated in Fig. 5(*a*); and the uncharged or neutral body, which is composed of an equal number of electrons and protons, is pictured, Fig. 5(*b*), as being surrounded by a resultant *electron field* superposed on an equal oppositely directed resultant *proton field*. The term resultant is usually dropped in speaking of any resultant field. Since the electron and proton fields about a neutral body neutralize each other's action on electric charges, the neutral body usually is represented as not being surrounded by any field.

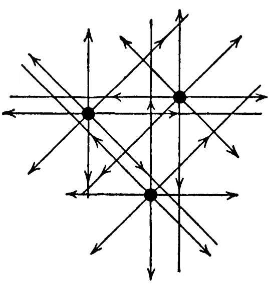
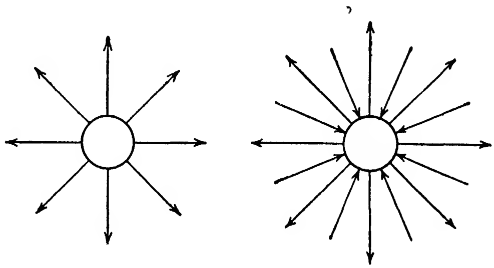


FIG. 4.—Superposed elemental fields (shown in one plane only)



(*a*) (*b*)

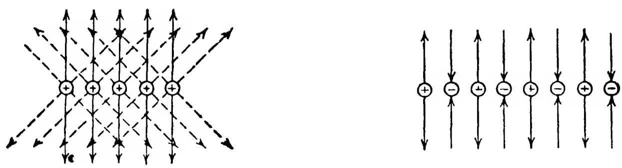
FIG 5—(*a*) Resultant field about a positively-charged body. (*b*) Electron and proton (resultant) fields about an uncharged body

If a large number of protons or of any positive charges are placed in line, the electric fields of the individual charges are superposed and together produce a field which is everywhere perpendicular to the row of charges. The broken lines, Fig. 6(*a*), represent the electric lines of force about each of several such positive charges, and the full lines the resultant field. Both sets of lines are shown in one plane only.

A row of electrons or of any number of negative charges is surrounded by a similar field except that the direction of the lines of force is reversed.

A metallic conductor or any neutral body may be considered as being composed of a large equal number of positive and negative charges arranged in rows. The resultant of the superposed electric fields of the positive charges diverges outward, and that of the negative charges converges toward the conductor. These

two resultant fields are shown, in one plane only, in Fig. 6(*b*). They extend to an infinite distance and together produce a resultant field of which they are the components. These component fields at any point in space are equal in strength and oppositely directed; hence they neutralize each other's action on other electric charges at every point. The electric field about such a



(*a*) (*b*)

FIG 6.—(*a*) Resultant proton field of the superposed fields of a line of positive charges (shown in one plane only) (*b*) Superposed resultant electron and proton fields about a line of positive and negative charges or about a conductor (shown in one plane only)

conductor, therefore, has zero intensity (*i.e.*, there is no effective electric field); however, for purposes of analysis in this text, the two component equal electric fields are treated as having an existence independent of each other. The electron field will be said to neutralize (the effect of) the proton field.

The electric lines of force can be traced roughly by means of a strip of paper which is made conducting and is suspended by a silk thread. The free electrons in the conducting surface are forced to one side by the electric field, causing the two ends of the strip to become charged with unlike kinds of electricity. The strip then is forced to take the direction of the lines of force as shown in Fig. 7, the + charge being urged by the field in the direction of the lines of force and the − charge in the opposite direction, as represented by the arrows *ff*.

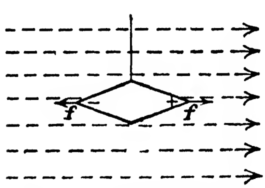


FIG. 7—Suspended conducting paper indicating the direction of an electric field.

The direction of an electric line of force at any point in the neighborhood of two or more charges is that of the resultant of all the electric forces that would act on a positive point-charge placed at that point. Experiment shows that

An electric field acts with the same force on any given electric charge regardless of whether this charge is in motion or at rest. **(3)**

Each of several fixed charges exerts the same force on a given charge as though it acted alone. **(4)**

A test point-charge at P, Figs. 8 and 9, then, can be pictured as being acted on by two forces, f_1 and f_2, due to the superposed electric fields of charges A and B and represented by the broken lines. The resultant, F, of these forces gives the direction of

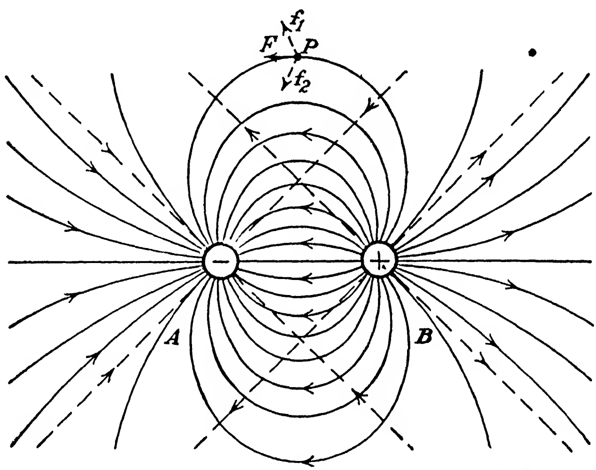


FIG 8.—Electric field about two equal unlike charges.

the resultant field at P. The loci of all such resultants are the full lines shown in the figures; these lines by convention are called simply the electric lines of force about the charges and are said to represent the electric field surrounding the charges. The lines in the figures necessarily show only those lines of force which are in the plane of the paper.

By knowing that the two charges in Fig. 8 attract, and those in Fig. 9, repel, a study of the lines of force between them leads to the following statement:

Electric lines of force tend to become as short as possible and to repel one another. **(5)**

It should be noted here that lines of force are spoken of as though they were real entities, while in fact they are imaginary

and represent only a condition in the space which causes + electric charges to be urged in the direction indicated by the lines. In the sense used here, the number of lines that may be drawn about charges is indefinite; but for some purposes it is advantageous to restrict their number either to one or to 4π for each electrostatic unit of charge (Arts. III–1, 2).

An electric line of force conveys the information that a force acts in the direction of the line on any positive charge placed at any point on that line and that it acts in the opposite direction

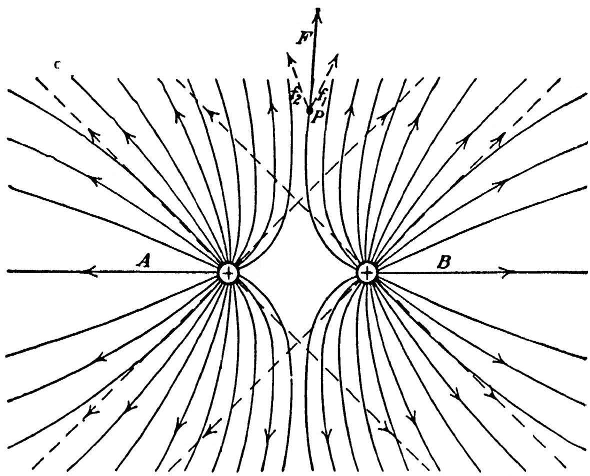


FIG. 9.—Electric field about two equal positive charges

on a negative charge. Work has to be done to move either of these charges against this force. These charges, when displaced, acquire potential energy equal to the work done upon them. If the charges are allowed to move freely in the electric field, the field does work upon them, and their potential energy gradually changes to kinetic as the velocity given them increases. This is restated in the following law:

An electric charge in an electric field possesses potential energy and is urged by the field toward points where its potential energy will be less. (6)

The term *electrostatic field* is sometimes used for the term *electric field* to emphasize the fact that the field is produced by a charge at rest or to distinguish such a field from an electric

field in motion or from the so-called nonconservative electric field (Chap. XIII–5).

The concepts of electric field, electric lines of force, and Laws 3 to 6 are no new facts in nature not included under B.P.1; they are elaborations, necessary consequences, or conventions that describe certain observed facts more conveniently than the more general concept. The B.P.1 is general and includes all that has been considered in this chapter.

Questions

1. Give the simple standard method for producing each of the two kinds of electric charges.

2. State B.P.1 (electric forces).

3. Which are the mobile charges in solid conductors?

4. Explain why an uncharged body is attracted by both kinds of electricity. In what respect does the explanation for the case of an attracted conductor differ from that of an attracted nonconductor?

5. Define electron; proton; elemental charge.

6. What are the relative magnitudes of the charges associated with the electron and with the proton? Of the masses?

7. What is the relation of the electron and the proton to the structure of material bodies? What is meant, in view of this, by a positive and by a negative charge on the body? How is the production of two dissimilar charges by the contact of two solids explained?

8. Define an electric field, and state what is meant by the direction of the field.

9. Define electric lines of force, and give their conventional direction. Are they real entities?

10. What is the direction of the electric field about each of the two kinds of charges? What is a proton field? An electron field?

11. What is the relation of the electric field to an electric charge?

12. Distinguish between elemental, resultant, and component electric fields.

13. Give the qualities that must be attributed to the elemental fields (Law 2).

14. What is the concept regarding the electric fields about an uncharged conductor?

15. Does the electric field, when at rest, act on any given charge with the same force regardless of whether the charge is at rest or in motion?

16. Is the force of one fixed charge acting on another fixed charge altered when other charges are also acting on it at the same time?

17. Draw the resultant lines of force between two charges that attract, and between two charges that repel. What law relating to the properties of lines of force is derived from these observed directions?

18. How is the direction of a line of force in the neighborhood of two charges determined experimentally?

19. What is meant by component and resultant electric lines of force?

20. Can a line be considered a component of some line and a resultant of other lines?

21. Give, in terms of the change in the potential energy of a charge, the direction of the force with which an electric field acts on the charge (Law 6).

Experiments

1. Sealing wax rubbed with fur attracts bits of paper, and a sheet of paper rubbed with fur adheres to the walls of the room.

2. Electricity produced either on sealing wax by fur or on glass by silk attracts a suspended pith ball and then repels it.

3. A pith ball charged with the electricity on sealing wax is repelled by the electricity on the sealing wax and is attracted by the electricity on the glass. The existence of + and − charges demonstrated.

4. Two pith balls charged with unlike kinds of electricity approached with + and − charges.

5. Electric lines of force between like and unlike charges traced by means of a suspended conducting paper needle.

6. Model of an elemental charge and its electric field

7. Model of interpenetrating elemental fields when their charges are at rest and when in rectilinear and in circular motion.

CHAPTER II

ETHER AND MATTER

1. The Ether.—What is the mechanism by which an electric charge exerts a force on another charge through intervening space? How is light propagated? These and similar questions have been asked for generations; have been "answered"; and all proposed solutions have been rejected. The corpuscles of Newton gave way to the nonviscous ether. This ether was finally endowed with all kinds of properties, and serious efforts were made to make it fit the increasing volume of observed facts. Electrical phenomena as well as those of light were explained in terms of it.

The ether was assumed to fill all space and wave motion was propagated through it. Electric fields were "strains" produced in the medium by the electric charges. The ether was more rigid than steel; yet all material bodies moved through it without friction. This ether, however, cannot account for the propagation of energy by quanta or for the magnetic field about a uniform current. It leads to many paradoxes, and all attempts that have been made to demonstrate its existence have failed. These inadequacies and failures have led to the almost complete abandonment of the hypothesis.

The aid of an ether need not be invoked if one is content to observe forces transmitted through open space without concerning one's self about the mechanism which propagates them. The observed laws of action are expressed quite as definitely when no thought is given to their manner of transmission. Such a procedure employs "action at a distance" without denying the possibility of a transmitting mechanism. The term "ether" is still used, more or less, especially in connection with wave propagation, but is not the old ether. It is employed, for convenience, only because the wave aspect of energy propaga-

tion is best represented by means of it; perhaps it is used also because there still lurks in the minds of many the idea that energy is propagated through space by some kind of a medium whose properties and relation to matter are now a mystery. A mathematical equation alone does not fully satisfy.

The treatment of electric phenomena in this text makes no use of the old ether. The elemental electric fields which are inseparably associated with the electric charges are given the functions of the ether. These fields are assumed to interpenetrate freely and are endowed with inertia and with the power to act on electric charges (Law I–2). These properties are either observed facts or necessary assumptions to account for the observed facts. No question is asked regarding the mechanism that gives these qualities to the fields. The "fields," however, move with the charges and are never (except apparently) separated from them. The electron with its associated electron field, then, may be considered as one entity and the proton with its associated field as another. Superposed fields may neutralize one another's action on electric charges, but they do not destroy one another. The foregoing treatment does not require the postulation of an ether of any kind as a separate entity from matter and is intended to be used at present, at least, only in connection with gross electric phenomena (principally outside the atom and molecule), to which it is known to be applicable. Every point in space, according to this concept, contains the superposed fields of all the electrons and protons in the universe. The electric fields perform the functions of the old ether and in that sense may be loosely called the ether.

2. Atoms.—All experimental evidence confirms the belief that atoms are composed of electrons and protons. An electrically neutral atom, then, must have the same number of electrons as protons. The atom of hydrogen is found experimentally to have the same mass as the proton. Because an electron added to a proton increases the mass less than measurement can detect, and for other reasons, an electrically neutral atom of hydrogen is believed to consist of one proton and one electron. The two charges may be assumed to rotate about their common center of mass because such a rotation only can be imagined to keep the attracting charges apart. The electron is said to rotate about

the proton because the latter is much nearer the center of the mass.

Heavier atoms contain a larger number of electrons and protons. Experiments lead to the conclusion that the protons are concentrated at the center with a smaller number of binding electrons and are kept in a close group by electric and magnetic forces. This concentrated group of protons and electrons is called the *nucleus* of the atom. The nucleus always has a positive charge with the proper number of electrons rotating about it in various orbits to make the whole atom electrically neutral in its normal state. For example, the nucleus of the atom of oxygen is believed to consist of 16 protons and 8 nuclear electrons, with 8 orbital electrons revolving about the nucleus. The latter are called *orbital electrons* to distinguish them from the *nuclear electrons* which aid in binding the protons. Each atom, consisting of electrons revolving about a nucleus, may be considered as a sort of a solar system. Very little is known about these orbits; so at present it is convenient merely to assign the orbital electrons to different energy levels (Art. XXVII-1).

The diameters of atoms vary from 1×10^{-8} to 5×10^{-8} cm. The diameter of the electron, calculated on the assumption that the whole mass is due to self-induction (to be explained later), is 3.8×10^{-13} cm, and the diameter of the proton, if it is assumed to be a simple charge like the electron, is 2.0×10^{-16} cm. It should be noted that, although the electron has a mass less than that of the proton, its radius is proportionately greater. If a hydrogen atom be imagined to be magnified so that its nucleus, a single proton in this case, has a diameter of 1 mm, the electron would have a diameter of 1.8 meters and would be at a distance of about 54 km (34 miles).

Nothing is known concerning the ultimate structure of electrons and protons. They are the smallest particles with which normally electric charges are found to be associated. Ultimately it may be shown that the proton is a stable union of a neutron and a positron (Art. I-2), but at present such a statement is a conjecture only.

3. Molecules and Field of Molecular Action.—In order to understand electric phenomena it is desirable to have some concrete picture of intermolecular forces and of the positions of atoms within a solid.

A molecule is composed of one or more atoms and may vibrate or rotate as a single body. When several atoms form a molecule, they may also vibrate or rotate as individuals within the molecule. Molecules, in this sense, are not supposed to exist in a solid.

Molecules or atoms at a distance attract one another with a force that varies inversely as the square of their distance; but when they are brought very close together, the force appears to become much greater than can be explained by this law of gravitation. In this region the force may be assumed to vary inversely as some variable higher power of the distance. This region is called the *field of molecular action.* As the atoms approach still closer, a point is reached where they appear to repel one another. At a temperature of absolute zero the atoms are assumed to be at rest at the point of equilibrium. If one of them is struck and thereby forced closer to another atom, the repellent action forces the atoms apart, giving them velocities that carry them beyond the point of equilibrium. The energy of the vibration is distributed in this manner between the neighboring atoms and is designated as heat. The intermolecular forces acting are believed to be electric and magnetic forces. The field of molecular action is not well understood, and much doubt exists, except in some simple crystals, regarding the variation of the force of attraction with distance.

The diameter of an atom in a solid is usually assumed to be the distance between the centers of two adjacent atoms. The outermost orbits of two adjacent atoms, however, may be far apart. Each atom, as already stated, corresponds roughly to a solar system, and the molecule (usually) to a group of these. A solid body, therefore, may be considered as consisting of "miniature solar systems," the electrons and protons of which occupy only an insignificant portion of the space within the solid. The dominant forces are electric and magnetic and not gravitational.

In a gas there are 27.09×10^{18} molecules/cm^3 at N.P.T.; in 1 cc of copper there are 8.51×10^{22} atoms.

4. Free Electrons in Solid Conductors (Conduction Electrons).—Various phenomena point to the existence of free electrons in a solid conductor. The existence of such free electrons is here hypothesized. These electrons perhaps move from

atom to atom and like comets swing around their nuclei. But for purposes of simplicity they are imagined to have left their orbits in the atoms of the solid and to have within the solid chaotic motions like the atoms of a monatomic gas. The atoms that have lost an electron are positively charged and, though vibrating because of temperature, are not free to move about in the solid. They are held in definite positions by molecular forces.

A body is charged with positive electricity when, as already stated, it has lost some of its free electrons, and with negative electricity when it has acquired an abnormal number of them.

The circles, Fig. 1, represent neutral atoms in a solid; the crescents, atoms that have lost an electron; and the dots, the free

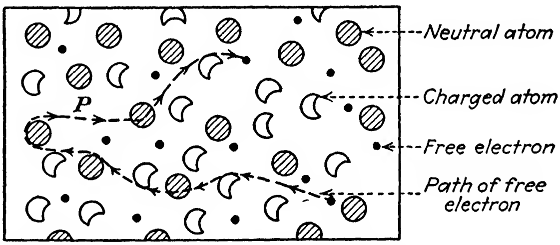


FIG. 1.—Representation of the chaotic motion of a free electron in a conductor.

electrons. The free electrons may recombine with atoms, but in the same material their number per cubic centimeter is assumed to be almost universally constant; when one electron recombines, another becomes free. At ordinary temperatures the free electrons do not break through the surface of the metal. Any electron moving outward at the surface is drawn back by the force of attraction of a neighboring positive charge from which it is escaping. The free electrons then must be assumed to move about within the surface like the atoms of a monatomic gas contained in a closed vessel. The fastest of the electrons force their way in large numbers through the surface only at temperatures higher than 500°C.

The free electrons that take part in the transfer of electricity in conductors also take part in the conduction of heat. They are called *conduction, free,* or *flow electrons.* The refraction of light in metals and the smaller forces binding the outermost or valence electrons of the atoms indicate that their number is approxi-

mately that of the atoms in the conductor multiplied by the valence of the atoms. By assuming the valence of copper in the solid state to be 1, the assumption gives 8.51×10^{22} conduction electrons (about 13,530 coulombs) to each cubic centimeter of copper.

Questions

1. What is a hypothesis? A theory? An established fact?

2. What is the ether? May the idea of an ether be discarded?

3. How are electric fields treated in this text?

4. What are the relative diameters of electrons, protons, and atoms?

5. Why is the atom of hydrogen believed to be composed of one proton and one electron?

6. What is the nucleus of an atom? What is its composition? Is it always positively charged? What are nuclear and orbital electrons?

7. Describe the construction of the oxygen atom.

8. In what respects does an atom resemble the solar system?

9. What is a molecule? What is meant by the field of molecular action?

10. What is meant by two atoms in contact?

11. What is meant by free or flow or conduction electrons?

12. Explain the sign of the charge on a body in terms of the number of its electrons.

Problems

1. Calculate the number of years it would take all the people of the earth to count the number of molecules in 1 cc of a gas at N.P.T. Assume each individual to count 12 hr. a day at the average rate of 1 unit/sec. The population of the earth is 1.75×10^9, and the number of seconds in a year is 31.56×10^6.

$$t = \frac{N}{n} = \frac{27 \times 10^{18}}{{}^{12}\!/_{24}(31.56 \times 10^6) \times 1.75 \times 10^9} = 978 \text{ years.}$$

2. How many protons together have a mass of 1 milligram?

$$n = \frac{M}{m} = \frac{0.001}{1{,}847 \times (9 \times 10^{-28})} = 600 \times 10^{18}.$$

3. How many electrons placed in line would cover a distance of 0.001 mm?

$$n = \frac{s}{d} = \frac{0.0001}{3.8 \times 10^{13}} = 2.63 \times 10^8.$$

The solutions of these problems are given for the purpose of illustrating the form in which all problems in this text should be solved.

Experiments

1. Model of a hydrogen atom.

2. Model of a helium nucleus.

3. Model showing atoms in a solid.

CHAPTER III

ELECTROSTATIC UNITS, ELECTRIC FIELD, POTENTIAL

1. Unit Electric Charge (Statcoulomb)—Law of Attraction and Repulsion between Charges.—It is necessary that some definite amount of electricity be called a unit quantity of electricity. Three such units are in use. The electrostatic unit (e.s.u.) of quantity of electricity (also called a *statcoulomb*). is normally the most convenient unit to employ when electricity at rest is referred to. It is defined as follows:

The electrostatic unit of quantity of electricity, the statcoulomb, is that quantity of electricity which, when concentrated at a point in a vacuum, repels an equal like charge at a distance of 1 centimeter with a force of 1 dyne.

The quantity of electricity measured in terms of this unit is represented in this text by Q'' or q'' (in an exceptional case by e''). Physical intuition, confirmed by experiment, shows that between any two point charges the force of attraction or repulsion varies as the product of the charge magnitudes. This relationship when expressed by mathematical symbols is

$$f \propto Q_1''Q_2''.$$

It is found experimentally by means of a torsion balance that for the same two charges the force varies inversely as the square of the distance between the charges; *i.e.*,

$$f \propto \frac{1}{d^2}.$$

Then

$$f \propto \frac{Q_1''Q_2''}{d^2},$$

or

$$f = \pm \frac{1}{K}\frac{Q_1''Q_2''}{d^2}. \qquad (1)$$

This basic equation is the quantitative expression for B.P.1 and is known as *Coulomb's law.* It also illustrates the *law of inverse squares,* one of the most important laws in nature. The $\pm$ sign indicates that the force is one of repulsion or of attraction depending on whether the charges are like or unlike, and the constant is written as a reciprocal because in that form the K has a specific meaning. The K then is the dielectric constant (Art. XII–4) of the particular medium into which the charges may be placed. It follows from the definition of a unit charge that in a vacuum $K = 1$, so that Eq. (1) becomes

$$f = \pm \frac{Q_1''Q_2''}{d^2} \text{ dynes.}$$

In material media the dielectric constant K always has a value greater than unity. In air $K = 1.000586$, which, for most purposes, is practically unity; forces existing between charges in air therefore are practically equal to those in a vacuum. In water the dielectric constant $K = 81$, and therefore the forces between charges in water are smaller than in a vacuum by that factor. Why a material dielectric diminishes the force acting between two charges is explained in Appendix V (1).

The elemental charge, *i.e.*, the charge of an electron or proton, in literature is represented by the letter e'' (usually written without the primes) in place of q''. Its magnitude as determined experimentally is

$$\mathbf{e'' = 4.770 \times 10^{-10} \text{ statcoulombs,}} \qquad (2)$$

from which the number of elemental charges (electrons or protons) in a statcoulomb is calculated to be

$$n = 2.096 \times 10^9.$$

This number is somewhat greater than the population of the earth (1.75×10^9) and is equal to the number of seconds in 70.7 years.

2. Intensity of the Electric Field—Unit Electric Field.—Since the electric field acts on electric charges, its *intensity* or *strength* naturally is measured by the force with which it acts on a unit electric charge placed in it; then

An electric field of unit intensity is a field which acts on a statcoulomb of electricity with a force of 1 dyne.

This unit field has no special name other than e.s.u. (electrostatic unit). The intensity of any field expressed in these units is represented by the letter F'' (and in literature also by the letter E). The force acting on any charge Q'' in a field of intensity F'' is

$$f = Q''F'' \text{ dynes.} \tag{3}$$

The magnitude of this *electric force* is independent of the state of motion or rest of the charge or of the field.

The foregoing statement applies equally to forces acting on electric charges in the so-called *nonconservative electric field* (Art XIII-5) but excludes the so-called magnetic force (Chap. V) which appears in addition to the electric force when the electric field and the charge immersed in it both are in motion with respect to the observer

Since by Eq. (1) the force acting on a unit charge at a distance d from another charge Q'' is

$$f_1 = \frac{Q'' \times 1}{d^2} = \frac{Q''}{d^2},$$

and since the force acting on a unit charge is a measure of the intensity of the field, the intensity of the electric field at any distance from an electric point-charge is

$$F'' = \frac{Q''}{d^2} \text{ e.s.u.} \tag{4}$$

Because the intensity of an electric field at any point is measured in terms of a force, a vector quantity, the field itself must be a vector quantity. It has direction and magnitude. It then follows that

The resultant of several superposed electric fields is their vector sum. (5)

A charge which is uniformly distributed over the surface of an isolated sphere has its lines of force diverging outward from the surface. These lines if extended back into the sphere meet at its center. At any point in space, therefore, the electric field due to a charged sphere is that due to an equal point-charge located at the center of the sphere. The intensity of the electric field and the potential (Art. 3) at any point outside the charged sphere, therefore, may be calculated as though the whole charge on the sphere were concentrated at its center. *The charge on a*

sphere then is equivalent to an equal point-charge at the center of the sphere.

At a distance of 1 cm from a unit point-charge the field intensity, F'', is 1 e.s.u. and is the same at all points about the charge on the surface of an imaginary sphere of 1-cm radius. The area of such a sphere is 4π cm². A unit field is assigned, by convention, one line of force per square centimeter. There are then 4π lines, all emanating from the unit charge, piercing this sphere. The number of lines of force emanating from any charge Q'', then, is $4\pi Q''$. Another convention assigns only one line of force to each unit charge. Inspection shows that parallel lines of force represent fields of uniform intensity throughout and divergent lines fields which vary in intensity from point to point except in directions at right angles to the lines.

3. Work Done in Moving Electric Charges in an Electric Field—Potential—Potential Difference—Statvolt.—Assume the electric field, Fig. 1, to be one of uniform intensity F'', such as exists between two oppositely charged parallel plates. A unit + charge (1 statcoulomb) at the point A feels a force

$$f_1 = F'' \text{ dynes.}$$

If the charge is moved by this force a distance l to the point B, the work done on it *by the field* is

$$w = f_1 l = F''l \text{ ergs.}$$

If the + charge be moved from B to A, *external energy* has to be supplied to move it against the electric force. No work against electric forces is required to move an electric charge at right angles to the lines of force, *i.e.*, from D to E. In moving the charge from C to D the work done is the product of the force acting on the charge and the distance CE, which is the component of the distance CD in the direction of the lines of force.

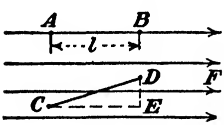


Fig. 1.—Energy expended in moving electric charges along electric lines of force.

If the field is not uniform, the work done is the product of the average force acting on the charge and the distance the charge is moved in the direction of the lines of force.

If a unit + charge is moved from B to A, work is done upon it, and its potential energy increases in the same manner in which the potential energy of a unit mass of matter increases when it is raised above the surface of the earth. By analogy, the point A may be considered as being at a higher electric level than the point B. The external energy supplied to move the charge

from B to A becomes a part of the potential energy of the charge; but when the charge moves from A to B and the "electric field does the work," the potential energy of the charge is changing into kinetic energy.

It is convenient to use the term *potential* to designate the electric level of any conductor or point in space. The measure of its magnitude is the work required to move a unit + charge to the conductor or point from some location that is designated as having zero potential, if it is assumed that the unit charge causes no redistribution of charges on the neighboring conductors. It is convenient sometimes to assign zero level (zero potential) to the dome of infinite space, and at other times to the earth. The potential at the point A, then, is measured by the work, in ergs, required to move a unit + charge to that point either from an infinite distance or from the earth. That the magnitude of the potential is the same regardless of which zero level is used will be shown in Art. IV–5. The actual approach of the unit + charge alters the potential at the point, but the work required to transfer the unit charge, nevertheless, is measured by the original potential at the point. The unit charge and its electric field are considered as being moved in the resultant of the electric fields of all the other charges in the neighborhood. Any alteration of the original field due to the field of the charge being moved does not affect the work done on the charge.

The *potential difference* between two conductors or two points in space is the difference between the potentials of the given conductors or points and therefore in the electrostatic system of units is measured by the work in ergs required to move a unit + charge through the space between the conductors or points. The potential at a point may be regarded as the potential difference between that point and either the earth or the infinite dome. The unit of potential or of potential difference (fall of potential or potential drop) in the electrostatic system is called a *statvolt*.

The electrostatic unit of potential difference, the statvolt, exists between two points when 1 erg of work is expended in moving a statcoulomb of electricity from one of the points to the other.

The potential in the electrostatic system is represented in this text by V'' or by E'', and the potential difference by the same

symbols or by $(V_1'' - V_2'')$. It then follows that the potential difference between any two points is

$$V'' = w_1 = f_1 l = F'' l \text{ statvolts,}$$

and that the work required to move any charge Q'' between the two points is

$$w = Q''V'' \text{ ergs.} \qquad (6)$$

4. Positive and Negative Potential.—Energy from an external source is required to move a unit + charge from the earth G, Fig. 2(*a*), to the + charge of conductor R. The potential of the charged conductor (and likewise that of any point about it) then is higher than the zero potential of the earth and by convention is called a *positive* or a + *potential*. In the case of the negatively charged conductor S, energy is expended in moving

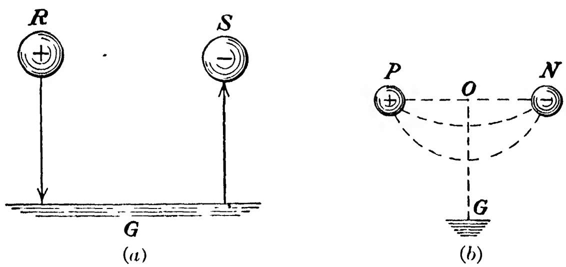


FIG 2—(*a*) The potential of the + charge on conductor R is higher than the zero potential of the earth G, and that of the − charge on conductor S is less than zero The arrowed lines represent the directions of the electric fields between the charges and the earth (*b*) Potentials due to the two equal unlike charges have a negative sign at any point in the space on the right-hand side of the line OG and a positive sign on the left-hand side

a unit + charge in the reverse direction, *i.e.*, from the conductor to the earth. The potential of the conductor then is less than the zero potential of the earth and consequently is called a *negative* or a − *potential*.

The electric lines of force between the oppositely charged spheres P and N, Fig. 2(*b*), are crossed by the line OG drawn perpendicular to all the lines of force. Since no work is done in moving charges at right angles to the lines of force, the potential at the point O is zero.

Since, by convention, the *point of higher potential* is the point at which the potential energy of a unit + charge is greater,

the potential energy of such a charge, and therefore the potential at the successive points along the line *NOP*, increases from *N* to *P*. Since the potential at the point *O* is zero, and yet higher than that at *N*, that at *N* must be less than zero, *i.e.*, negative. The potential energy of the unit + charge at *P* is greater than it is at *O*; therefore, the potential at *P* is greater than zero and is a positive potential.

It then follows that in the neighborhood of an isolated free charge the sign of the potential at any point is that of the charge, and at any point in the neighborhood of two unlike charges (free or bound) the sign of the potential is that of the charge which contributes most in magnitude to that potential.

Points of + and of − potential have potentials respectively higher and lower than that of the earth.

If a + charge moves from another free + charge toward a free − charge, it moves from points of higher to points of lower + potential until it reaches a point having zero potential. From this point it continues to move from points of lesser − potential to points of greater − potential. Since the greater − potential is a "lower" potential, the charge is continuing to move from points of higher to points of lower potential. This motion corresponds to the falling of a mass from a height into a well. The mass falls from a higher level to zero level and then continues falling from zero level to a still lower level.

A + charge is moved by the field from points of higher to points of lower potential, and a − charge is moved by the field from points of lower to points of higher potential. In any case, as already stated (Art. I–3), the field moves either kind of charge toward points at which the potential energy of the charge is less. Restated:

An electric field moves positive charges from points of higher to points of lower potential, and negative charges from points of lower to points of higher potential. Potential energy therefore is given to a positive charge when it is moved to points of higher potential and to a negative charge when that is moved to points of lower potential. **(7)**

This law is often invoked in explaining phenomena and is usually a more useful point of view than the statement that + charges are urged by the field in the direction of the lines of force and that − charges are urged in the reverse direction.

The energy expended by an external agent in moving a charge between two points having different potentials, in the electrostatic fields considered above, is conserved and becomes available when the charge returns to its original position. Such a field is called a *conservative field* whenever it is desirable to distinguish it from the nonconservative electric field of Arts. XIII–5, XVI–2.

5. Four Points of View of the Action on an Electric Charge.—Let the charge A in each case, Figs. 3(a)(b), be regarded as fixed and the charges B and C free to move. The charges B and C with their electric fields (not shown) are in the electric field of the charge A, which field is represented in each of the two cases by the long arrowed lines. The charges B and C are assumed to be placed in the field one at a time, so that the short arrowed lines

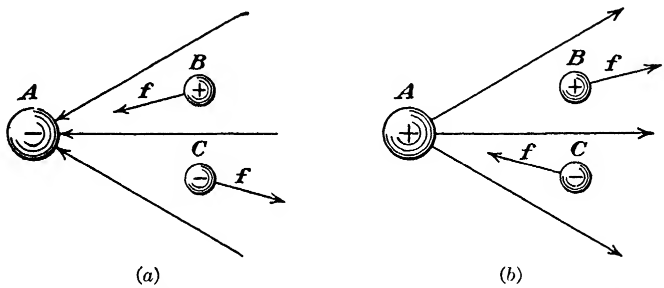


FIG. 3.—Forces f acting on charges B and C in the electric field of the charge A, the interaction between charges B and C being disregarded.

on B and C show the directions of the forces evoked by the charge of A alone. In Fig. 3(a) the + charge B is known to be urged by the charge A from right to left; in Fig. 3(b) the same + charge B is urged from left to right. From what has been learned this action on the charges B and C can be "explained" in any one of the following four ways:

1. The charge B is attracted or repelled by the charge A (B.P.1).

2. A + charge is moved by the electric field in the direction of the lines of force, and a − charge is moved in the opposite direction.

3. The + charge moves freely from the higher to the lower potential, and the − charge, from the lower to the higher. The potential energy of a system always tends to become less.

4. Lines of force tend to shorten and to repel one another. (See Figs. I–8, 9.)

Point of view 3 is the most useful; however, in many cases it is more convenient to employ some one of the other points of view.

6. Electric Fields and Potential Differences—Potential Gradient.—Wherever there is an electric field, work is done, either by the field or by an external force, in moving charges along the electric lines of force. In other words, a potential difference necessarily exists between any two points on an electric line of force. Wherever there is an electric field, potential differences exist; and wherever there are potential differences, there is an electric field.

The potential difference per unit distance is given the name *potential gradient*. When expressed in the electrostatic system, the potential gradient and the average intensity of the electric field are numerically identical, for

$$V'' = w_1 = f_1 l = F''l \text{ statvolts,} \tag{8a}$$

in which f_1 is the force acting on a unit charge and by definition (Art. 2) is numerically equal to the intensity of the field, F''. Then

$$F'' = \frac{V''}{l} \text{ e.s.u.,} \tag{8b}$$

in which V''/l is, by definition, the potential gradient.

7. Potential Due to a Point-charge.—Let $+Q''$, Fig. 4, represent an electric charge concentrated at a point, and let the distances of the points A, B, C, D, be a, b, c, d cm from this charge. Also let the distances between the points A, B, C, D be very short or infinitesimal.

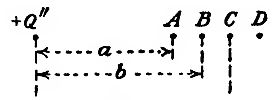


FIG. 4.—Work required to move a unit charge between consecutive points on an electric line of force.

To find the potential at the point A, it is necessary only to imagine a unit + charge at that point and to calculate the amount of work expended by the field in moving this charge to an infinite distance.

The force acting on a unit charge at A is

$$f_a = \frac{Q'' \times 1}{a^2} = \frac{Q''}{a^2} \text{ dynes;}$$

When the charge reaches the point B at a distance b from Q'', the force is

$$f_b = \frac{Q''}{b^2} \text{ dynes.}$$

The work required to move the charge from A to B is the average force times the distance. If B is imagined to be at an infinitesimal distance from A, the average value of the force, Appendix V (3), is

$$f_{ab} \cong \frac{Q''}{\frac{a^2 + b^2}{2}} \cong \frac{Q''}{ab} \text{ dynes;}$$

the work done,

$$w_{ab} = \frac{Q''}{ab}(b - a) = Q''\left(\frac{1}{a} - \frac{1}{b}\right).$$

Similarly, in moving from B to C,

$$w_{bc} = \frac{Q''}{bc}(c - b) = Q''\left(\frac{1}{b} - \frac{1}{c}\right).$$

In moving from C to D,

$$w_{cd} = \frac{Q''}{cd}(d - c) = Q''\left(\frac{1}{c} - \frac{1}{d}\right).$$

Then

$$W = \Sigma w = Q''\left(\frac{1}{a} - \frac{1}{d}\right).$$

The total work done in moving the unit charge from A to D is the sum of the amounts of work done in moving from point to point. It is seen from the foregoing equations that, regardless of the number of short distances considered, all terms in the brackets cancel except $1/a$ and $1/d$ (a being the distance of the first point and d that of the last). If the distance through which the charge is moved is extended to the end of the field, *i.e.*, until the distance d is infinite in length, then

$$W = Q''\left(\frac{1}{a} - \frac{1}{\infty}\right) = \frac{Q''}{a}.$$

By definition, this equation is a measure of the potential at the point A. The potential at any point which is at a distance d from a point charge then is

$$V'' = \frac{Q''}{d}.$$

This potential is either + or −, depending on the sign of the charge Q''. To represent this + or − potential, the equation is written

$$V'' = \pm\frac{Q''}{d} \text{ statvolts.} \qquad (9)$$

This equation is derived more easily by calculus as follows. The force acting on the conventional unit point-charge at any distance r from the charge Q'' is Q''/r^2. The work expended in moving the unit charge an infinitesimal distance is

$$dw = \frac{Q''}{r^2}dr.$$

The potential at any point at a distance d from the charge Q'' then is

$$V'' \equiv W = \int_d^\infty \frac{Q''}{r^2}dr = \left[-\frac{Q''}{r}\right]_d^\infty = \frac{Q''}{d}.$$

The ± sign then is added to indicate that the potential may be either positive or negative.

8. Potential of a Sphere.—The electricity on a charged isolated sphere is distributed uniformly over the surface, but the intensity of the electric field at any point outside the sphere is the same as though the whole charge were concentrated at the center (Art. 2). The potential at any point outside the sphere, then, is calculated by considering the whole charge as though it were concentrated at the center of the sphere. The surface of the sphere is at the distance R from the center; so at the surface, $d = R$. Therefore the potential of an isolated sphere due to its charge is

$$V'' = \pm\frac{Q''}{R} \text{ statvolts.} \qquad (10)$$

9. Equipotential Surfaces—Potential a Scalar Quantity.—It follows from Eq. (9) that any spherical surface which is concentric with a charged sphere is an equipotential surface. Since the potential is the same at all points, no work against electric forces is required to move an electric charge from one point to

another on such a surface. An indefinite number of such equipotential surfaces, in general not spherical, may be imagined about any charged body. Let A, Fig. 5, be a point on one equipotential surface, and B a point on another of higher potential The work in ergs required to move a statcoulomb of positive electricity from A to B is numerically equal to the potential difference $(V_1'' - V_2'')$ between the surfaces. The same amount of work is done in moving the charge over any path as, for example, over path a or b, because work is being done only while the charge is moved through that component of the displacement which is perpendicular to the surfaces. For the same reason the same amount of work is required to move a unit + charge from any point C on the outer sphere to the point B. If the unit charge is imagined to be moved over the path d, the energy expended against electric forces is that expended in moving the charge over the path a. Additional energy is expended in moving the unit charge from the surface of the inner sphere to the charged body. This energy, however, is returned by the field in urging the charge from the charged body to the other side of the inner sphere. While the charge passes through the charged sphere, no work against electric forces is done because the potential at all points within the sphere is that at the surface. When the outer surface has an infinite radius, the same amount of work likewise is required to move a unit + charge from any point on it through the infinite distance to the given point B. Potential, then, is a scalar quantity because the work required to move a unit charge from an infinite distance to any given point is independent of the direction from which the charge is moved; likewise potential is a scalar because work is a scalar and potential is measured in terms of work.

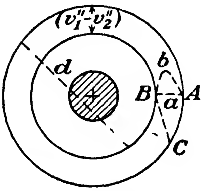


FIG. 5—Equipotential surfaces.

Potential is a scalar quantity. **(11a)**

The potential at the point P, Fig. 6, therefore, is the algebraic sum of the potentials due to each of the three charges, $+Q_1''$, $-Q_2''$, and $+Q_3''$. The individual potentials contributed by the charges are positive or negative depending on the sign of the individual charges. The potential at P due to the three charges

then is

$$V'' = \frac{Q_1''}{a} + \frac{-Q_2''}{b} + \frac{Q_3''}{c}$$

and in general is represented by

$$V'' = \sum \frac{q''}{r}. \tag{11b}$$

This reads, *The potential V'' at any given point is equal to the algebraic sum of such terms as q''/r,* in which q'' and r represent each individual charge and its distance from the point.

If the conventional unit + charge is imagined to be brought from an infinite distance to the point P, external force has to do the work in moving the charge against the repellent action of the + charges; the − charges, however, assist the external force and do part of the work for it. The work done by the external force in moving the unit + charge to the point P, therefore, is less than it would be if the charge $-Q''$ were removed.

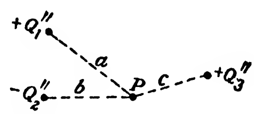


FIG. 6.—The potential at a point P due to several charges.

It also follows that the potential at any point increases with the increase of like charges in the neighborhood.

10. Dependence of the Potential of a Body on Its Dimensions.—An isolated body having large dimensions has a lower potential than a smaller body of the same shape and with the same charge. The potential on a conductor must be the same at all points and is equal to $\Sigma q''/r$. In the larger body the corresponding values of r are larger; therefore each corresponding element q'' of the charge contributes less to the resultant potential.

Questions

1. Define the e.s.u. of quantity of electricity (statcoulomb).

2. How large is the elemental charge in statcoulombs?

3. How many electrons in a statcoulomb of negative electricity? How many protons in a statcoulomb of positive electricity?

4. State the law of attraction and repulsion between electric quantities in a vacuum; in a material medium.

5. Is the attraction between charges in material media larger or smaller than in empty space? How large is this difference in air? In water?

6. Define unit electric field; field of unit intensity; field of unit strength.

7. Give the expression for the force acting on any charge, Q'', in an electric field of F'' units. Does this electric force depend on the state of motion or rest of the charge? Of the field?

8. Give the expression for the strength of the electric field in terms of the distance from a point-charge.

9. Distinguish between electric field, gravitational field, and field of molecular action.

10. Show that the intensity of the electric field outside a charged sphere is the same as though the whole charge were a point-charge at the center of the sphere.

11. Define potential, potential difference, statvolt.

12. From what planes of reference are potentials measured? Do the potentials as measured from one plane of reference have the same magnitudes as when measured from the other plane?

13. Give the expression for the work required to move a given charge through a given potential difference.

14. Derive the expression for the potential at any point in the vicinity of an electric charge and compare it with the expression for the intensity of the electric field.

15. (*a*) Distinguish between + and − potentials. (*b*) When a − charge is being moved from the earth to a point of + potential, does the electric field or an external agent furnish the energy? (*c*) Which furnishes the energy when a + charge is being moved from the earth to a point of − potential?

16. State from four points of view the direction in which a charge is acted on in the space surrounding a neighboring charge.

17. State, with reference to higher or lower potential, the direction in which positive and negative charges are urged by an electric field.

18. Is −20 a lower potential than −10? Is −10 lower than +20?

19. If an electron is being moved in a uniform electric field from a point whose potential is −10 to one whose potential is +20, is the field or an external agent supplying the energy? Is the force continually in the same direction? Does its magnitude vary?

20. Can a potential difference exist between any two points in a space in which there is no conservative electric field?

21. Does the existence of an electric line of force in a conservative electric field involve the existence of a potential difference along its length? Explain the relation a line of force has to potential difference.

22. Give the expression for the intensity of the electric field in terms of potential difference and distance. Define potential gradient.

23. Can the potential at some one point on an electric line of force be zero?

24. Give the expression for the potential of a charged sphere.

25. Define an equipotential surface and show that no work is done against electric forces in moving an electric charge along such a surface.

26. Show that in moving a charge from one equipotential surface to another or from one point on such a surface to a point on the other the amount of work required is independent of the path taken.

27. Show that the work required to move a unit charge from an infinite distance to any point in an electric field is independent of the direction from which the charge is moved. In what other way can it also be shown that potential is a scalar quantity?

28. Give the expression for the potential at a point due to any number of electric charges.

29. Give the expression for the intensity of the electric field outside a charged sphere.

30. What is a "point-charge"?

31. What would be the potential of any charge if it could actually be concentrated at a point, *i.e.*, on a sphere whose dimensions are negligible?

32. Show why the resultant of superposed electric fields is the vector sum of the component fields and the resultant of superposed potentials is the scalar sum of the component potentials.

33. Why does the potential of a conductor depend on the extent of its surface?

Problems

1. Two + charges of 10 and 25 statcoulombs, respectively, are 5 cm apart in a vacuum. What is the force of repulsion between them? What would be the direction of the force if both charges were negative? If one was positive and the other negative?

2. With what force do 2 electrons 1 cm apart repel each other?

3. (*a*) What would be the force between the charges of Prob. 1 if they were in air? (*b*) In pure water?

4. What are the intensity and direction of the electric field at a distance of 10 cm from an isolated + charge of 200 statcoulombs? From a − charge of the same magnitude?

5. A + charge A of 100 statcoulombs is placed 10 cm from a + charge B of 200 statcoulombs. Calculate the magnitude of the force acting on the charge A, using the equation for the force of repulsion between charges and then assuming the charge to be in the electric field produced by the charge B.

6. With what force is a charge of 20 statcoulombs acted on in an electric field whose intensity is 40 e s.u.?

7. Calculate the potential at a point 30 cm distant from a charge of 90 statcoulombs.

8. (*a*) What is the potential energy of a + point-charge of 100 statcoulombs placed 20 cm from a + point-charge of 200 statcoulombs? (*b*) How much kinetic energy is given to either charge while it moves freely to the end of the electric field of the other charge?

9. What is the potential difference between two points 20 and 40 cm distant from a charge of 120 statcoulombs?

10. How much work is expended in moving a + charge of 100 statcoulombs between two points which are respectively 50 and 20 cm from a + charge of 400 statcoulombs?

11. An equilateral triangle whose sides are 100 cm in length has a + charge of 500 statcoulombs at its upper corner and a − charge of 500

statcoulombs at the lower right-hand corner. Find (*a*) the intensity and direction of the electric field and (*b*) the potential at the left-hand corner.

12. Three charges of +10, +30, and −20 statcoulombs are respectively 10, 20, and 5 cm distant from a given point. The lines joining the charges and the point make angles of 30° with each other. What is the potential at the point?

Experiments

1. Coulomb's balance (shown only).

2. Motion of a charge through a p.d. illustrated by moving a mass from one level to another.

CHAPTER IV

DISTRIBUTION OF ELECTRIC CHARGES ON CONDUCTORS

1. Conductors and Insulators.—The materials through which free electrons flow readily are called *conductors;* those through which they do not are called *dielectrics, insulators,* or *nonconductors.* The ability to conduct electricity varies in the different materials. There are good and poor conductors, and good and poor insulators. In order that a conductor may retain its charge, it must be held in place by an insulator to separate it from other conductors or from the earth. The following lists show the important conductors and insulators in approximately the order of their efficacy as conductors and as insulators.

Conductors	Insulators
Metals	Sulphur
Graphite	Amber
Aqueous solutions	Paraffin
	Mica
Semiconductors	Hard rubber
Linen	Shellac
Cotton	Silk
Dry wood	Dry paper
	Glass

In this text it is assumed that conductors contain free electrons (Art. II–4) which can move about freely between the atoms. It follows, then, that when a conductor is subjected to a potential difference, these electrons move from points of lower to points of higher potential until their redistribution equalizes the potentials, and that, if such a conductor connects two charged conducting bodies having unequal potentials, electrons are transferred from one body to the other (Law III–7) through this conductor. If a conductor connects a charged body to the earth, the electrons move in one direction or the other through the conductor until

the potential of the body becomes that of the earth, *i.e.*, zero. A dielectric has so few free electrons that the discharge through it is inappreciable or very slow.

2. Potentials at Different Points in a Cloud of Like Charges.—Let the dots, Fig. 1, represent + electric charges. At the center, A, the potential due to these charges is higher than that at a point B on the outer edge because the average distance to A from the individual charges is less than that to B (Eq. III–11). If the + charges are free to move, they move from the higher to the lower potential, *i.e.*, from A to B or toward the outside.

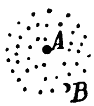


FIG. 1.—Two points, A and B, in a cloud of electric charges.

Similarly, if the dots represent − charges, the negative potential at the center is greater than that at the edge; *i.e.*, the potential at A is lower than that at B. Hence negative charges, which are urged from points of lower to points of higher potential, also move from the center to the outside.

In any cloud of like charges the charged particles, if free to move, flow to the outer surface of the space they occupy.

3. Distribution of a Free Charge on Conductors—Absence of Electric Field within Charged Conductors.—In any uncharged body the protons and electrons are equal in number and when considered on a large scale are uniformly distributed; hence at any point the + potential due to the + charges is neutralized by an equal − potential due to the − charges.

If some electrons are added to an uncharged conductor, it becomes negatively charged. Such charges must appear on the surface (Art. 2). They cannot, however, move outward beyond the surface because the electrons on the surface of even a highly charged conductor are comparatively far apart and therefore an escaping electron, although repelled by the charge on the conductor, is drawn back more strongly by the adjacent induced + charge (Art. 5) from which it is escaping.

If some electrons were removed from an inner portion of a conductor, and if the excess positive charge produced by the removal were to remain within the conductor's surface, it would be equivalent to a cloud of positive charges which, if free to move, would come to the surface. These + charges, however, are immobile; but they raise the potential of the interior of the

conductor above that of the surface. The mobile surface electrons then are forced inward toward this higher potential (Law III–7) until the interior positive charges are neutralized. The departure of the surface electrons leaves a corresponding + charge on the surface.

If an electric field existed within a conductor, the free electrons in it would move along the lines of force, from the lower to the higher potential, until the redistribution of the electric charges neutralized the field. The potential must therefore be the same at all points within a conductor if it is made of the same material throughout (see Art. XIV–1). For the same reason the potential on the surface of such a conductor must also be the same at all points. The charge on a conductor refers normally to the excess charge only and is always small compared with the total of the neutralized charges within the conductor.

From these considerations

A free charge on a homogeneous conductor distributes itself over the outer surface only, and in such a manner that no potential difference exists between any two points within or on the surface. **(1)**

If the inner portion of a charged solid conductor is imagined to be removed, a charged hollow conductor results which has the surface charge distribution of the solid conductor. The potential $\left(V'' = \sum \frac{q''}{r}\right)$ at all points in the space within the hollow conductor, therefore, is shown also to be equal to that at the surface. It is also seen that the inner surface of the charged hollow conductor normally can have no charge, and experiment shows that even a metal beaker, which is a hollow conductor with a large opening, has practically no charge on its inner surface at some distance from the opening. A bound charge, however, may be induced on the inner surface by placing an insulated free charge into the space within the hollow conductor (Fig. 9).

If a charged carrier (proof plane) touches the inside surface of a hollow conductor, the whole charge of the carrier "flows" to the outer surface of that conductor. This is due to the carrier's becoming a part of the inside portion of the conductor whose whole static charge, as already explained, is forced to appear on the outer surface. The charge itself, however, does not in reality

pass to the outer surface. All the free electrons within the conductor, regardless of the sign of the charge which passes, are only displaced slightly in one direction or the other. This displacement causes the charge which appeared on one side to appear on the other without the charge's moving the whole distance. The magnitude of this displacement can be calculated from the estimated number of free electrons per cubic centimeter (8.51×10^{22} in copper) and the known number (2.096×10^{9}) of them in a statcoulomb. To transfer a statcoulomb from a surface of 1 cm^2 to the opposite equal surface requires an electron displacement within the conductor of only 2.46×10^{-14} cm, which is about 0.000001 the distance between the centers of two adjacent atoms. A charge of any magnitude consists of the adding or subtracting

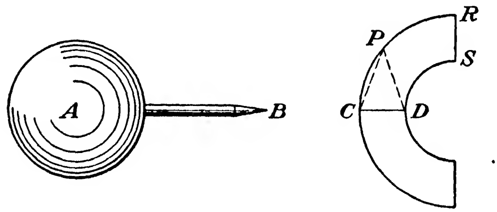


FIG. 2.—(a) A pointed conductor. (b) A side view of a curved strip of metal.

of but a minute fraction of the total number of the free electrons in a conductor.

The distribution of electricity over the surface of a pointed conductor is not uniform: the density is greatest at, and in the neighborhood of, the pointed end. If a + charge, for example, is *assumed* to be uniformly distributed over the surface, the bulk of the charge is on the larger part of the conductor. Any point, A, on or within the spherical part, Fig. 2(a), is nearer the bulk of the charge than the point B on the pointed end. This being the case, the $\Sigma q''/r$ is greater for the point A than for the point B; *i.e.*, the potential at A under such an assumed condition is higher than that at B. Electrons then are displaced from B to A until the potentials are equalized. This redistribution of the electrons leaves more than the average density of the + charge in the neighborhood of B at the pointed end.

The density of electric charge is greatest at the pointed end of a conductor and especially at sharp points. (2)

The density of charge is greater also on the outer surface of a curved strip of metal. Again assume, Fig. 2(*b*), a uniform distribution of a + charge. Inspection shows that, when the point *P* is equally distant from the symmetrically placed points *C* and *D*, every point on the surface above *CD* and to the right of *PD* is nearer *D* than *C*, while only points on the smaller area whose one side is *CP* are nearer *C* than *D*. It follows that with uniform distribution of the charge the inside surface has a higher potential than the outer surface. To establish potential equilibrium, electrons must be displaced from the outer toward the inner surface and thereby increase the density of the + charge on the outer surface at the expense of that on the inner surface.

4. Redistribution of Electricity on a Charged Conductor When Another Charge Is Brought into Its Neighborhood.—When the positively charged spheres *A* and *B*, Fig. 3, are far apart, the charges on them are uniformly distributed over the surfaces. But when *B* is in the neighborhood of *A*, the attraction of the + charge of each sphere for the free electrons of the other causes the electrons to be displaced and the density of the + charges to be increased thereby at the more distant parts of the spheres. The equivalent point-charges (Art. III-2) are displaced thereby to points beyond the centers of the spheres. If the charges on *A* and *B* are unlike, the density on the nearer sides is increased.

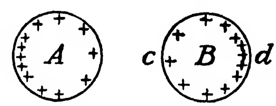


FIG 3.—Redistribution of electricity due to neighboring charges.

From these facts it follows that two spheres with charges of the same sign repel each other with a force less than that with which they would if their charges were concentrated at their centers. If their charges are unlike, they attract with a greater force.

5. Electrostatic Induction—Bound and Free Charges—Electrostatic Screening.—When the neutral conductor *AB*, Fig. 4(*a*), is brought into the neighborhood of a charge $+Q''$, it occupies a space in which there is a potential difference due to the presence of the charge $+Q''$. The potential within the nearer side *A* of the conductor is higher than that within the more distant side *B*.

The free electrons, then, are forced (Law III–7) from the lower toward the higher potential (from B to A) and continue to be displaced until the potentials within the conductor are equalized. The potential of the conductor AB, after the induced charges are established, then is the same at all points and is a + potential which existed at some intermediate point P before the introduction of the conductor into the space. This equal potential is at every point the algebraic sum of the superposed potentials contributed by the inducing charge $+Q''$ and the induced charges $-q''$ and $+q''$.

The equal induced charges of unlike sign on AB also affect the potential of the sphere containing the inducing charge Q''. The nearer charge produces a larger effect than the more distant

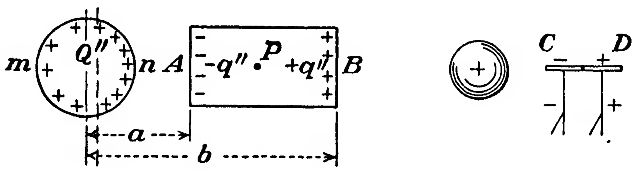


FIG. 4 —Induced charges

one. In the case shown, the potential on the side n is lowered more by the induced charges than on the side m. Electrons then are displaced from n toward m until they equalize the potentials. This redistribution of electrons produces a greater density of the + charge at n than at m.

The induced charges on AB cause a potential difference between the ends of AB, which is equal but reverse in direction to that produced there by the inducing charge. Therefore the resultant potential difference, as already explained, is zero and no electric field exists within the conductor charged oppositely by induction. This condition was shown (Law IV–1) to exist in a conductor with a free charge and may be shown to be the condition in any case when the charge on the conductor is at rest.

Since there is no potential difference in the space within a hollow conductor and practically none within a cage of metallic netting, there can be no inductive action on bodies located therein by charges on or outside the conductor or netting. Such inclosed bodies are said to be *electrically screened.*

If the inducing charge Q'' is removed from the neighborhood of the conductor AB, the potential difference due to it in AB disappears; the potential difference produced by the equal induced charges on AB, no longer being neutralized, causes the displaced electrons to move back from A to B into their original position.

Imagine the charge Q'' again to be in place and the conductor AB again to have both a + potential and the induced charges. If now the conductor is grounded by means of a conducting wire, electrons move from the earth toward the body irrespective of the point at which the wire touches the conductor. When finally the conductor attains zero potential at all points, the + charge on the side B is completely neutralized. If now, after AB is disconnected from the earth, the inducing charge again is removed, the induced negative charge is redistributed over AB and gives it a negative potential. The body then is said to have been charged by induction with the opposite kind of electricity to that of the inducing charge.

Of the two induced charges on AB, the one at A is called the *bound charge* because it is held there by the inducing charge and under no condition can escape. The charge at B is called the *free charge* because it disappears when the conductor is grounded. It is "free to move" elsewhere. It must be remembered, however, that, when the free charge is positive, the charge does not actually flow away, but electrons move to the space occupied by it. This neutralization of the + charge is equivalent to its moving to other parts.

The separation of the induced charges may be shown by making one conductor of two electroscopes, C and D, Fig. 4(*b*). After the charges are induced, the electroscopes are separated and the signs of their induced charges determined.

It should be noted that, when an electric field passes through a conductor, the electric fields of the induced charges neutralize the inducing field within the conductor and strengthen the field adjacent to the charges in the space outside. The resultant field within the conductor is "broken," as represented by its lines of force in Fig. 5(*a*).

The earth E, Fig. 5(*b*), cuts the field of the charge Q'' in a similar manner. The intensity of the resultant field within

the earth is zero, and, on account of the great diameter of the earth, that beyond the point B is negligible. Since the same amount of work is done in moving a unit + charge to any point P in the neighborhood of the charge Q'' by any path from an infinite distance (Art. III–9) such as the paths $XBAP$ or YP, and as no appreciable amount of work is done along the path

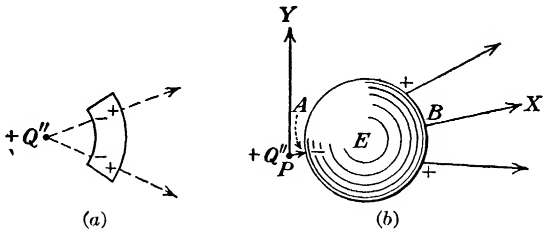


FIG. 5—(a) A conductor in an electric field shown apparently "breaking" the resultant electric lines of force. (b) The earth E "breaks" the electric lines of force in the same manner.

$XBAP$ except in the part AP, the calculated potential has the same value whether the infinite dome or the earth is taken as having zero potential.

It should also be noted that a charge near the surface of the earth is not an isolated charge because of the induced charge of opposite sign on the earth. The potential at any point near such

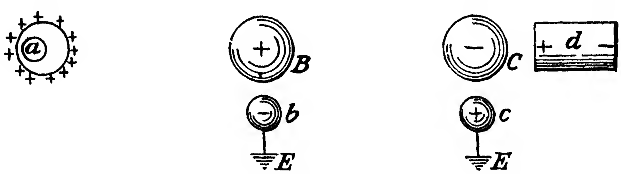


FIG. 6.—The potential and induced charges on a, b, c, d.

a charge, then, is due in part to this induced charge, which reduces the amount of work required to move a unit test charge from space to the point P while it adds to it when the test charge is carried to the same point from the earth.

The sign of the potential of a body usually corresponds to the sign of the free charge on it (Art. III–4) regardless of how the charge is produced. In the case of the bound charge, however, the potential of the conductor, if it is not zero, has a sign opposite that of its charge. A neutral body a within a hollow charged conductor, Fig. 6, has the potential of the conductor and no

charge (Law IV–1). The body b, before being grounded, has a + potential because of the + charge on B; but when the body is grounded, electrons are forced into it and lower its potential to zero. The body then has a − charge and zero potential. Similarly, the body c has a + charge and zero potential. The body d has a − potential and both + and − charges.

6. Electroscope.—In its simplest form the electroscope, Fig. 7(a), consists of two gold or aluminum leaves attached to an insulated conductor. An electric charge may be collected from a charged piece of sealing wax by means of a proof plane, which consists of a piece of metal on an insulating handle. This charge can be transferred to the electroscope by touching its plate with the plane. The leaves, becoming charged with the same kind of electricity, repel each other. The amount of their divergence

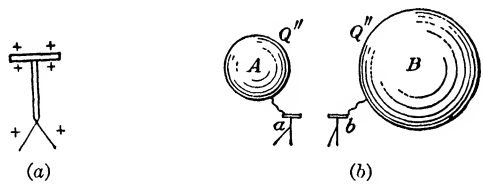


FIG 7 —(a) Electroscope. (b) Electroscope measures potentials.

indicates the magnitude of the charge on the electroscope and therefore its potential. The electroscope may also be charged by induction in the same manner as the conductor in Art. 5.

If an uncharged electroscope is connected to any charged body, it takes from that body such a quantity of electricity as is required to equalize the potentials. The quantity of electricity taken depends on the potential of the charged body and not on the quantity of electricity that it may contain.

The sphere B, Fig. 7(b), has twice the radius of the sphere A; therefore, if the two have equal charges, the potential of B is only one-half that of A (Art. III–8). If the spheres are connected to similar electroscopes which are imagined to be small so as not to change appreciably the original potentials of the spheres, the potential of the electroscope b is only one-half that of the electroscope a and is charged with only one-half as much electricity. Because of this smaller charge the divergence of the leaves in electroscope b is less than that of those in electroscope a.

The quantities of electricity on the two systems are the same, but the electroscopes take different charges which produce different amounts of divergence. The divergence of the leaves then depends on the potential of the spheres and not on the magnitude of the charges.

An electroscope measures the potential and not the quantity of electricity on the system of which the electroscope is a part. (3)

The potential of the electroscope, as well as that of any conductor, is affected by the presence of electric charges in the neighborhood. The electroscope, therefore, is usually inclosed in a grounded metal case with openings only for the purpose of taking observations. Inside such a metal case the electroscope is screened (Art. 5) and therefore is not affected by charges in the neighborhood outside the case.

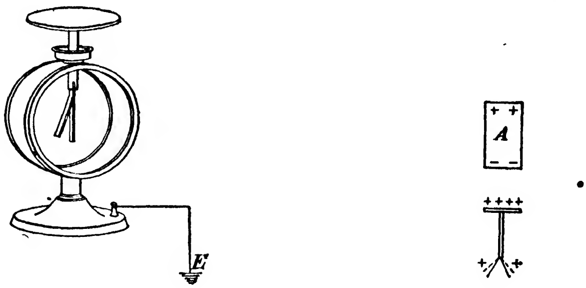
If the metal case surrounding the leaves of the electroscope is disconnected from the earth and then charged together with the leaves to some same potential, $+V''$, the leaves show no divergence even though their potential is $+V''$. If now the potential of the electroscope (the leaves) is changed, the divergence of the leaves indicates the potential difference between the electroscope and the metal case. The electroscope then is said to indicate the potential difference between itself and the metal case and can be depended on to measure the potential of a system only when the enclosing case is grounded.

For qualitative work the electroscope has usually only a grounded metal cylinder surrounding the leaves, Fig. 8(a), or simply two grounded pieces of wire gauze, one on either side of the leaves. The "plate" of the electroscope is not protected.

In order to test the kind of electricity on a charged electroscope, a proof plane carrying a charge of known sign is brought into the neighborhood of the plate. If a part of the charge on the electroscope is "repelled" into the leaves and thereby increases their divergence, the charges on the electroscope and proof plane have like signs. Likewise if an electroscope is charged with a known kind of electricity, divergence of the leaves indicates that the charge brought near it is one having the same sign.

Convergence of the leaves is not a sure test for charges of unlike signs because it can be produced by a neutral body. When the

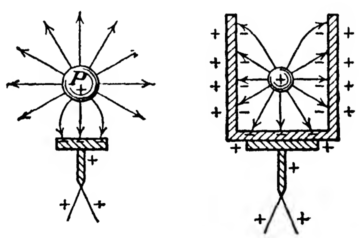
neutral body A, Fig. 8(b), is brought into the neighborhood of the charged electroscope, it becomes charged by induction (because of the charge on the electroscope). The induced $-$ charge, in the illustrative case, is nearer the electroscope than the induced $+$ charge, and therefore the two charges together pro-



(a) (b)

FIG. 8.—Electroscope, showing method of screening in protecting case (b) Neighboring "uncharged" conductor affecting the potential of an electroscope.

duce the same convergence of the leaves as would a single $-$ charge brought into the neighborhood. If the conductor A is grounded, the induced free $+$ charge is neutralized; hence a still greater convergence of the leaves takes place. This effect is considered again under the subject of capacitance (Art. XII-3).



(a) (b)

FIG. 9.—Magnitude of induced charges.

7. Equality of Induced and Inducing Charges.—The resultant lines of force starting at the $+$ charge on the proof plane P, Fig. 9(a), end in $-$ charges, of which some are on the plate of the electroscope and others on the walls of the room. When the electroscope is grounded, only the bound $-$ charge remains on its plate. That the magnitude of this induced charge is less than

that of the inducing charge on the proof plane can be shown by touching the proof plane to the electroscope. Some of the excess + charge "passes" to the electroscope and causes the leaves to diverge.

Figure 9(*b*) shows a metal beaker on the plate of an electroscope. If the charged proof plane is brought into the beaker without touching it, practically all the lines of force end on the walls of the beaker, and practically none on the walls of the room. When the beaker and the electroscope are now momentarily grounded, the + free charge is neutralized by electrons from the earth, and the leaves collapse. That the bound charge is present may be shown by withdrawing the proof plane from the beaker: the leaves diverge owing to the negative charge. When the proof plane is placed back into the beaker, the leaves again collapse because the − charge again becomes bound and all the lines of force from the charge on the proof plane end on the walls of the beaker. If the beaker is now touched with the proof plane, the leaves remain collapsed, showing that the + and − charges are equal. This experiment is called *Faraday's ice-pail experiment* because it was first performed by Michael Faraday, who employed an ice pail in place of the beaker. It proves that

For every free charge there is induced an equal opposite bound charge on neighboring conductors or on the earth. (4)

The object of the beaker or pail is to cause all the resultant lines of force to end on the vessel connected to the electroscope so that all the induced charge is on the conductor of which the electroscope is a part.

A resultant line of force, then, begins at a + charge and ends at an equal opposite charge. This refers to the observed resultant lines of the superposed fields and not to those of the elemental fields, which always extend radially to an infinite distance.

8. Equality of the Two Kinds of Charges Produced by Friction.—If a piece of fur is wound around one end of an insulating rod and is then rubbed with a hard-rubber rod, the fur is found to be charged positively and the hard-rubber rod negatively. The equality of these charges is found by placing them both, at the same time, in a beaker attached to an electroscope. The two together produce no divergence of the leaves; hence two equal

The equality of the charges is explained by attributing them to electron transfer from one body to the other. This transfer leaves an equal + charge on the body from which the electrons are taken. Thus

The production of a charge necessitates the production of an equal opposite charge. (5)

9. Why Electrons Are Believed to Be the Moving Charges in a Conductor.—Why electrons are assumed to be the moving charges in any electric transfer is shown from the following:

1. The atom of a solid is held in place by molecular forces, and all its protons are concentrated in the nucleus. The light orbital electrons far removed from the heart of the atom are the charges most easily separated from the atom.

2. Electrons flow out of hot metals in a vacuum (Art. XXV–5).

3. In a circuit containing a Coolidge x-ray tube, for example, the electrons pour out of one electrode and, after spanning the space between the electrodes as a current, enter the other electrode (Art. XXVII–2).

4. When a brake is applied to check suddenly the rotation of a closed loop of wire, the inertia of the mobile charges is expected to cause them to continue to move in the direction of the retarded rotation. The observed direction of the electric current produced by such a retardation shows that the negative charges are the mobile charges.

Questions

1. What are conductors? Insulators? What causes the difference in their electric properties?

2. Explain why the potential is higher at the center of a cloud of positive charges than at the outer edge Why is it lower at the center of a cloud of negative charges? Show that in both cases the inner charges are urged outward

3. Explain why the whole electric charge is on the surface of a conductor and why, when the conductor is charged negatively, the excess electrons do not escape from the conductor

4. Explain why there can be no electric field and no potential differences within any charged conductor or in the space within a charged hollow conductor.

5. Show that a bound charge exists on the inner surface of a charged hollow sphere when an insulated charge is placed within the sphere.

6. Explain why the density of the charge is greatest at the pointed end of a conductor. What would be the density on the wall of a cavity near the pointed end?

7. Explain why the density of charge is greater on the outer than on the inner surface of a curved conductor.

8. Explain how electricity redistributes itself on two charged spheres when brought near each other. How does this affect the attraction or repulsion?

9. Explain, using the concept of potential, why a neutral conductor brought into the neighborhood of an electric charge acquires an induced + charge on one side and an induced − charge on the other. What happens when the conductor is grounded? Will the leaves of the electroscope to which this conductor may be connected then diverge? Why will they diverge when the inducing charge is removed?

10. What is a "bound charge"? A "free charge"?

11. Show that the bound charge on a body does not, as a rule, correspond in sign to the potential of the body.

12. Show that a conductor brought into an electric field "breaks" the lines of the resultant field.

13. Prove that the potential at any point has the same magnitude regardless of whether it is measured from the earth or from the infinite dome.

14. How can an electroscope be charged with + electricity? With − electricity? Give two methods.

15. How can the kind of electricity on a body be tested by its effect on a charged electroscope? Why is the convergence of the leaves not necessarily a test?

16. Explain why the divergence of the leaves of an electroscope measures the potential and not the quantity of electricity on the system to which the electroscope is connected.

17. Explain why the leaves of a charged electroscope converge when a large uncharged body is brought into the neighborhood, and why they converge still more when the body is grounded.

18. Explain why the electroscope is inclosed in a grounded metal case when used for quantitative measurements. Why does the electroscope measure the potential difference between itself and the enclosing case?

19. What is the quantitative relationship between the induced and the inducing charges? Explain Faraday's ice-pail experiment.

20. Explain why, when a positive charge is produced by friction (or contact), there is always produced an equal negative charge.

21. What is the potential of an insulated uncharged man within a highly − charged hollow sphere? What would happen if he touched the sphere? If he touched a grounded wire which projected into the sphere through an opening?

Experiments

1. Various kinds of electroscopes shown. Charging the electroscope by transfer of electricity and by induction.

2. Testing of kind of electricity on a body by means of an electroscope.

3. Conductors and insulators.

4. Electroscope measures potential and not the quantity of electricity on a conductor. Chinese pulley used to change the surface and therefore the potential of a roll of tin foil without changing the charge.

5. When the shielding case is insulated and charged, the leaves of the electroscope diverge although their potential is zero. The electroscope measures the potential difference between the leaves and the outer casing.

6. Influence of an insulated and of a grounded conductor on the divergence of the leaves.

7. No charge can be induced on an electroscope inclosed in a metal vessel or in a coarse metal screen (screening effect).

8. Distribution of charge on and within hollow conductors tested by means of proof plane and electroscope; also on pointed and curved surfaces.

9. Two kinds of induced charges on a conductor and on two connected electroscopes.

10. Faraday's ice-pail experiment.

11. Equality of the two kinds of charges produced by friction.

CHAPTER V

MAGNETIC FORCES (BASIC PHENOMENON 2)

1. Magnetic Forces—Magnetic Field.—The free electrons in the conductor of Fig. II–1, because of their chaotic motions, have at any instant velocities in all possible directions. On account of their large number, the average velocity of those moving in any one direction is equal to the average velocity of the equal number of them moving in the reverse direction. The free electrons as a group, therefore, have no velocity in any one direction and therefore as a group may be designated as electrons at rest. These electrons as a group may be given an additional, orderly velocity along the conductor, in which case they are said to *flow* or to *drift* along the conductor. Although this electron group, as such, appears to, and therefore is said to, move or to accelerate as a unit, each individual electron can progress only by jerks. The impacts with obstructing atoms produce intermittent and sudden retardations of the motion. These individual speed variations as well as the chaotic motions which the electrons still possess may be, and generally are, disregarded whenever the electron flow or drift is under consideration. This *electron flow* is also called an *electric current* (Art. VIII–1).

In a conductor containing no electron flow the equal oppositely directed electron and proton fields (Art. I–3) are at rest, as represented in Fig. 1. In these superposed fields neither the electric charge A, which is at rest, nor the charges B, C, and D, which have the indicated velocities, feel any effective force. The superposed electric fields completely neutralize each other's action on any electric charge within them, regardless of whether this charge is in motion or at rest. If, however, the free electrons are caused to flow along the conductor so that the part of the electron field contributed by them is in motion, as represented by the uppermost arrows in Fig. 2, the stationary electron A again feels no effective force, while the moving electrons B, C, and D,

which have the average velocities v, feel an effective force f urging them in the indicated directions. A proton in the same field moving with the same velocity as an electron feels an equal but oppositely directed force.

The electric force which the moving electron field exerts on charges as such is *not changed by the field's motion* and continues to oppose the equal electric force exerted on the charges by the stationary proton field; but charges *in motion* within the superposed fields now are urged, in a direction depending on the direction of their own motion as well as that of the electron field,

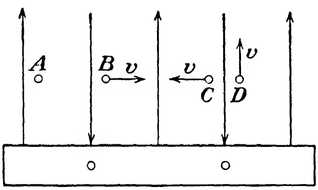


FIG 1—No effective force acts on electrons in motion or at rest in the superposed electric fields about a conductor when both the fields are at rest.

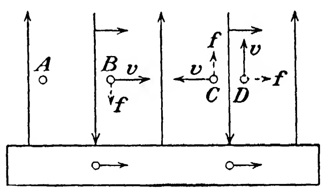


FIG 2—The forces, f, acting on moving electrons because of their motion in a space where one of two equal oppositely directed electric fields is in motion.

by a force which did not exist when the electron field was at rest. Hence it is seen, or rather *assumed*, that to a stationary observer

Motion endows an electric field with a property which gives it power to exert magnetic forces on electric charges because of their motion. **(1)**

These forces are called *magnetic forces* to distinguish them from the electric forces; a space which contains the moving electric field that evokes the magnetic forces is called a *magnetic field.*

The magnetic field may exist (1) in a space where the moving electric field has its action on electric charges, as such, neutralized by an equal electric field of opposite sign, as is the case in conductors carrying a current, or (2) in a space in which the action of the electric field on charges, as such, is not neutralized by an oppositely directed electric field, as in the case of a stream of electrons ejected from an electron gun (Art. XXV-9). In the first case the magnetic field alone is effective, and, since reference to the electric field which produces the magnetic field is ordinarily not necessary, the space is said to contain only a magnetic field.

CHAPTER V

MAGNETIC FORCES (BASIC PHENOMENON 2)

1. Magnetic Forces—Magnetic Field.—The free electrons in the conductor of Fig. II–1, because of their chaotic motions, have at any instant velocities in all possible directions. On account of their large number, the average velocity of those moving in any one direction is equal to the average velocity of the equal number of them moving in the reverse direction. The free electrons as a group, therefore, have no velocity in any one direction and therefore as a group may be designated as electrons at rest. These electrons as a group may be given an additional, orderly velocity along the conductor, in which case they are said to *flow* or to *drift* along the conductor. Although this electron group, as such, appears to, and therefore is said to, move or to accelerate as a unit, each individual electron can progress only by jerks. The impacts with obstructing atoms produce intermittent and sudden retardations of the motion. These individual speed variations as well as the chaotic motions which the electrons still possess may be, and generally are, disregarded whenever the electron flow or drift is under consideration. This *electron flow* is also called an *electric current* (Art. VIII–1).

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energized suspended rectangle is shown reacting with the energized wire A, which is part of a separate circuit. The electrons enter and leave the rectangle through the mercury cups. The force f_2 $(= f_1)$ that is evoked by the electron motions, although small, causes the rectangle to turn because of the large leverage.

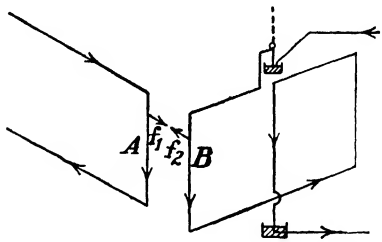


FIG. 4.—Apparatus for demonstrating the forces evoked between wires carrying electric currents.

At present the evoking of the magnetic forces cannot be explained satisfactorily in terms of B.P.1. For this reason this text assumes the evoking of these forces to be a basic phenomenon and refers to it as *basic phenomenon* 2 (B.P.2).

Basic phenomenon 2 may be stated as follows:

Electron flows in two parallel conductors evoke forces of attraction between the conductors when both flows have the same direction, and forces of repulsion when the flows have reverse directions; in conductors at right angles to each other, the electron flows evoke torques which tend to turn the conductors until they are parallel and their electron flows have the same direction. (B.P.2) (3)

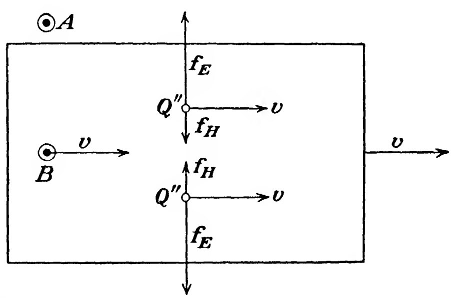


FIG. 5.—Magnetic forces observed by observer A and not by observer B.

In the preceding considerations, and always in this text, the observed facts are what appear to be facts to a stationary observer, *i.e.*, to an observer who measures all motions relative to himself. It is not the velocity of the charges with respect to the earth or with respect to any other heavenly body that are

taken into consideration, but only their velocity relative to the observer. The notion of velocity is relative, and such a thing as absolute rest does not exist.

Imagine the stationary observer A, Fig. 5, observing two like electric charges, Q'', located on a passing train moving with the velocity v. The charges relative to this observer also have the velocity v and consequently appear to attract each other (B.P.2) with the magnetic forces f_H. The charges also repel (B.P.1) with the forces f_E, because they are like electric charges. The observer B, who is on the train and who therefore is moving with the velocity v relative to the observer A, is at rest relative to the train and the two charges. He observes, in common with A, the same electric forces, f_E, repelling the charges; but, because he and the charges are relatively at rest, the charges appear stationary, and therefore to him the magnetic forces of attraction observed by A do not exist. This consideration illustrates the relative character of magnetic forces and also that of the notion of force itself. It is the subject of Einstein's theory of relativity and is referred to again in Art. (XIII-5).

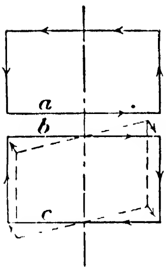


FIG. 6.—Loops which are free to turn take the positions indicated because of the magnetic forces.

All large-scale electric phenomena, including that of magnetism, are manifestations of one or more of three basic phenomena and, if the conservation of energy is assumed, may be explained wholly or in part in terms of them. It must be understood that many phenomena are not yet fully explained in detail, and that certain additional postulates regarding the conditions existing within and about the atoms must be made in order to attempt to account in detail for their behavior.

2. Currents in Loops.—Under ordinary conditions the electrons in a metallic conductor flow in a complete circuit. The adjacent wires a and b, Fig. 6, then are parts of loops. If the planes of the loops are at an angle with each other, the evoked magnetic torques (B.P.2) turn the loops until the nearer sides, a and b, are parallel and have the electrons in them flowing in the same direction (as represented in the figure). The smaller opposing torques caused by the more distant sides of the loops,

as c acting on a, affect the magnitude but not the direction of the resultant torque. Loops of any form turn into similar positions when they are energized by electron flows

3. Magnetic Loop (Elemental Magnet, Amperian or Electron Whirl)—Magnetic Shell.—An isolated loop of wire suspended near the surface of the earth so as to turn freely will turn when a current is flowing through it into a position in which the electrons in the lower side flow approximately from west to east. The earth's rotation from west to east produces (Art. XXX–1) what is equivalent to an electron flow from west to east within the earth as represented by W-E in Fig. 7(a). The loop then turns (B.P 2) until the electrons in its lower side flow from west to east. In such a position, one side of the plane of the loop faces toward the magnetic north and the other toward the magnetic south. Figure 7(b) represents the loop in section as viewed from the west. The cross × (feathered end of an arrow) represents electrons flowing away from the observer, and the dot (the pointed end of an arrow), in the upper cross section of the wire, electrons flowing toward the observer. The face pointing north is called the *north-seeking*, the *north*, or the + *magnetic pole;* the other face is called the *south-seeking*, the *south*, or the − *magnetic pole*. The loop stream of electrons is a *magnetic loop*, an *elemental magnet*, or an *Amperian* or *electron whirl* and is equivalent to a *magnetic shell*, *i e*, to a thin cross-sectional slice of a magnet. Inspection shows that

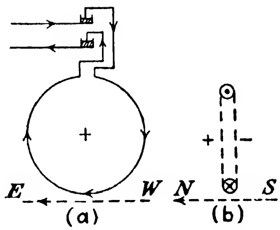


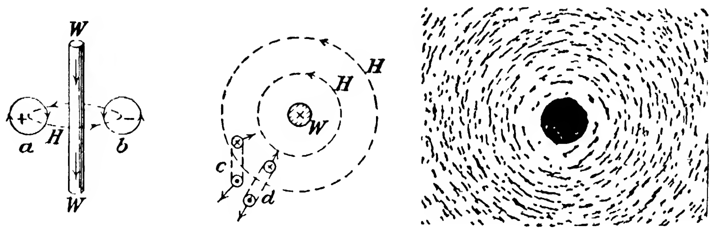
FIG. 7 -Two views of a magnetic loop showing the relation between the two magnetic poles and the direction of electron flow The line W-E represents the direction west to east

The north pole of a magnetic loop (elemental magnet) is the pole which when faced by the observer appears to be due to a clockwise electron motion in the energized loop. (4)

4. Magnetic Field and Its Magnetic Lines of Force about an Electron Flow.—The magnetic loops, a and b, on either side of the energized wire W, Fig. 8(a), because of the magnetic field about the wire, turn (B.P.2) until the plane of each loop includes the wire, as shown. In each case the electrons in the side of the loop adjacent to the wire are moving in the direction of those

in the wire, hence the + magnetic pole of the loop *a* and the − pole of the loop *b* face the observer.

If the magnetic loop *a* be moved around the wire in a manner such that its plane revolves about the wire as an axis, the center of the loop describes a circular line, *H*, about the wire. This line is called a *magnetic line of force*. Although such lines show only the direction of the perpendicular to the plane of a magnetic loop in its position of equilibrium, they are spoken of as though they were real entities and represent the magnetic field and its direction. It follows from the foregoing that



(*a*) (*b*) (*c*)

FIG. 8. (*a*) Magnetic loops *a* and *b* as determinants of the direction of the magnetic line *H* about an electron flow. (*b*) Direction of magnetic lines of force about an electron flow when the observer is looking in the direction of the electron motion. The test loop *c*, drawn in cross-section, shows the direction of the torque acting on an energized loop in a magnetic field and the loop *d*, the position of equilibrium of such a loop. (*c*) Photograph of the magnetic lines of force as marked roughly by iron filings on a smooth surface.

A magnetic field exerts forces on conductors carrying electron flows and thereby torques on magnetic loops until the + poles of the loops face the conventional direction of the magnetic lines of force. **(5)**

By convention the direction of a magnetic line of force at any point is the direction in which the + pole of a magnetic loop is forced to face at that point. This direction, in referring to the flow in the wire *W*, is such that

When one looks in the direction of an electron flow, the direction of the magnetic lines of force about the flow is counterclockwise. **(Law A_1) (6)**

This fact, referred to as Law A_1, is represented in Fig. 8(*b*). A magnetic loop *c* placed in the magnetic field surrounding the wire, because of the magnetic forces (B P.2), turns until, as shown at *d*, its plane is perpendicular to, and its + magnetic pole is

facing in, the conventional direction of the lines of force. Two such adjacent magnetic loops (elemental magnets) in their final positions of equilibrium have their electrons flowing in the same direction around the loops and therefore attract each other (B.P.2). It will be shown (Art. VI–6) that iron filings are composed of such elemental magnets and therefore when placed on a smooth surface about the electron flow adhere to each other (Art. VI–9) and roughly mark the directions of the magnetic lines of force as shown in Fig. 8(*c*).

Figure 9 represents the superposed electron and proton fields about a conductor carrying an electron flow (electric current). In Fig. 9(*a*) the observer is looking in the direction of electron flow; the crosses represent the direction of motion of the moving

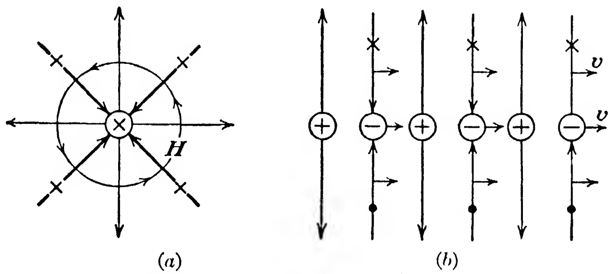


FIG. 9.—The two superposed electric fields with the accompanying magnetic field when the free electrons in a conductor are in motion (shown in one plane only), when viewed (*a*) in the direction of electron flow and (*b*) at right angles to that direction.

part of the electron field and of the free electrons, while the arrowed circle *H* represents the direction of the magnetic field. In Fig. 9(*b*) the observer is looking in a direction at right angles to that of electron flow. The arrows here represent the direction of motion of the moving part of the electron field and of the free electrons, while the crosses and dots represent the direction of the cylindrical magnetic field surrounding the conductor. In each case the fields are shown in one plane only.

The comparatively insignificant electric field or potential gradient (Arts. III–6, IX–2) which causes the electron flow through the conductor is not involved in the foregoing considerations.

The direction of the magnetic lines of force (and thereby that of the magnetic field) about moving positive charges is

the reverse of the direction of the lines about moving negative charges.

The term *magnetic field,* as already stated, is used to designate a property of an electric field in motion by virtue of which property the electric field evokes magnetic forces. But it is convenient to speak of the magnetic field as though it were a separate entity, which, however, is always associated with and inseparable from a moving electric field. The magnetic lines of force which represent the magnetic field are referred to as *magnetic flux.*

Henry A. Rowland (1848–1901), professor of physics, Johns Hopkins University, Baltimore, Md., proved conclusively (1876) by the rapid rotation of a condenser plate that static electricity in motion is surrounded by a magnetic field and therefore that static electricity and current electricity are identical. Before his positive proof, the identity of the two was only inferred from the fact that a discharge from a condenser or from the electrodes of an electrostatic machine decomposes water, deflects a galvanometer coil, and magnetizes a piece of steel inserted in a solenoid.

It must be remembered that both electric and magnetic fields are only general abstract concepts in terms of which electric and magnetic phenomena are conveniently explained. Although we must endow these fields with certain properties possessed by physical entities, we know nothing concerning their physical nature.

5. Direction of the Force Acting on an Energized Wire and of Torques on Energized Loops in a Magnetic Field—Electro-

magnetic Reaction.—It is found convenient, in place of thinking of two wires energized with electric currents as attracting or repelling each other, to take another point of view that often greatly simplifies the determination of the direction of the forces acting on conductors.

The flow in the wire A (shown in cross section in Fig. 10) has its magnetic field represented by the full-line arrowed circles. If the direction of the flow in another wire, B, is the same as that in A, its magnetic field, which is superposed on that of A,

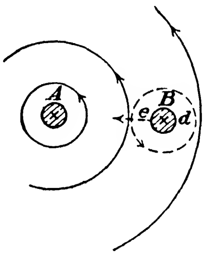


FIG. 10.—Law A_2 and B P 2 shown to express the same fact.

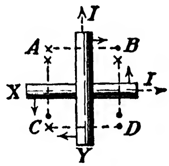


FIG. 11.—Torques between two currents at right angles to each other shown to be included under Law A_2.

necessarily weakens that field on one side, e, and strengthens it on the other. Under these conditions the wire B is urged toward A (B.P.2), but the direction of this force is also from the strengthened toward the weakened part of the field. If the direction of the flow in B and with it the accompanying field is reversed, the field due to A is strengthened at e, and the wire B is repelled from A. From these considerations it follows that

An electric charge moving at right angles to the lines of force in a magnetic field is urged from the strengthened toward the weakened part of the field. (Law A_2) (7)

This statement is observed to apply to moving charges (electric currents) in any magnetic field and holds because all magnetic fields are produced in the same manner: by charges in motion. It should be noted that Law A_2 just expresses in a convenient form the observed facts of B.P.2.

The force acting on an energized conductor in a magnetic field is usually referred to as being due to *electromagnetic reaction. The directions of the force, the electron flow, and the magnetic field are always mutually perpendicular.*

Law A_2, derived here only for parallel currents, applies equally well to currents at right angles to each other. Let X and Y, Fig. 11, represent two wires with the electrons flowing in the directions of the broken-line arrows I. The crosses and dots, joined by broken lines, indicate the directions in which the magnetic lines produced by the two currents enter and leave the plane of the paper. It is observed that the magnetic fields strengthen each other in the quadrants A and D and weaken each other in the quadrants B and C. Forces act on the two wires, by Law A_2, as indicated by the full-line arrows. These forces produce the

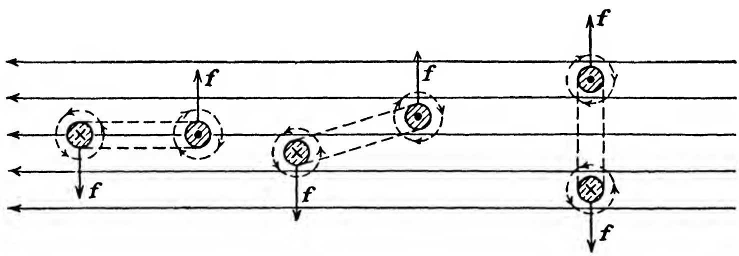


FIG. 12.—Existence and direction of torque acting on a magnetic loop in a uniform magnetic field.

torques which turn the wires into the same plane and so that their electrons flow in the same direction.

The existence of a torque which acts on an energized loop in a magnetic field was explained [Art. 4 and Fig. 8(*b*)] in terms of B.P.2. It can now also be ascribed to electromagnetic reaction (Law A_2). The forces acting on the loop as ascribed to this law are shown for three positions of the loop in Fig. 12. The loop is forced to turn until its + pole faces in the direction of the magnetic lines.

It is possible to assume as basic the observed fact that moving electrons produce a magnetic field and that other electrons moving in this field are urged from the strengthened toward the weakened part of that field. From this point of view Law A of this text would be a basic phenomenon, and the B.P.2 would be a law derived from it. It is of no material consequence which point of view is taken. The author chooses the point of view of this text because all magnetic forces are resolvable into interactions between moving electric charges, and the concept of the magnetic field is developed from the more elemental observed facts. After the concept of the magnetic field is developed, magnetic phenomena are almost invariably referred to it and not to B.P.2.

6. Resultant of Two Superposed Magnetic Fields.—The wire *W*, shown in cross section in Fig. 13, is at right angles to the lines of force of the magnetic field *H* and has its free electrons moving

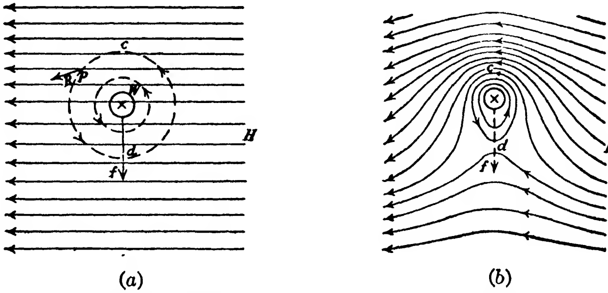


FIG. 13.—(*a*) A circular magnetic field about an electron flow superposed on a uniform magnetic field. (*b*) The resultant of the two superposed fields.

toward the paper. In Fig. 13(*a*) the circular magnetic field due to this electron motion is shown superposed on the field *H*. It strengthens the field *H* on the side *c* of the wire and weakens it on the side *d*. The moving electrons and the wire containing

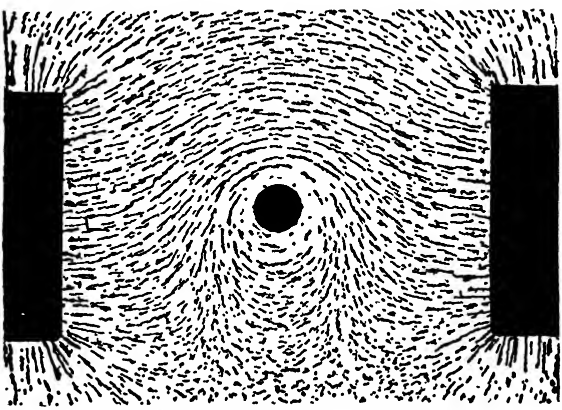


FIG. 14.—Photograph of the resultant magnetic field when a circular field about a current is superposed on the field between two magnetic poles of opposite sign. Lines of force shown roughly by iron filings on a smooth surface.

them are then urged in the direction of the force *f* in conformity with Law A_2. The *observed* magnetic lines of force, however, cannot cross as shown because at the crossing the + pole of a

magnetic loop would have to face in two directions at the same time. The perpendicular to the magnetic loop must take the direction of the resultant of the superposed fields. The arrowed line R therefore represents the direction of the observed or resultant field at one point P.

The whole resultant field, which is called simply "the magnetic field," is represented in Figs. 13(b) and 14. The lines crowd on the side c at the expense of those on the side d.

The resultant of any two or more magnetic fields may be determined in a similar manner because each field produces the same effect as though it were acting alone. For purposes of analysis the superposed component fields may be treated individually.

A magnetic field exerts no force on a charge at rest, *i.e.*, on a charge that has no magnetic field of its own. Since two magnetic fields are involved wherever magnetic forces act, the observed force acting on any wire is ascribed to the *reaction between the fields* or to *electromagnetic reaction*, as already stated in Art. 5.

7. Electric and Magnetic Forces Acting between Isolated Charges in Motion.—If two isolated charges (or two streams of such charges) are moving parallel to each other, the electric forces between them are not neutralized by oppositely directed electric fields which, it has been shown, exist in connection with similar electron streams within conductors (Art. 1). Each of such isolated moving charges, therefore, is acted on by the electric as well as the magnetic field of the other charge. The force acting on each of such like charges moving in the same direction then is the resultant of two forces: one of repulsion (electric force due to the charges themselves) and the other of attraction (magnetic force due to the motions of the charges). The magnetic force between two such charges is always less than the electric force and increases as the square of their common velocity. *The magnitude of the electric force is practically unaltered by motion except at velocities approaching that of light.*

Two streams of unlike charges moving in the same direction are repelled by the magnetic forces which their motion evokes and are attracted by the electric forces.

In (a), (b), and (c) of Fig. 15 are shown the directions of the two kinds of forces (f_E = electric, f_H = magnetic) acting between isolated electric charges moving in the same direction with a

velocity v. If the charges or streams of such charges were moving in opposite directions, the direction of the magnetic forces alone would be reversed.

The magnetic forces acting between charged particles moving with ordinary speed are so small that they are masked by the much greater electric forces. It is therefore difficult to show

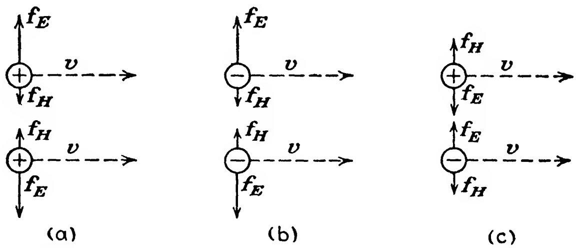


FIG. 15.—Direction of magnetic forces f_H and of electric forces f_E, between charges moving with the velocities v

this effect directly by observing charged bodies in motion. The effect of the magnetic forces, however, can easily be observed in wires carrying electric currents, as already described (Art. 1).

Two isolated electrons moving together in a magnetic field are repelled from each other by electric forces and are urged by magnetic forces in a direction at right angles to the magnetic field, as shown in Fig. 16, and feel a force of attraction (B.P.2).

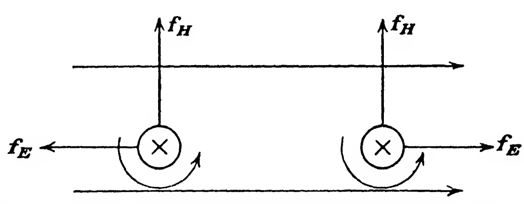


FIG. 16.—Forces acting on each of two isolated electrons moving together in a magnetic field.

8. Directional Relationship of Magnetic to Electric Fields.—An electron is surrounded by an electric field (Art. I–3) whose lines of force converge as represented in one plane by the lines F, Fig. 17(a). When the electron is moving from the observer, the direction of the magnetic lines of force about it, Law A_1, is counterclockwise (shown by the circular lines H). The electric and magnetic fields then co-exist in the same space and are at

right angles to each other. One acts on electric charges as such, and the other acts on them only by virtue of their motion.

Figure 17(*b*) shows an enlarged view of a small section *S* of Fig. 17(*a*). The facts represented by the figure may be expressed as follows:

A moving electric field and its associated magnetic field always exist together and are so related that, if they are moving from the observer with the electric lines pointing upward, the magnetic lines point from left to right. (Law B) (8)

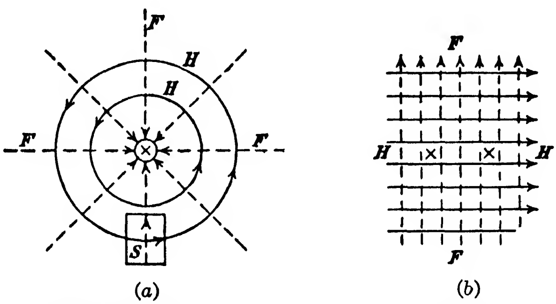


FIG. 17.—(*a*) Electric and magnetic lines of force about a moving electron, shown in one plane only. (*b*) Directions of the associated electric and magnetic fields when the fields are moving from the observer.

If a proton is moving, the directions of both the electric and the magnetic lines of force about it are the reverse of those about the electron, but the foregoing statement still applies.

9. Effect of a Moving Electric Field and Its Associated Magnetic Field on Electric Charges through Which They Pass.—The electric field *E* and the associated magnetic field *H*, Fig. 18(*a*), are represented as moving from the observer toward the paper and passing through the two kinds of charges. The electric field gives to these charges the velocities *C* in reverse directions (Art. I–3). The magnetic fields about these oppositely moving unlike charges have the same direction, and in each case they strengthen the cutting magnetic field on the cutting side and weaken it on the opposite side. Both charges, therefore, are moved by the electromagnetic reaction (Law A) in the direction of motion of the cutting fields.

The actual motion of the charges which are cut is the resultant of the two velocities given each charge by the two kinds of fields.

This is represented in Fig. 18(*b*), in which the electric field is shown to be moving toward the right and the accompanying magnetic field pointing toward the observer. The lines *C* represent the velocities given the charges by the electric field in some given time; and the lines *D*, the velocities given them in the same time by the magnetic field by virtue of the motions acquired by the charges through the action of the electric field. The charges are thereby given the resultant velocities *R*.

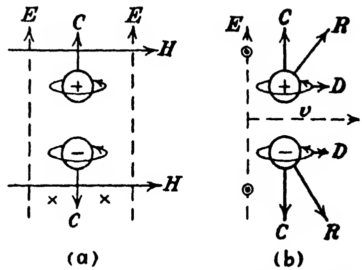


FIG 18.—(*a*) Electric field *E*, with its associated magnetic field *H*, moving toward the paper through opposite kinds of charges. (*b*) The fields moving toward the right through the same charges.

When such moving fields pass through a conductor, the free electrons only are given an appreciable velocity. The force acting on these electrons, owing to their motion in a magnetic field, is transmitted to the mass of the conductor which then is urged in the direction of the moving fields.

10. The Basic Phenomena and Two Basic General Laws Restated.—The basic phenomena 1 and 2 and Laws A and B are referred to often and are here restated, for reference, in a somewhat modified form. Laws A_1 and A_2 are placed together as Law A. It must be understood that all three laws are derived from and are only convenient expressions of one or both of two basic phenomena. The basic phenomena alone cannot explain all the observed facts, but all the observed large-scale facts, as far as they are completely established, conform to these phenomena and to the laws derived from them.

B.P.1 (Electric Forces).—Electrons and protons and the attractions and repulsions between them are the basic entities and observed facts in electricity. Like electric charges repel and unlike charges attract one another.

B.P.2 (Magnetic Forces).—When electrons are flowing in two conductors, the parallel components of their motions evoke forces of attraction or repulsion depending on whether the flows have the same or opposite directions; and the perpendicular

components evoke torques which tend to turn the conductors so that the electrons in them flow in the same direction.

B.P.3 (Nonconservative Electric Field).—This basic phenomenon will be described in Art. XIII-5.

Law A (Electromagnetic Reaction).—When looking in the direction of moving electrons, the direction of the magnetic lines of force about them produced by the motion is counterclockwise; an electric charge moving at right angles to the lines of force in a magnetic field is urged from the strengthened toward the weakened part of the field.

Law B (Directional Relation of Magnetic to Electric Fields).—A moving electric field and its accompanying magnetic field necessarily move together and are so related that, if they are moving from the observer with the electric lines of force pointing upward, the magnetic lines of force point from left to right.

Law C (Electromagnetic Pulse).—Another general law, called Law C, will be derived in Art. XVI-5.

Questions

1. What are magnetic forces? What is a magnetic field?

2. Distinguish between electric and magnetic fields by means of the forces the fields exert on electric charges placed in them

3. Show that if a stream of electrons is flowing through a conductor and therefore is mingling at all points with an equal number of similar stationary + charges, the magnetic field about the stream appears without an associated electric field. (The electric field that generally causes the electrons to move is not considered an associated field)

4. Explain why only the magnetic forces act between two wires carrying a steady electric current.

5. What is meant by an electric charge at rest? In motion?

6. What is meant by a stationary magnetic field?

7. State the law of attraction and repulsion between parallel currents and currents at right angles to each other.

8. What forces operate between two currents at any angle with each other? If they are free to turn, what positions will the wires finally take?

9. State B.P.2.

10. If the wires are parts of loops, one or both of which are free to turn, what positions with respect to each other will the loops be forced to take?

11. A loop of wire free to turn and energized with an electron flow turns into what position with respect to the earth? To explain this by B.P.2. what must be the direction of the electron flow in the earth relative to the observer? What is the direction of the electron flow in this loop when viewed from the north? From the south?

12. What is a magnetic loop? What are the north and south magnetic poles of a magnetic loop? What is an elemental magnet? A magnetic shell?

13. What position will a magnetic loop take in the neighborhood of a wire carrying an electric current?

14. Define a magnetic line of force. What relation has the line to the magnetic field?

15. What direction will a magnetic loop take with respect to the magnetic lines of force in a magnetic field? Show that this conforms to B P.2 and follows from Law A.

16. When looking in the direction of moving electrons, what is the direction of the observed magnetic lines of force? What is the direction when the electrons are viewed moving toward the observer? State Law A_1.

17. Are magnetic lines of force real entities? What is their nature?

18. Show that an electron flow at right angles to a magnetic field necessarily strengthens the field on one side and weakens it on the other side.

19. Explain in terms of B P 2 why an electron flow at right angles to a magnetic field is urged from the strengthened part of the field toward the weakened part and at right angles to the magnetic lines of force.

20. State Law A_2 (electromagnetic reaction).

21. Draw the resultant field and show that the wire carrying an electron flow is urged from the part of the field where the lines are crowded.

22. Explain in terms of Law A_2 the torques evoked by electron flows in two conductors at right angles to each other. What relation has this explanation to B P 2.?

23. State Law B (directional relation of electric and magnetic fields), and show how it is derived

24. What happens to stationary + and − charges when the two associated moving fields pass through them?

25. Why must the moving electric field set the charge into motion before the magnetic field exerts any force upon it?

Experiments

1. Deflection of a stream of electrons in a magnetic field.

2. Attraction and repulsion of parallel wires carrying electric currents: (*a*) rectangle turned, (*b*) coils attracted or repelled.

3. Wires at an angle with each other turn until they are parallel and the currents in them flow in the same direction.

4. A suspended loop carrying a current turns in the earth's magnetic field. The + and the − poles of a magnetic loop.

5. Tracing magnetic lines of force about a current by means of a magnetic loop and by means of a magnetic needle. Compasses and iron filings about a current.

6. Iron filings about a current stick together in circles and thereby appear to adhere to the conductor carrying the current.

7. A current flowing in a wire placed at right angles to the lines of force in a magnetic field feels a force urging it from the strengthened to the weakened part of the field.

8. Distortion of a weak magnetic field by a current flowing at right angles to magnetic lines of force (projected). The fields shown separately and then superposed.

9. Deflection of a magnetic needle by a current (Oersted's experiment).

10. Magnetic field shown to exist between the poles of an electromagnet by means of a magnetic loop and by means of a magnetic needle.

11. Model of a moving electron showing the directional relationship of the magnetic to electric field.

CHAPTER VI

MAGNETS AND MAGNETIC FIELDS

1. Forces of Attraction or Repulsion between the Poles of Magnetic Loops.—A magnetic loop was shown (Art. V–3) to have a + and a − magnetic pole. Two such loops placed with their planes parallel and their free electrons flowing in the same direction attract each other (B.P.2); but, as shown by loops A and B, Fig. 1, they necessarily have their unlike poles facing each other. The attraction then can be referred to the poles in place of the currents by the statement that *unlike magnetic poles attract each other.*

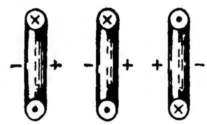


FIG. 1.—Sections of magnetic loops viewed at right angles to their planes showing the signs of the magnetic poles and the direction of the electron flows.

The magnetic loops B and C have their electrons flowing in opposite directions and the like + poles facing each other. The loops repel (B.P.2), and this repellent action may again be attributed to the poles by the statement that *two like magnetic poles repel each other.*

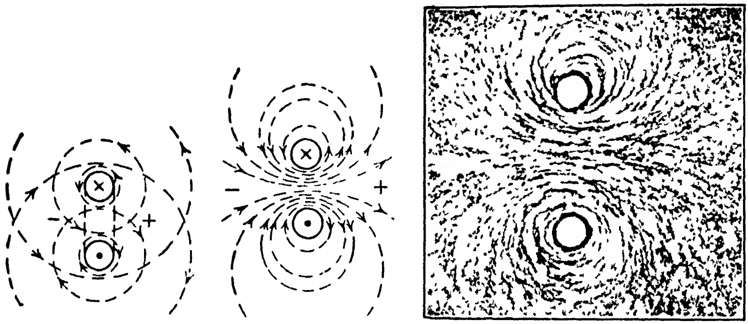
If the two statements are placed together, the law of attraction and repulsion between magnetic poles is formulated:

Unlike magnetic poles attract, and like magnetic poles repel one another. (1)

2. Magnetic Lines of Force Link Magnetic Loops.—The two full-line circles, Fig. 2(a), represent a cross section of a magnetic loop. The magnetic lines of force due to the electron flow in each of the two cross sections of the wire shown are represented by the broken-line, arrowed circles. It should be observed that the relative directions of the superposed fields vary in different regions, and that within the loop both fields have the same general direction. The resultant of these superposed magnetic fields is shown in Figs. 2(b),(c). A similar set of lines may be drawn for any similar section of the magnetic loop. It then follows that

Every magnetic loop is linked by magnetic lines of force which are closed loops and which emerge from the positive-pole side of the loop. (2)

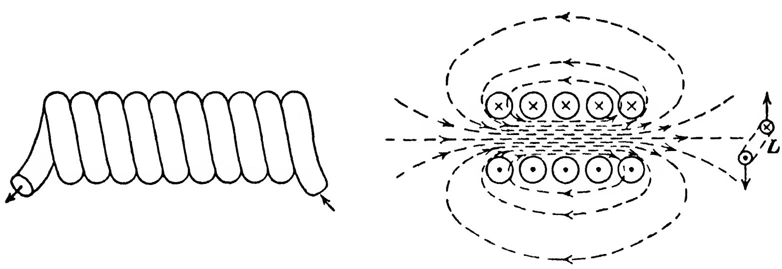
3. Solenoid.—A helix of wire, Fig. 3(*a*), composed of many loops is called a *solenoid.* The *spiral* motion of the electrons in



(*a*) (*b*) (*c*)

FIG. 2—(*a*) Cross section of a magnetic loop showing superposed magnetic fields due to the elements of the loop that are in the plane of the paper (*b*) The resultant magnetic field due to these elements or to the whole loop (*c*) Photograph of the field as shown by iron filings placed on a smooth surface in the field

it can be resolved into a linear current and a series of magnetic loops. The progressive part of the flow constitutes a linear current, and the rotary part magnetic loops. Such a solenoid,



(*a*) (*b*)

FIG. 3.—(*a*) A solenoid. (*b*) Cross section of a solenoid showing the magnetic field.

cut to show the loops in section, is shown in Fig. 3(*b*). Each loop of the solenoid, when considered as a perfect loop, produces a magnetic field like that shown in Figs. 2(*b*),(*c*). The magnetic fields of the several loops overlap and produce a resultant field represented by the arrowed closed curves in Fig. 3(*b*).

Imagine one end of the solenoid to be facing a magnetic loop *L*. Each component loop of the solenoid acts on this external loop in the same manner, except that the nearer ones exert a greater torque. The resultant effect is greater than that of any one loop alone; hence the ends of a solenoid exhibit comparatively strong magnetic poles. The solenoid is then a magnet with a positive and a negative pole and with a comparatively strong magnetic field linking the loops.

The external magnetic loop *L* is turned by the magnetic field of the solenoid until its plane is perpendicular to the magnetic lines of force. Such loops when placed within the solenoid are turned in the same manner, and, when finally their faces are perpendicular to the lines of force, their electron flows have the direction of the flow in the loops of the solenoid. Their magnetic fields then strengthen that of the solenoid.

When an energized solenoid is suspended perpendicular to its axis in the earth's magnetic field, it feels a torque which is the resultant of the torques which act on its individual loops. The solenoid, therefore, turns until its axis lies in the magnetic meridian.

Since the electron flow in all the loops of the solenoid has the same direction, the solenoid, for purposes of analysis, is assumed to be approximately equivalent to a cylindrical whirl of electrons. The component of the current which constitutes the progressive part of the flow produces a comparatively small additional effect which usually may be disregarded.

4. Toroid.—A solenoid curved so that its succession of loops forms a circle is called a *toroid*.

In the cross section of such a toroid, Fig. 4, all the magnetic loops may be paired, as illustrated by the loops *A* and *B*. The direction of the electron flow in these loops is represented in the conventional manner. The direction of the magnetic field due to the loop *A* is shown by the arrowed broken lines in different regions, and that due to loop *B* by the arrowed full lines. The fields reenforce each other in the regions *X* (within the loops) and oppose each other in the regions *Y* and *Z*. Each of the other loops of the toroid also contributes to the field in these regions. In the region *X* all these superposed loop fields have components in the direction indicated for the fields of loops *A* and *B*; and by

reenforcement these components produce a strong magnetic field within the loops of the toroid. Inspection shows that in the regions Y and Z a smaller number of the nearer loops contribute to the field in one direction at any point P than the larger number of more distant loops contribute in the other direction. It can be shown that these superposed fields neutralize one another completely (proof not given in text); hence the only field in these regions is the one produced by the "linear" component of the spiral motion. This linear component of the flow is equivalent to

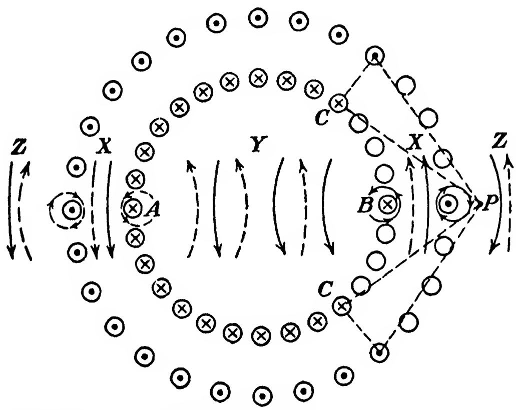


FIG. 4.—Cross section of a toroid showing the superposed magnetic fields due to two paired loops A and B. All the loops within the limiting loops CC contribute to the field at the point P in one direction and all the other loops in the reverse direction.

a single magnetic loop whose radius is that of the toroid. The field of this "linear" component may be neutralized by causing one of the current leads to pass once around the toroid within the loops and to carry the flow in a direction opposite that of the progressive component of the flow in the toroid.

5. Electromagnet—Temporary and Permanent Magnets.—An iron rod placed within a solenoid becomes a magnet which is many hundred times stronger than the solenoid magnet itself. The strong magnet, composed of the solenoid and the inclosed soft-iron or soft-steel rod, is called an *electromagnet*. The + pole of the magnetized iron, Fig. 5, is on the + pole side of the solenoid, and the magnetic field about the magnetized iron or steel rod has the same distribution as the field of the solenoid.

Soft iron, soft steel, and some iron alloys magnetize easily but lose most of their magnetism as soon as the magnetizing field is removed. They form *temporary magnets.* Hard steel magnetizes with difficulty; however, pounding it to jar the atoms aids in its magnetization. Hard steel retains most of its magnetism after removal from the solenoid and forms what are called *permanent magnets.* The temporary magnets, because they are stronger and for other reasons, are more generally useful than the permanent magnets.

Nickel and cobalt can be magnetized considerably; also an alloy of copper, manganese and aluminum, known as "Heusler alloy," although its component elements are practically non-

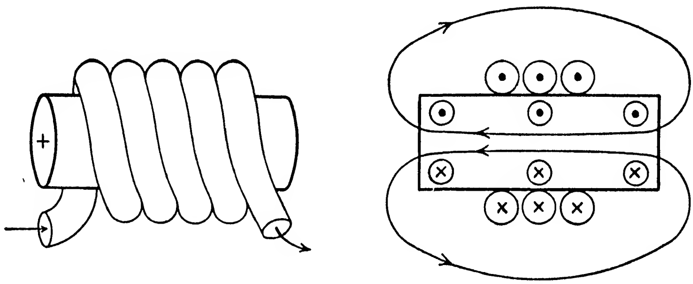


FIG. 5.—Electromagnet (a) Relation of the magnetic poles of the iron to the poles of the magnetizing solenoid (b) Cross section of electromagnet showing direction of the magnetization whirl of the magnetized iron in relation to the direction of flow in the magnetizing loops.

magnetic. Iron, nickel, cobalt, and this alloy are known as *ferromagnetic substances.*

Magnetite or loadstone, an oxide of iron (FeO, Fe_2O_3), has in nature a considerable amount of permanent magnetism. Pieces of iron or steel also are always slightly magnetized because of the earth's magnetic field.

Some substances like platinum, aluminum, and manganese are only slightly attracted by a magnet. They show only a trace of magnetization and are known as *paramagnetic substances.* Other substances like bismuth and antimony are slightly repelled by a magnet. They too show only a trace of magnetization, but in a direction opposite that in which ferromagnetic and paramagnetic substances would be magnetized in the same field. These are called *diamagnetic substances.*

6. Molecular Theory of Magnetism.—A permanent magnet when cut in two at right angles to its axis produces two complete magnets. If each of these magnets is again cut, each new part again becomes a complete magnet. If the cutting is imagined to be continued until sections perhaps only an atom in thickness are reached, it is believed that each such section, called a *magnetic shell,* would be a magnet and would behave like a magnetic loop with its circular flow of electrons. If this thin section be imagined to be cut repeatedly, at right angles to the former direction of cutting, smaller and smaller magnets of the same length would be produced until individual atoms or some small coherent groups of them were reached. These are the *elemental magnets* or *molecular magnets* or *magnetons* out of which it is believed all larger magnets are formed. The orbital electrons of an atom, or either the revolving electrons or protons within the nucleus of an atom, or the electrons revolving in the same direction about an axis of a coherent group of atoms, or even spinning electrons or protons, may form elemental magnets because they produce the same kind of magnetic field as exists about the stream of electrons in a magnetic loop. In nonmagnetic media the orbital electrons are assumed to be revolving in planes at such angles with one another that their magnetic fields practically neutralize.

When iron is not magnetized, its elemental magnets probably arrange themselves in circuits with the unlike poles facing each other; hence in each circuit they form a ring of loops like that of a toroid (Art. 4) and exhibit no external magnetic field. When the iron is placed into a magnetic field, this field forces the + and the − poles of the elemental magnets in opposite directions until finally, if the magnetizing field is sufficiently intense, all the + poles point in the direction of the field. This turning of the elemental magnets produces a + magnetic pole at one end of the bar and a − pole at the other in the same manner as a series of magnetic loops makes the solenoid a magnet (Art. 3).

The following experiments and considerations confirm the molecular theory of magnetism:

1. Each part of a long thin bisected magnet becomes a complete magnet, as already described.

2. The two poles of a normal magnet are found to be exactly equal in strength. There are necessarily as many + poles of molecular magnets facing in one direction as there are − poles facing in the other.

3. There is a limit to the amount of magnetization, because after all the elemental magnets have been completely turned so that their + poles face in the direction of the field the amount of magnetization cannot be increased.

4. In a uniformly changing magnetizing field, magnetization changes by successive jerks. This can be shown by the murmuring sound in an amplifier produced by the changing e.m.f. induced by the magnetization in a coil surrounding the magnet.

5. Magnetic properties disappear at red heat. The energy of molecular motion, then, is too great for the magnetic field appreciably to turn the molecular magnets.

6. Magnetization can be satisfactorily explained only by such a theory.

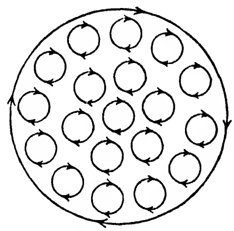


FIG. 6.—The molecular magnets in a magnetic shell shown to be equivalent externally to one larger whirl represented roughly by the large circle.

7. Equivalence of a Magnet to a Cylindrical Whirl of Free Electrons within a Conductor (Magnetization Whirl).—In the thin cross-sectional piece of magnet (magnetic shell), Fig. 6, the elemental magnets (Art. V–3) are shown with their + poles facing the observer. Each of these elemental magnets contributes its share to the production of one magnetic field; and all of them together, therefore, are externally equivalent to one larger magnetic loop, which is represented by the large circle. The whole magnet is composed of a series of such adjacent sections, and therefore its magnetic action is equivalent to that of a *cylindrical whirl* of free electrons (*magnetization whirl*) within a conductor or approximately to that of a solenoid. No practical solenoid, however, can be constructed to carry enough current to produce a magnetic field approaching in strength that of a well-magnetized piece of iron.

When the magnetization of a magnet is changing, the turning elemental magnets have angular accelerations in the same general direction; hence the conventional magnetization whirl which

represents these elemental magnets also has positive or negative acceleration.

For purposes of analysis, therefore,

A stationary magnet is equivalent to a cylindrical electron whirl about the axis of the magnet; and a magnet of changing magnetization is equivalent to an accelerating whirl. **(3)**

8. Magnetic Needle.—If a bar magnet be placed in a magnetic field, all its elemental magnets, some of which are represented by the loops *m*, Fig. 7, feel torques tending to turn them until their + poles face in the direction of the magnetic lines of force (Arts. V–3, 4). All these torques together are equivalent to one torque whose forces act at the points *PP*, called the *point-poles* of the magnet. The point-poles of the bar magnet, although imaginary, are treated as though they were real isolated point-poles which are urged in opposite directions by the magnetic field, the + pole in the direction of the lines of force and the − pole in the reverse direction.

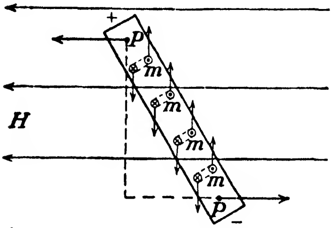


FIG. 7.—A bar magnet in a magnetic field *H*, showing cross sections of a few of the elemental magnets *m* (greatly magnified) and the conventional point-poles *PP*. The actual torques acting on the elemental magnets and the resultant torque attributed to the conventional point-poles are represented by appropriate arrows.

Such a bar magnet, then, if suspended in a magnetic field, is usually treated as being acted on by this one resultant torque which turns the magnet until its axis takes the direction of the magnetic lines of force. The suspended or pivoted bar magnet marks, in this manner, the direction of the magnetic lines of force. It is called a *magnetic needle* and, when placed in an appropriate case, a *magnetic compass*, Fig. 8. The needle of the magnetic compass in the earth's magnetic field, however, is turned on its vertical axis by the action of only the horizontal component of the field (see Art. VII–6).

9. Magnetization in Magnetic Fields.—The magnetization of iron or steel within a solenoid has already been considered (Art. 5). In a similar manner iron or steel is magnetized in any magnetic field. Pieces of iron placed in the magnetic field near the pole of a magnet become magnetized as shown in

Fig. 9(*a*). The unlike poles face each other; hence the magnet attracts the pieces, and the pieces attract one another. In this manner, iron filings are caused to stick together and roughly mark the direction of the lines of force in any magnetic field. In the magnetic field about a current the iron filings form rings as shown in Figs. 9(*b*) and V–8*c* which have sufficient coherence

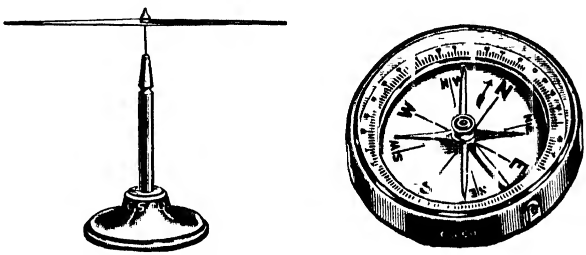
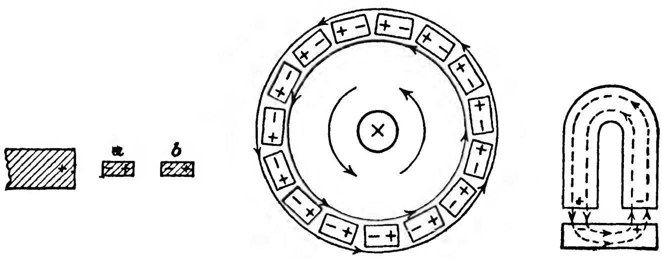


FIG. 8.—Magnetic needle and magnetic compass.

to hang on the wire. A piece of iron placed opposite the poles of a horseshoe magnet becomes magnetized as shown in Fig. 9(*c*) and is attracted by both poles of the magnet.

Magnetic fields penetrate nonmagnetic substances, such as glass and wood, without appreciable change.



(*a*) (*b*) (*c*)

FIG. 9—(*a*) Magnetization of pieces of iron in a magnetic field. (*b*) Iron filings in the magnetic field about an electron flow. (*c*) Soft-iron armature of a horseshoe magnet.

When an alternating current is flowing in a solenoid, the direction of the magnetic field within and in the neighborhood of the solenoid alternates. If magnetized steel is placed in such a field, it soon loses its original magnetism but becomes magnetized in alternate directions in conformity with the alternating field. If the steel is slowly removed from the field, the alternating magnet-

izations gradually diminish until finally the steel loses all of its magnetism.

10. Magnetic Lines of Force about Magnets.—In the solenoid, Fig. 3, the magnetic lines of force form complete loops which are partly inside and partly outside the solenoid. In the bar magnet the magnetic lines may be traced by means of a magnetic needle only on the outside; but they also are assumed to make complete circuits through the body of the magnet, *i.e.*, through the elemental magnetic loops of which the magnet is composed. The direction of the magnetic lines of force outside a magnet may

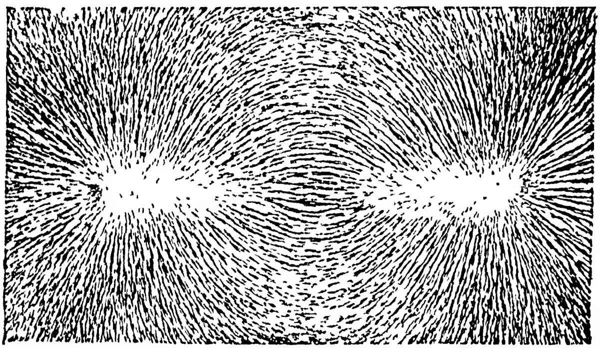


Fig. 10.—Photograph of magnetic lines of force about a bar magnet as shown by iron filings on a smooth surface.

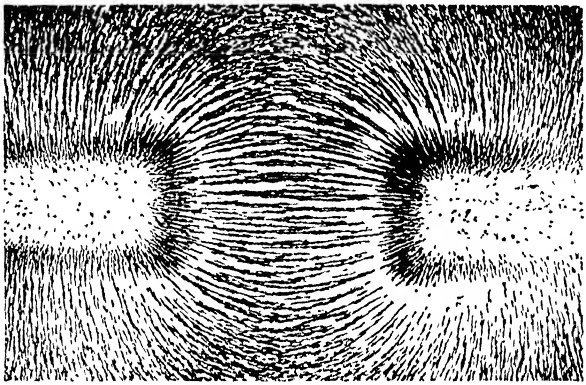
be shown by means of iron filings on a plate of glass placed above the magnet, Fig. 10.

The resultant lines of force between unlike poles of two magnets are shown in Fig. 11(*a*) and between two like poles in Fig. 11(*b*). A study of these lines, and the known attraction and repulsion of the poles leads to the following statement concerning the apparent properties of magnetic lines:

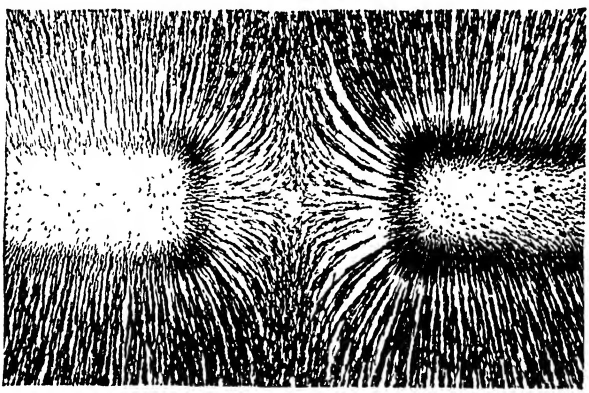
Magnetic lines of force tend to become as short as possible and to repel one another. (4)

If an iron ring be placed within the loops of a toroid, its elemental magnets turn until their loop planes are in those of the adjacent loops of the toroid. These elemental magnets together are equivalent (Art. 7) to a toroidal electron whirl within the iron, stronger, but similar to that in the loops of the magnetizing toroid. Such an iron ring has all its magnetic lines of force within the iron and exhibits no external magnetic field (Art. 4).

If such a magnetized ring be cut across its diameter and the two halves separated a short distance, the magnetic lines of force complete their circuits through the two intervening spaces. If one of the halves is removed entirely, the magnetic lines of force



(*a*)



(*b*)

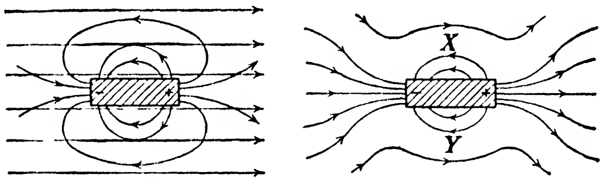
FIG. 11.—Magnetic lines of force between (*a*) unlike and (*b*) like magnetic poles (photographs of iron filings).

of the other half complete their circuits through the space between its poles; this, in a more or less modified construction, forms what is called a *horseshoe magnet*.

When two magnets are placed in line with their + poles together, they form a magnet with two − poles at the ends and a

+ pole at the center. A single bar of iron or steel can be magnetized to have such *consequent poles.*

11. Screening Effect of Iron—Permeability.—A piece of iron in a magnetic field, Fig. 12(*a*), becomes magnetized as shown and has its own magnetic field superposed on the magnetizing field. The resultant field, Fig. 12(*b*), shows a crowding of the lines of force through the space occupied by the iron and a weakening of the magnetic field in the regions X and Y outside the iron. In the regions about the ends of the bar the field is strengthened. The lines of force appear to pass more readily through the iron than through the air. The iron, therefore, is



(*a*) (*b*)

FIG. 12.—(*a*) A magnetizing field and that of a magnetized piece of iron superposed. (*b*) The resultant of the superposed fields.

said to be more "permeable" than air, or to have a greater *permeability* (Art. XX–7).

If a hollow iron cylinder is placed in a magnetic field, it "screens" the space within the cylinder from most of the original field. Several such concentric cylinders screen the space much more completely than one cylinder of equivalent thickness.

12. Superposed Electric and Magnetic Fields.—A magnetic field, even though it is evoked only by an electric field in motion (Law B), exists without an effective accompanying electric field when it is produced by a steady current in a conductor (Art. V–1). It follows that the magnetic fields about solenoids and magnets, if the solenoids and magnets are not in motion, are also free from the accompanying electric field; conventionally they are stationary fields. Such a magnetic field may exist simultaneously in the same space with an electric field produced by some stationary electric charge. One field acts on electric charges only because of their motion (Art. V–1), and the other acts on them because they are electric charges regardless of whether they are in motion or at rest (Art. I–3).

Questions

1. Give the law for the direction of the forces acting between the poles of magnetic loops and show how it follows from B.P.2.

2. Describe the form of the magnetic field linked with a magnetic loop; with a solenoid; with a toroid; with a bar magnet; with a horseshoe magnet.

3. Describe what is believed to take place when a piece of iron is being magnetized. How are the elemental magnets arranged in an unmagnetized piece of iron? In a magnetized piece? Which pole of the magnet necessarily faces in the direction of the magnetizing field? What is an electromagnet? Why cannot its strength be indefinitely increased?

4. Give the molecular theory of magnetism.

5. Explain why when a bar magnet is broken the two parts are complete magnets.

6. Explain why a bar magnet may be considered an electron whirl within a conductor. What is a magnetization whirl?

7. Explain, in terms of its elemental magnets and the torques acting on them, why a bar magnet feels a torque in a magnetic field. Explain the concept of the magnetic point-poles of a magnet.

8. What is a magnetic needle? Why does its + pole point north?

9. Explain why nails and iron filings stick together and align in a magnetic field.

10. Sketch the directions of the magnetic lines of force between like and unlike poles and show how the law concerning the properties of magnetic lines is derived from these observations.

11. Explain why magnetic lines of force appear to pass more readily through a piece of iron than through open space. What is meant by permeability? By the screening effect of iron?

Experiments

1. Attraction and repulsion of magnetic loops and of poles of solenoids and magnets

2. Magnetic lines of force about a magnetic loop, a solenoid, and a bar magnet.

3. Toroid.

4. A rod of iron within the solenoid becomes a much stronger magnet than the solenoid. Lifting power of an electromagnet. Slides of commercial applications

5. An iron tube is drawn into a solenoid by the action of the magnetic field; and if the circuit is broken when the velocity of the tube is the greatest, the tube is thrown out at the other end of the solenoid (electric gun).

6. Pieces cut from a bar magnet are complete magnets.

7. Loss of magnetic property by heating.

8. Model illustrating molecular theory of magnetism; also magnetized steel filings in a glass tube.

9. Magnetic needle.

10. Magnetization of iron and steel between the poles of an electromagnet and in the field near a strong electromagnet.

11. Magnetization changes by jerks (Barkhausen effect).

12. Temporary and permanent magnets Various forms of magnets.

13. Attraction of iron through nonmagnetic solids.

14. Rings of cohering iron particles suspended on a wire carrying a current.

15. Magnetic lines of force between the poles of magnets.

16. Permeability of iron.

17. Demagnetization of a magnetized watch by withdrawal from an alternating magnetic field.

CHAPTER VII

MEASUREMENTS IN MAGNETISM

1. Unit Magnetic Pole.—The term "magnetic poles" refers to the faces or ends of loops, solenoids, and bar magnets that exhibit the property of attracting or repelling the faces or ends of similar loops, solenoids, and bar magnets. These face or end-effects, as already explained, are due to the attraction or repulsion between electron whirls (B.P.2). It is only for convenience that the term magnetic pole is used. It is not an entity in itself and has no exact location on the magnet except that assigned to it for mathematical analysis. The pole, in the mathematical sense, is a *point-pole* (Art. VI–8) and is approximately where a short magnetic needle points from all positions about the end of the magnet.

The position of a magnetic point-pole is that position which makes the force between two such point-poles vary inversely as the square of the distance. **(1)**

The distance between the point-poles of a magnet is called the *equivalent length* or simply the *length of the magnet* and is shorter than the length of the bar. The length is usually represented by l or by $2L$.

Different magnetic poles act on each other with different forces and are therefore said to have different strengths. To enable the strength of magnetic poles to be measured, a magnetic pole of unit strength is arbitrarily defined as follows:

A unit magnetic pole is a point-pole which in a vacuum acts on another equal point-pole 1 cm distant with a force of 1 dyne.

The pole strength in these units is designated by the letter m, but the unit has no special name.

2. Magnitude of Forces Acting between Magnetic Point-poles (Law of Inverse Squares).—Let m and m' be the pole strengths of two magnetic point-poles at the distance d from each other. When the pole strengths vary, physical intuition and experiment show that the force acting between the poles,

$$f \propto mm'.$$

If the distance between the point-poles is varied, the force

$$f \propto \frac{1}{d^2}.$$

This latter relationship is proved approximately by direct experiment and conclusively by the fact that the assumption of the relationship makes all calculations agree with experimental data.

Then

$$f \propto \frac{mm'}{d^2},$$

and

$$f = K\frac{mm'}{d^2},$$

in which, when m and m' are each unit poles and d is 1 cm, the force f is 1 dyne by definition (Art. 1); then, from the relationship in the equation, $K = 1$. Hence in a vacuum or in a nonmagnetic medium, including air,

$$f_0 = \pm\frac{mm'}{d^2} \text{ dynes.} \tag{2a}$$

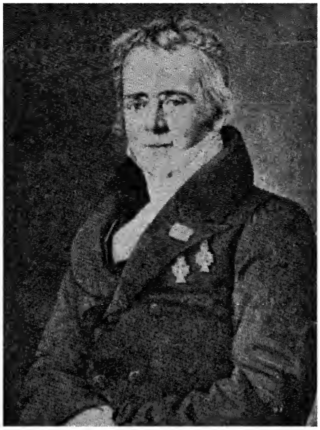
In material media the force acting between the poles [see Appendix V (2)] differs from that in this equation by the factor $1/\mu$, in which μ is the permeability (Art. XX–7) of the material. Then

$$f = \pm\frac{1}{\mu}\frac{mm'}{d^2} \text{ dynes.} \tag{2b}$$

In "nonmagnetic" media, including air, $\mu \cong 1$, so that Eq. (2a) is applicable when the magnets are immersed in them. This law is another illustration of the *law of inverse squares.* The $\pm$ sign indicates that the force between the poles may be one either of repulsion or of attraction.

The magnetic poles of actual magnets, as already explained, are not points. Two such poles, however, act on each other as if they were point-poles located at some distance from the ends of the magnets. These point-poles can represent the magnet for analytical purposes.

3. Unit Magnetic Field (Oersted).—*A magnetic field has unit strength or unit intensity if it acts on a unit point-pole in the field with a force of* 1 *dyne.* The proposed name for this unit is the *oersted.* The strength of a magnetic field in oersteds is repre-



Hans Christian Oersted (1777–1851), professor of physics, University of Copenhagen, Denmark; discoverer of the fact that a magnetic field surrounds an electric current (1820) It is proposed to name the unit of magnetic field intensity, the *oersted*, in his honor

sented by the letter H or by the symbol $\mathcal{H}$. The force acting on a magnetic pole of m units in a magnetic field of H oersteds then is

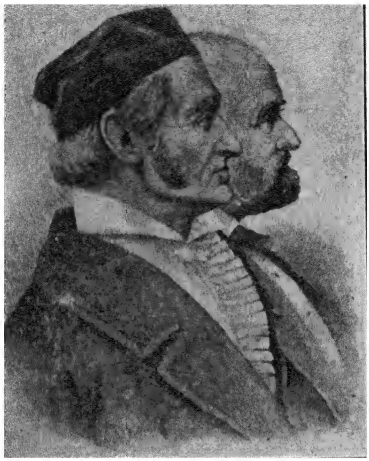
$$f = Hm \text{ dynes.} \tag{3}$$

In this equation, H is the field strength at the particular point at which the pole m is placed. Most fields are not uniform in strength and therefore in such fields the magnitude of H varies from point to point.

The unit point-pole in terms of which the intensity (or strength) of a magnetic field is measured is a magnetic pole by convention only, and therefore one may be imagined at any desired point as an isolated conventional entity.

The intensity of the field at any point outside a bar magnet and therefore the magnitude of the torque acting on a neighboring magnet or solenoid are calculable in terms of the point-poles. The concept of the point-poles, however, has a limited application

and is used only because it simplifies the calculations in cases to which it is applicable. The same calculations can be made without invoking the aid of point-poles by means of the universally applicable interaction between the elemental electron whirls which form the magnet or magnets. It should here be observed



Karl Friedrich Gauss (1777–1855), professor of mathematics and director of the observatory, Gottingen, Germany, and Wilhelm Edward Weber (1804–1891), professor of physics, Gottingen; Gauss devised the present system of magnetic units (1832) and Weber, the present system of electric units (1846). The unit of magnetic flux density, *the gauss*, is named after Gauss (on left of photograph). The *weber*, a multiple unit of magnetic flux, is named in honor of Weber.

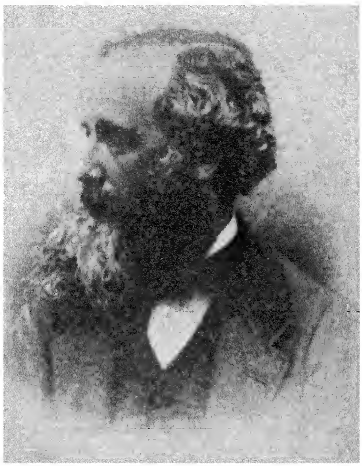
that the magnetic lines about a conventional isolated point-pole are straight divergent lines, while actual magnetic lines are always closed loops.

A magnetic field, defined in Art. V–4, can now be redefined:

A magnetic field is a space which (1) exerts forces on moving electric charges because of their motion, (2) turns magnetic loops into positions in which their + poles face the direction of the field, and (3) urges the conventional + magnetic point-poles in the direction of the field and the − point-poles in the reverse direction. (4)

The action of any magnetic field on a magnet is the action of the original field into which the magnet and its field are placed. The resultant of the superposed fields, however, becomes *the field* for any magnet brought later into the region.

4. Magnetic-flux Density—Quantity of Flux—Gauss—Maxwell.—It is convenient, for purposes of analysis, to assign to a magnetic field a definite number of lines of force proportional to the strength and to the cross-sectional area of the field. The number of lines assigned to each square centimeter (at right angles to the lines) is by convention numerically that of the intensity of the field in oersteds. This number of lines per square centimeter is called the *density of the magnetic flux* and is represented by the letter B (see also Arts. XX-6, 7).



James Clerk Maxwell (1831–1879), mathematical physicist, professor of physics in succession at Marischal College (Aberdeen), King's College (London), and Cambridge University (Cambridge), England. Perhaps the greatest of mathematical physicists, devised the now famous "Maxwell's equations" and propounded the electromagnetic theory of light (1864). His reasoning (subsequently experimentally verified by Hertz) showed that electromagnetic waves radiate from oscillating charges. The unit of magnetic flux, *the maxwell*, is named in his honor.

One magnetic line of force is called a *maxwell*, and the unit of magnetic-flux density (one line of force per square centimeter) a *gauss*. A *weber* is a multiple unit of magnetic flux and is equal to 10^8 maxwells.

The total magnetic flux in any magnetic field is

$$\phi = BA \text{ maxwells} \tag{5}$$

or

$$\phi_w = \frac{BA}{10^8} \text{ webers,}$$

where B is the flux density, and A the area in square centimeters.

A magnetic field which exerts a force of 3 dynes on a unit point-pole, for example, has an intensity of 3 oersteds and a flux density of 3 gauss. It has 3 maxwells/cm² and therefore, if its cross section is 100 cm², contains 300 maxwells or 0.000003 webers of magnetic flux.

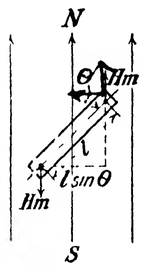


FIG. 1.—Forces acting on a deflected magnet in a magnetic field.

5. Torque Acting on a Deflected Magnet—Magnetic Moment. Figure 1 shows a magnet in a magnetic field with its axis at an angle θ with the magnetic lines of force, which are represented by the lines NS. The forces Hm acting on the point-poles of the magnet exert the torque

$$T = Hm \times l \sin \theta = Hml \sin \theta.$$

The term ml, because of its frequent appearance in equations relating to magnets, is represented by the single letter M and is called *magnetic moment*. Then the torque

$$T = HM \sin \theta. \tag{6}$$

This equation shows that M represents the torque to which the bar magnet would be subjected if it were placed at right angles to the lines of force in a unit magnetic field.

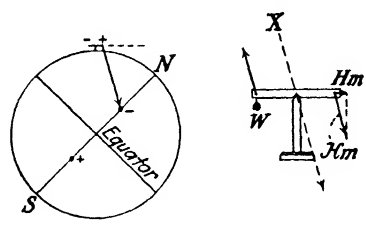


FIG. 2.—(a) The direction in which the earth's magnetic point-pole acts on the + pole of a magnetic needle in latitude 45°N (b) Forces acting on a magnetic needle in the earth's magnetic field.

6. Horizontal Component of the Earth's Magnetic Field.—The earth is a spherical magnet having its − pole near the north pole of the earth. In any locality the direction and the intensity of the earth's magnetic field depend on its distance from the magnetic poles and on the magnetic substances in the neighborhood. In Minneapolis the direction of the earth's

magnetic field makes an angle of about 75° with the horizontal, as illustrated in Fig. 2(*a*).

The line X, Fig. 2(*b*), represents the direction of the earth's magnetic field, whose intensity is $\mathcal{H}$. In this field the supported magnetic needle, weighted by W at its south pole, rests in a horizontal position. Each pole of the needle is acted on by a force of $\mathcal{H}m$ dynes. The component of this force in the horizontal

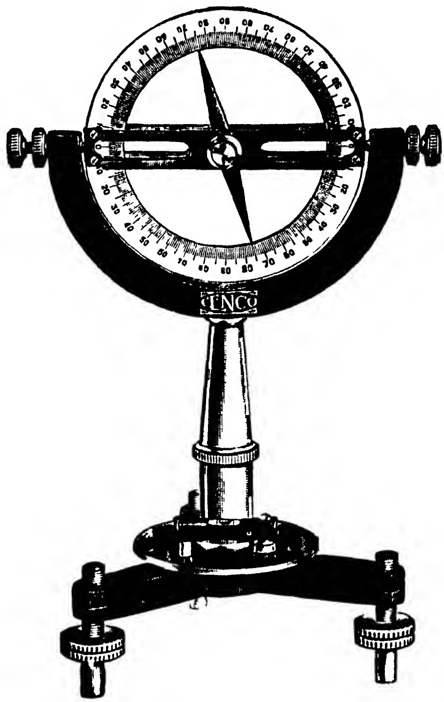


FIG. 3.—Dipping needle.

direction is Hm, in which H, called the *horizontal component of the earth's magnetic field*, represents the force with which the earth's magnetic field acts on a unit point-pole in a horizontal direction.

When a magnetic needle is deflected in a horizontal plane from its normal north and south position, it is only the horizontal component of the earth's field that tends to return the needle into its original position. Hence, in the case of a magnetic

needle in the earth's magnetic field, the H in the equation $T = HM \sin \theta$ refers to the horizontal component of the earth's magnetic field.

A *dipping needle* consists of a magnetic needle pivoted in the center of a circular scale, as shown in Fig. 3. When the plane of the scale is vertical and in the magnetic meridian, the needle points in the direction of the lines of force of the earth's magnetic field, and its reading on the scale gives the angular dip of the field with respect to the horizontal plane.

7. Time of Vibration of a Bar Magnet Suspended in the Earth's Magnetic Field.—The torque acting on a suspended magnet in the earth's field was shown (Arts. 5, 6) to be $T = HM \sin \theta$.

By neglecting the small effect of the torque due to the suspension and remembering from the study of mechanics that in rotary motion the torque is equal to the product of the moment of inertia, I_0, and the angular acceleration produced by the torque,

$$T = HM \sin \theta = -I_0\alpha.$$

Whence

$$-\frac{\sin \theta}{\alpha} = \frac{I_0}{HM}.$$

When the angle is small, $\sin \theta \cong \theta$; then

$$-\frac{\sin \theta}{\alpha} \cong -\frac{\theta}{\alpha} = \frac{I_0}{HM}.$$

The time of a complete double vibration in simple harmonic motion, derived in the study of mechanics, is

$$t = 2\pi\sqrt{-\frac{\theta}{\alpha}}.$$

Since for magnets which vibrate through small angles θ/α is practically constant, the motion of the magnet through such angles is simple harmonic. Hence the time for a complete double vibration is

$$t = 2\pi\sqrt{-\frac{\theta}{\alpha}} = 2\pi\sqrt{\frac{I_0}{MH}} \text{ sec.} \tag{7}$$

8. Intensity of the Magnetic Field at Any Point on the Line of the Axis of a Bar Magnet.—Since the intensity of a magnetic field is measured by the force the field exerts on a unit point-pole placed in it, such a unit pole is imagined at the point P, Fig. 4, in the line of the axis of the magnet. Let d be the distance of the point from the center of the magnet; L, the distance of each point-pole from the center of the magnet; and m and $-m$, the strengths of the poles. The magnetic moment M then equals $2Lm$.

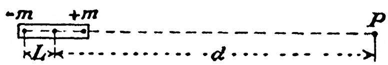


FIG. 4.—A bar magnet and the intensity of its magnetic field at the point P.

The pole m acts on the imagined unit point-pole at P with a force $\frac{m \times 1}{(d - L)^2}$ dynes, and the pole $-m$ acts on it in the reverse direction with a force $-\frac{m \times 1}{(d + L)^2}$ dynes.

The resultant of the two forces acting on the unit point-pole is, by definition, the intensity of the magnetic field at the point P. Then

$$\mathcal{H} = \frac{m}{(d - L)^2} - \frac{m}{(d + L)^2} = \frac{4dLm}{(d^2 - L^2)^2} = \frac{2dM}{(d^2 - L^2)^2} \text{ oersteds.}$$

At some distance from the magnet L^2 becomes negligible compared with d^2, and the equation may then be written

$$\mathcal{H} \cong \frac{2M}{d^3} \text{ oersteds.} \tag{8}$$

This equation shows that at some distance from a bar magnet the intensity of the field varies inversely as the cube of the distance, and that the strength of the field becomes negligible at comparatively short distances from the magnet.

It can be shown that the law of inverse cubes holds for any direction from the magnet. Such a law applies in all cases in which two equal sources of opposite sign, some distance apart, exert on external objects forces that follow the law of inverse squares. That which evokes the forces is called a *doublet*. The two point-poles of a magnet form a *magnetic doublet*, and two

equal opposite electric charges, usually very near each other, form an *electric doublet.*

9. Measurement of the Horizontal Component of the Earth's Magnetic Field and the Magnetic Moment and Pole Strength of a Magnet.—The bar magnet $m'm'$, Fig. 5, is so placed that the line which passes through its poles also passes through the pivot of a distant magnetic needle and is perpendicular to the magnetic meridian. The magnetic needle is acted upon by two opposing torques: one due to the bar magnet, and the other due to the horizontal component of the earth's magnetic field.

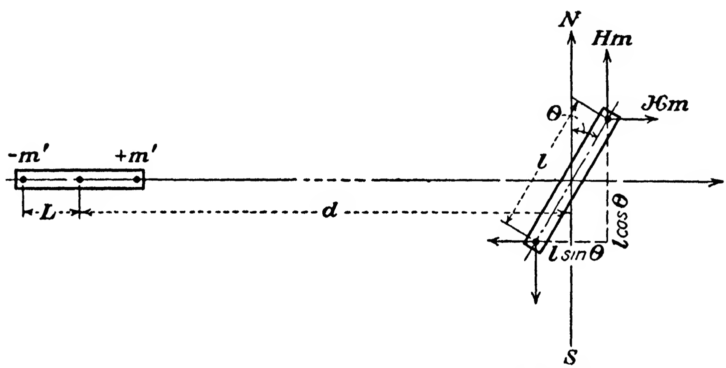


FIG. 5.—Torques acting on a magnetic needle in two magnetic fields superposed at right angles to each other

The former decreases and the latter increases as the angle of deflection becomes greater. The needle continues to turn until the two opposing torques are equal.

The center of the needle is at the distance d from the center of the bar magnet. When the needle is short compared with this distance, the strength and the direction of the field of the magnet are practically the same at the poles of the needle as they are at the pivot. The strength of this field at the pivot of the needle was shown (Art. 8) to be

$$\mathcal{H} = \frac{2dM}{(d^2 - L^2)^2}.$$

The forces acting on the poles of the deflected needle, then, are $\mathcal{H}m$ (due to the bar magnet) and Hm (due to the horizontal

component of the earth's magnetic field). When the torques are in equilibrium, the torque

$$T = \mathcal{H}\cancel{m}l \cos \theta = H\cancel{m}l \sin \theta,$$

from which

$$\mathcal{H} \cos \theta = \frac{2dM}{(d^2 - L^2)^2} \cos \theta = H \sin \theta.$$

$$\frac{M}{H} = \frac{(d^2 - L^2)^2}{2d} \tan \theta = A.$$

The unknown quantities M and H necessarily must appear in the equation, for the amount of the deflection is dependent on both of them. The pole strength of the needle, however, does not appear because the magnitude of the deflection is independent of it. The terms on the right-hand side of the equation can be measured so that that side of the equation has a known numerical value expressed for convenience by A.

In order to determine the values of either M or H, another experiment must be performed which involves these quantities in some other relationship. If the bar magnet used in the foregoing experiment replaces the magnetic needle, it becomes a needle suspended in a field whose horizontal intensity is H. The time of a complete double vibration of this bar-magnet needle depends (Art. 7) on both M and H and is

$$t = 2\pi \sqrt{\frac{I_0}{MH}};$$

whence

$$MH = \frac{4\pi^2 I_0}{t^2} = B.$$

The moment of inertia, I_0, of the magnet is calculated from its mass and dimensions; t is obtained experimentally. The quantities on the right-hand side of the equation then are known so that that side of the equation has a known value expressed by B. From the equations

$$\frac{M}{H} = A, \qquad \text{and} \qquad MH = B,$$

$$H = \sqrt{\frac{B}{A}}; \qquad M = \sqrt{AB}.$$

This method may be used to measure any weak fields whose lines of force lie in a horizontal direction; otherwise, as in the case of the earth's field, it measures their horizontal component only. The magnetic moment of any bar magnet may be measured by this method.

Since $M = 2Lm$, the pole strength of the bar magnet is

$$m = \frac{M}{2L}.$$

10. Comparison of the Horizontal Components of Weak Magnetic Fields.—1. The angle through which a bar magnet deflects a magnetic needle depends on the magnitude of the horizontal component of the magnetic field in which the needle is placed. The magnitudes of the deflections of a magnetic needle produced by the same bar magnet at the same distance in different fields, then, give the data necessary to compare the horizontal components. The desired relationship is expressed by the equation (Art. 9) for M/H, from which

$$H = \frac{2dM}{(d^2 - L^2)^2} \times \frac{1}{\tan\theta} = K \cdot \frac{1}{\tan\theta}.$$

For the purposes of comparison, all the quantities involved are constant except H and $\tan\theta$. The equation then states that H varies inversely as $\tan\theta$; otherwise expressed,

$$\frac{H_1}{H_2} = \frac{\tan\theta_2}{\tan\theta_1}.$$

2. It is usually more convenient, however, to make comparisons by taking the time of vibration of a bar magnet at the different stations. This method is capable of greater precision than method 1. From Eq. (7) for the time of vibration

$$H = \frac{4\pi^2 I_0}{M} \cdot \frac{1}{t^2} = K \cdot \frac{1}{t^2},$$

$$H \propto \frac{1}{t^2},$$

or

$$\frac{H_1}{H_2} = \frac{t_2^2}{t_1^2}.$$

These methods (1 and 2) are applicable to the measurement of weak magnetic fields only, because in the development of the equations the magnetic moment of the bar magnet is assumed to be constant. This is not the case in strong magnetic fields.

Questions

1. Define unit magnetic pole; unit magnetic field; oersted.

2. Have magnetic poles definite positions on a magnet? How is their location made definite for mathematical analysis?

3. Give the law of inverse squares for magnets.

4. Give the expression for the force acting on a magnetic point-pole in a magnetic field

5. How is the number of magnetic lines of force restricted?

6. How is density of magnetic flux represented?

7. Define unit of magnetic flux; unit of magnetic flux density; maxwell; gauss

8. Derive the expression for the torque acting on a deflected magnet in a magnetic field.

9. Define the magnetic moment of a magnet.

10. What is meant by the horizontal component of the earth's magnetic field?

11. Derive the expression for the intensity of the magnetic field at any point on the line passing through the axis of a bar magnet.

12. What is a magnetic doublet? An electric doublet?

13. What is the law of inverse cubes, and where does it apply?

14. Derive the expression for the time of vibration of a bar magnet.

15. Explain how H and M can be measured, and derive the necessary equations.

16. Show how m can be measured.

17. Show how the horizontal components of weak magnetic fields may be compared.

Problems

1. With what force does a magnetic point-pole whose strength is 100 units act on another point-pole of 200 units at a distance of 10 cm in a vacuum?

2. With what force is a magnetic pole of 10 units strength acted on in a magnetic field whose intensity is 100 oersteds?

3. What is the torque acting on a magnetic needle, whose magnetic moment is 150 c g.s. units, when the needle is deflected through an angle of 30° from the magnetic meridian in a location where $H = 0\ 20$ oersteds?

4. What is the total strength of the earth's magnetic field in a location where it makes an angle of 75° with the horizontal plane and has a horizontal component of 0 16 oersteds?

5. (*a*) What is the strength of the magnetic field at a distance of 40 cm from the center of a bar magnet in the line of its axis if the pole strength of

the magnet is 200 units and the distance between the poles is 10 cm? (*b*) With what force will a unit magnetic pole be acted on at that point? (*c*) Which pole of the magnet faces the point when the direction of the field at the point is toward the magnet?

6. What is the value of MH for the foregoing magnet whose moment of inertia is 120 c.g.s. units in a location where its double-vibration period is 10 sec?

7. The bar magnet of Prob. 5 is placed horizontally and in a position so that a line drawn through its poles is perpendicular to the magnetic meridian and passes through the pivot of a short magnetic needle. If the distance of the pivot from the center of the bar magnet is 35 cm when the needle is deflected 6°, what is the magnitude of M/H?

8. From the data of Probs. 5, 6, and 7 calculate H, M, and m.

9. The time of vibration of a bar magnet is 12 sec at a point where $H = 0.160$. What is H_1 at a point where this same magnet has a period of 15 sec?

10. A bar magnet with its axis in the magnetic east and west deflects a magnetic needle 30° in location I and 40° in location II. The distance between the needle and the magnet is the same in both cases. If $H = 0.16$ in location I, what is the horizontal component of the earth's field in location II?

11. The intensity of the earth's magnetic field in Minneapolis is 0 617 oersteds and the inclination is 75°. What is the magnitude of the horizontal component of the field?

12. The cross-sectional area of a magnetic field of 200,000 maxwells is 20 cm². (*a*) What is the average density, B, of the magnetic flux? (*b*) What is the intensity, H, of the field?

13. A magnetic field whose cross section is 20 cm² acts on a magnetic pole of 7 units with a force of 749 dynes. (*a*) What is the intensity of the magnetic field? (*b*) What is the total flux?

Experiments

1. Magnets with spherical poles, such as are used in the study of the law of inverse squares, shown.

2. Magnetic needle. Magnetic compass. Dipping needle.

3. Time of vibration of a suspended magnetic needle shown to vary with the intensity of the magnetic field.

4. Magnetometer.

CHAPTER VIII

ELECTRON FLOW (ELECTRIC CURRENT)

1. Direction of Electron Flow and of Electric Current.—In solutions of acids, bases, or salts (called *electrolytes*) the molecules of the solute are dissociated into electrically charged parts called *ions*. A molecule of the solute forms at least one positive and one negative ion, each of which contains one or more elemental charges depending on the valence and the grouping of the constituent elements.

The two oppositely charged plates, Fig. 1, are immersed in such a solution, and one of each kind of univalent ions of the solute is represented. These ions are moved by the electric field in opposite directions as shown.

Each positive univalent ion on reaching the negative plate "takes" one electron from it, and similarly each negative univalent ion "gives" its excess electron to the positive plate. The same number of electrons must enter as leave the electrolyte because any tendency toward variation from this condition would result in a reaction preventing inequality. The two oppositely directed streams of ions accomplish what is equivalent to a transfer of the same number of electrons from plate B to plate A as moves by every plane of the circuit of which the electrolyte is a part.

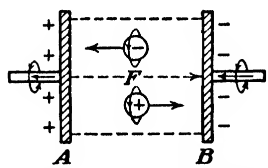


FIG. 1.—Ions of unlike charge moving in opposite directions have magnetic fields in the same direction.

The equal electric fields associated with each of the two kinds of moving ions are oppositely directed and therefore neutralize each other; the magnetic fields evoked by their motions reenforce each other. Therefore, as in the case of moving electrons in a metallic conductor, a cylindrical magnetic field without an effective moving electric field surrounds the moving ions and has the same intensity and direction as that about the metallic part of the circuit.

It is convenient to use the word *current* to designate the fact that electricity is being transferred from one plate to the other, and it is desirable to call one direction the *direction of the current*. This direction is given arbitrarily to that in which + charges are being transferred, regardless of the fact that in an electrolyte there is at the same time a motion of − charges in the reverse direction. The direction of the current in the electrolyte, Fig. 1, then, by convention, is from plate *A* to plate *B*. The nature of the flow of electricity within metallic conductors was unknown when the *direction of the current* in them was arbitrarily taken to be that in which + charges would be moving if they were the charges producing the observed magnetic field. But since the cause of the magnetic field in them is now known to be the flow of electrons (Art. IV–9), the direction of the electron flow is the reverse of the conventional direction of the current. The terms *direction of flow*, *electron flow*, and *electron drift* are used in this text to designate the actual direction of progressive electron motion (Art. V–1), the term *current* being used only in a general sense when the actual direction of flow is immaterial. In all cases concerning currents the arrows in the figures of this text show the direction of the electron flow and not the conventional direction of the current. The many advantages of this designation appear to outweigh any temporary confusion.

It should be noted at this time that, although a static electric charge is distributed only over the surface of a conductor (Art. IV–3), the elemental charges in the electron flow, except in the case of high-frequency currents (Art. XXVIII–9), move through the body of the conductor.

2. Units of Current.—The primary definition of the unit of electron flow (current) could have been given in terms of the silver deposited by the current from an electrolytic solution (Art. XI–5), in terms of a potential difference (Art. IX–7), or in terms of the power expended (Art. IX–9) in maintaining the flow in some given length of a standard conductor. The adopted and most satisfactory primary definition of the unit, however, is that in terms of its magnetic effect.

1. In any circular loop of wire, Fig. 2, each element of the wire is equally distant from the center of the loop; hence every element of the same length contributes equally to the strength

of the magnetic field $\mathcal{H}$ at the center; therefore the strength of this field at the center of the loop varies as the length of the wire; *i.e.*,

$$\mathcal{H} \propto l.$$

2. If the number of moving electrons in the loop is imagined to be doubled without changing the velocity, it is seen that each half of them must contribute equally to the magnetic field at the center. The magnetic field at the center then varies as the number of electrons which pass any plane of the loop per second; *i.e.*, $\mathcal{H} \propto Q'/t$. If, however, the number of moving electrons in the loop remains unchanged while their velocity is doubled, what is the change in the intensity of the magnetic field at the center of the loop? The elemental electron fields, which always move with their lines of force parallel to themselves, now cut through the center of the loop with twice their original velocity. Since the magnetic field is an aspect of the electric field due wholly to relative motion, it appears that doubling the velocity of the electric field should double the intensity of the magnetic. The positive proof, however, is experimental. If the + ions which deposit on the negative plate from the electrolyte of Fig. 1 are the silver ions of $AgNO_3$, the mass of the silver deposited in a given time is proportional to the number of atoms and therefore to the number of electrons that have to pass every plane in the metallic part of the circuit in order to neutralize the + charges of the deposited silver. If the current is increased so that the rate at which the silver is depositing is twice the original rate and the velocity of the electrons in the circuit is also twice the original velocity, the intensity or strength of the current of flow, I', is doubled and the intensity of the magnetic field at the center of the loop is also found to be doubled. Therefore in any case

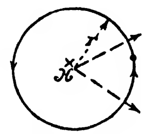


FIG. 2.—A loop of wire carrying an electric current.

$$\mathcal{H} \propto \frac{Q'}{t} \propto I'.$$

3. It is found that the magnetic field due to two loops of wire whose radii are twice that of a single loop, Fig. 3, neutralizes the magnetic field of the single loop in which the same current

flows in the reverse direction. The two larger loops together have four times the length of wire contained in the smaller loop; therefore each unit length of wire in the larger loops contributes only one-fourth as much to the intensity of the field as an equal length of wire in the smaller loop. The intensity of the magnetic field produced at the center, therefore, varies inversely as the square of the distance from each element of length of the wire carrying the current; *i.e*.,

$$\mathcal{H} \propto \frac{1}{r^2}.$$

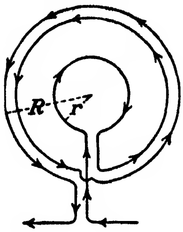


FIG. 3.—A single loop neutralizes the magnetic field at its center due to two concentric loops when $R = 2r$.

It follows from the preceding three relationships that in a vacuum or in air

$$\mathcal{H} \propto \frac{I'l}{r^2},$$

or

$$\mathcal{H} = K\frac{I'l}{r^2}.$$

If the current I' is arbitrarily called a unit current when r, l, and $\mathcal{H}$ are each unity, K is equal to 1. When the unit of current is defined as stated, the intensity of the magnetic field at the center of any coil in a vacuum or in air is

$$\mathcal{H} = \frac{I'l}{r^2} \text{ oersteds.} \qquad \textbf{(1}a\textbf{)}$$

The unit of current here used is the *electromagnetic unit of current* or the *abampere*.

The abampere is that current which when flowing in a loop of 1 cm radius produces at the center of the loop, which is in open space, a magnetic field of 1 oersted for each centimeter length of the conductor.

The practical unit of current, *the ampere*, is defined arbitrarily in terms of the abampere:

$$\textbf{1 amp.} \equiv \textbf{0.1 abampere.}$$

If the current is flowing in a conductor immersed in a magnetic medium, Fig. 4, the elemental magnets (magnetic loops) of the

medium are turned partly or completely into positions where their + poles face in the direction of the magnetizing space field $\mathcal{H}$. The resultant intensity of the magnetic field at any given point then is

$$\mathcal{H} + H_1 = \mu\mathcal{H} = \mu\frac{I'l}{r^2},$$

where μ is a factor by which the space field $\mathcal{H}$ must be multiplied to give the total intensity of the magnetic field in the magnetic medium. In such a medium and in any medium or space, even when $\mu = 1$, the flux density at the center of the loop is

$$B = \mu\mathcal{H} = \mu\frac{I'l}{r^2}\ \text{gauss.} \qquad (1b)$$

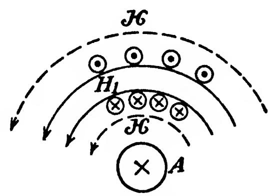


FIG. 4.—The magnetic field H_1, due to a magnetic medium, superposed on the space field H due to the flow in the conductor A.

When a current is flowing through a wire, the same number of electrons necessarily pass through every cross section of the wire in any given time. If this were not the case, the electrons would collect in some part and thereby repel others that were being forced in that direction.

The strength of the current is doubled if the progressive electron velocity is doubled, or if the cross-sectional area of the conductor is doubled while the electron velocity remains unchanged.. In either case twice the number of electrons pass any given plane per second. The current and the quantity of electricity passing any given plane per second, then, are so related that either can be defined in terms of the other.

In the electrostatic system of units, the current is defined in terms of the quantity of electricity which passes through every plane of the circuit in a given time because in this system, which is primarily employed for measurements pertaining to electricity at rest, the quantity of electricity is the basic measurement. In the electromagnetic system the measurement of current is basic and therefore the quantity of electricity is defined in terms of the current.

The electrostatic unit of current, the statampere, is the current when 1 statcoulomb per second is flowing through every plane of the circuit.

The current in statamperes then is

$$I'' = \frac{Q''}{t}. \tag{2a}$$

The *abampere* and *statampere* are comparatively new terms which are replacing the older designations *electromagnetic unit of current* and *electrostatic unit of current* (e.m.u. and e.s.u. of current). The prefixes *ab-* and *stat-* and the abbreviations are employed with all other electric units in the same manner.

André Marie Ampère (1775–1836), French physicist and mathematician; famous for his experiments on electromagnetism and his memoir on the magnetic effects of currents; demonstrated electromagnetic reaction (1820). The unit of current, *the ampere*, is named in his honor.

It is important to understand all three systems of units, as all are used in this text and in scientific literature.

3. Units of Quantity of Electricity.—The electrostatic unit of quantity, the statcoulomb, has already been defined (Art. III–1) as the primary or basic unit in the electrostatic system of units.

The electromagnetic unit of quantity of electricity, the abcoulomb, is the quantity which passes every plane of the circuit per second when the current is 1 abampere.

Then,

$$Q' = I't \text{ abcoulombs.} \tag{2b}$$

The practical unit of quantity of electricity, the coulomb, is the quantity which passes every plane of the circuit per second when the current is 1 ampere.

Then,

$$Q = It \text{ coulombs.} \tag{2c}$$

4. Relation of the Units of Quantity and of Current in the Three Systems.—Since the ampere is defined as 0.1 abampere, it follows from the definitions of the units of quantity that

$$1 \text{ coulomb} \equiv 0.1 \text{ abcoulomb.}$$

The relation of the abcoulomb to the statcoulomb is determined experimentally and is

$$1 \text{ abcoulomb} \equiv 3 \times 10^{10} \text{ statcoulombs.}$$

From which it follows that

$$\mathbf{1 \text{ coulomb} \equiv 3 \times 10^{9} \text{ statcoulombs.}}$$

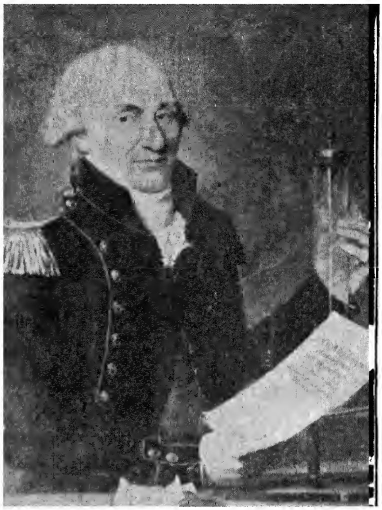
The factor 3×10^{10} is the velocity of light in centimeters per second and is usually represented by the letter c. The equality of this factor c to the velocity of light is not accidental but due to a fundamental relationship between electric and magnetic fields and the velocity of propagation of electromagnetic pulses.

Since one elemental unit (electron or proton) has a charge of 4.770×10^{-10} statcoulombs (Art. III-1), it follows that

$$1 \text{ statcoulomb} \equiv 2.096 \times 10^{9} \text{ elemental units.}$$
$$1 \text{ coulomb} \equiv 6.289 \times 10^{18} \text{ elemental units.}$$

The letters Q, Q', and Q'' represent the number of units of quantity of electricity in the practical, the electromagnetic, and the electrostatic systems, respectively; and I, I', and I'' represent the number of units of current. In this text the unprimed letters always represent practical units; the primed letters, electromagnetic units; and the double primed, electrostatic units. This priming of the symbols is not generally employed but is found to be useful and will be strictly adhered to in this text. What system of units is used in any equation, then, is never in doubt, and no statement concerning the system of units used is

To understand the relation between the *magnitudes* of the units in the different systems and of the *numbers* of these units in any given quantity, it must be noted that the *number* of units in any quantity *varies inversely* as the *magnitude* of the units. In any equation, for example, the letters Q, Q', and Q'' represent *numbers* and *not* the *magnitudes* of the units in the three systems.



Charles Augustin Coulomb (1736–1806), French physicist; proved conclusively that the law of inverse squares holds for electric and magnetic attractions and repulsions (1784), that electrostatic forces vary as the product of the quantities of electricity, and that electric charges exist only on the surfaces of conductors. The unit of quantity of electricity, *the coulomb*, is named in his honor.

If a given quantity has Q' abcoulombs of electricity, it has ten times that number ($10Q'$) of coulombs, because the coulombs are the smaller units. Hence

$$Q = 10Q'.$$

This algebraic equation reads, "The *number*, Q, of coulombs equals ten times the given *number*, Q', of abcoulombs."

The relations of the magnitudes of the units of current and of quantity are necessarily similar. Placing them together,

$$1 \text{ coulomb} \equiv 0.1 \text{ abcoulomb} \equiv 3 \times 10^9 \text{ statcoulombs.}$$
$$1 \text{ ampere} \equiv 0.1 \text{ abampere} \equiv 3 \times 10^9 \text{ statamperes.}$$

The relation between the *numbers* of the units in any given quantity of electricity or of current then is

$$Q = 10Q' = \frac{Q''}{3 \times 10^9}. \tag{3}$$

$$I = 10I' = \frac{I''}{3 \times 10^9}. \tag{4}$$

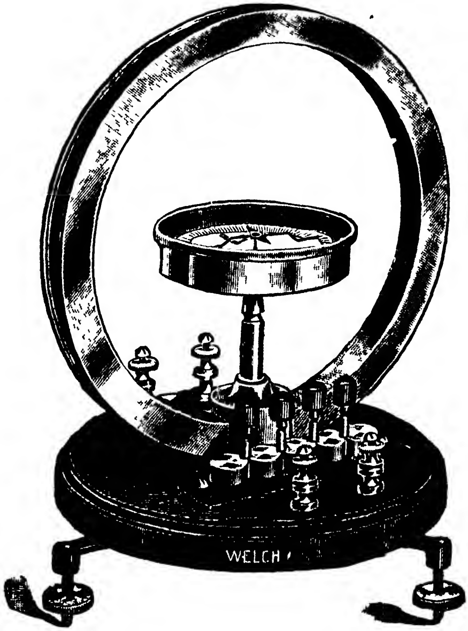


Fig. 5—Tangent galvanometer.

5. Measurement of an Electric Current by Means of the Tangent Galvanometer.—The measurement of an electric current directly in terms of the defining quantities is called an *absolute method of measuring the current.* The most direct of such methods employs the tangent galvanometer (Fig. 5). This instrument consists of a short magnetic needle at the center of a vertical coil of large diameter having N turns of wire. When in use, the coil is placed so that the plane of its loops is in the magnetic meridian, *i.e.*, in the plane of the magnetic needle NS, Fig. 6, before the coil is energized. The electron flow in the coil is a

magnetic loop whose magnetic field is perpendicular to the plane of the coil and therefore, in the adjusted position, is also perpendicular to the direction of the horizontal component of the earth's magnetic field.

The needle is short for the same reason as when measuring H (Art. VII-9) in order that the direction and the intensity of the magnetic field produced by the current may be practically the same at the magnetic poles of the needle as it is at the center of the coil Figure 7 shows the needle, enlarged and viewed from the top, as it is after being deflected by the current through the angle θ.

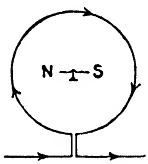


FIG. 6.—Diagram of tangent galvanometer.

At the center of the coil of N turns the intensity of the magnetic field, Eq. (1), produced by the current is

$$\mathcal{H} = \frac{lI'}{r^2} = \frac{2\pi r N I'}{r^2} = \frac{2\pi N I'}{r} \text{ oersteds.}$$

Each pole of the needle is then acted on by the field of this current with a force of $\mathcal{H}m$ dynes at right angles to the force Hm

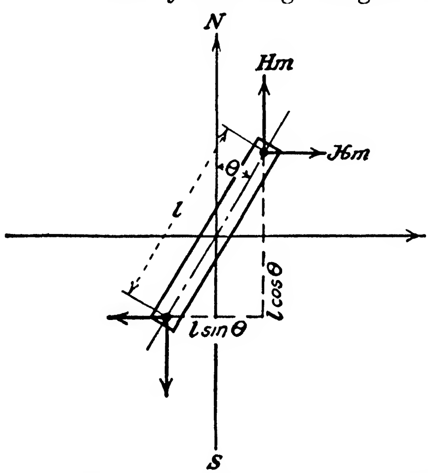


FIG. 7.—Forces acting on the needle of a tangent galvanometer.

dynes due to the horizontal component of the earth's magnetic field. The torques due to these forces oppose each other and are equal when the deflected needle is at rest. At equilibrium the torque,

$$T = \mathcal{H}ml \cos \theta = Hml \sin \theta,$$

from which, together with the preceding equation,

$$\mathcal{H} = \frac{2\pi NI'}{r} = H \tan \theta,$$

Then

$$I' = \frac{Hr}{2\pi N} \tan \theta$$

or

$$I = 10I' = \frac{5Hr}{\pi N} \tan \theta = K \tan \theta,$$

in which

$$K = \frac{5Hr}{\pi N}.$$

The constant K can be calculated for any position in which H has been determined; the angle θ may be measured on a circular scale by means of a long, light aluminum pointer attached at right angles to the magnetic needle.

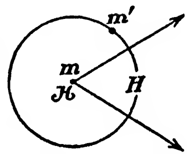


FIG. 8.—An energized loop in the magnetic field of the point-pole m.

In practice, currents are more conveniently measured by means of an ammeter (see cuts of ammeters with Art. XXI–4), the tangent galvanometer being important only because it measures the current in terms which define it and therefore is a basic instrument for measuring the current.

6. Force Acting on a Wire Carrying a Current in a Magnetic Field.—Imagine a magnetic point-pole of m units at the center of a loop of wire, Fig. 8, which is carrying a current I', and let r be the radius of the loop. The current in the loop then is in a magnetic field H due to the point-pole m. The intensity of this field is measured by the force, f_1, which the point-pole m would exert there on a unit point-pole. Then at the loop

$$H = f_1 = \frac{mm_1}{d^2} = \frac{m \times 1}{d^2} = \frac{m}{r^2} \text{ oersteds.}$$

The wire of the loop is everywhere perpendicular to the lines of force of this field.

At the center of the loop in open space or in air, the intensity of the magnetic field due to the current in the loop, Eq. (1a), is

$$\mathcal{H} = \frac{I'l}{r^2} \text{ oersteds,}$$

so that the point-pole m at the center feels a force

$$f = \mathcal{H}m = I'l\frac{m}{r^2} = HlI' \text{ dynes.}$$

Since the cause of this force is the current in the loop, the loop itself feels an equal force in the reverse direction (Newton's third law of motion). The loop then feels a force

$$f = [BlI'] \text{ dynes,} \tag{5a}$$

where B is the flux density of the magnetic field (Art. VII–4) within which the current (with its superposing field) is flowing. The current and its magnetic field are in a field whose density is B. It is becoming the practice always to use B in this equation in place of the numerically equal field intensity H.

Equation (5) applies to a conductor of any shape provided it is perpendicular at every point to the magnetic lines of force. If the conductor is not perpendicular to the lines, l represents the perpendicular component of the length. The brackets indicate that the component of B at right angles to I must be taken. *The vectors representing f, B, and I' are mutually perpendicular.*

Summarizing and combining with Law A_2:

A conductor carrying a current I′ at right angles to the magnetic lines of force feels a force of [BlI′] dynes acting at right angles to the lines from the strengthened toward the weakened part of the field. (5*b*)

The field due to the current in any short element of length of the wire exerts no force tending to displace the element as a whole, so that the force acting on this element of length can be calculated solely on the basis of the external field and the field due to the remainder of the wire. If the wire is straight, no part of it exerts a force on any other part. The torque tending to turn any coil carrying a current in a magnetic field is not affected by the action of one part of the coil on another.

7. The Amount Each Moving Electron Contributes to the Total Current in Any Given Length of a Circuit.—Let A, Fig. 9, be a section of a conductor of length l containing n electrons, each having a charge e' and moving with the average velocity v to the right. In the time $t = l/v$ all the electrons in section A move

into section B and other electrons occupy the section A. During that time every perpendicular plane in the section A, therefore, is crossed by n electrons, the number of them contained at any one time in the section A. Therefore the quantity of electricity which passes through every plane in the time $t = l/v$ is the quantity contained in the section A; *i.e.*,

$$Q' = ne'.$$

But

$$I' = \frac{Q'}{t} = \frac{ne'}{t} = \frac{ne'v}{l}.$$

And therefore the amount each of the electrons contributes to the current in the section l is

$$i' = \frac{I'}{n} = \frac{e'v}{l} \text{ abamperes.} \qquad (6)$$

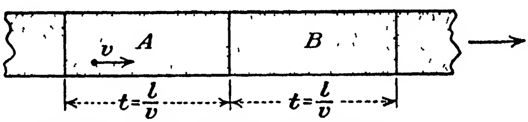


FIG. 9.—The part each moving electron contributes to the current in the section A of a conductor.

8. Force Acting on Each Moving Electron or Ion in a Magnetic Field.—The force acting on a current in a magnetic field, Eq. (5*a*), is at right angles to the field and of magnitude

$$f = [BlI'] \text{ dynes.}$$

But if i' is the contribution of each electron to this flow (Art. 7), $I' = ni'$. Then since $i' = e'v/l$ the force acting on the n moving electrons in any part of a conductor of length l is at right angles to the field and of magnitude

$$f_n = BlI' = Blni' = Bln\frac{e'v}{l} = nBe'v.$$

And therefore the force acting on each electron is

$$f_1 = \frac{f_n}{n} = Be'v = \frac{Be''v}{c} \text{ dynes,} \qquad (7)$$

where $c = 3 \times 10^{10}$.

When an electric charge, e'', is moving in a space in which an electric field and a magnetic field are superposed, the force acing on it, from Eqs. 7 and III-2, is

$$f_1 = \left[F''e'' + \frac{Be''v}{c}\right] = e''\left[F'' + \frac{[vB]}{c}\right] \text{ dynes,} \qquad (8)$$

in which e'' and F'' represent the magnitude of the moving charge and the intensity of the electric field, respectively. The brackets again indicate that the component of the velocity v at right angles to B must be taken.

Questions

1. Define positive ion; negative ion.

2. If two plates oppositely charged are placed in an electrolytic solution and any loss of charge is continually replaced, describe the motion of the ions and show that the magnetic fields produced by their motion have the same direction.

3. Explain how it is possible for a magnetic field to exist about the electrolyte without any apparent moving electric field.

4. Explain what is believed to be taking place in a metallic conductor when it is energized by an electric current.

5. Define "direction of current" and "direction of flow."

6. Define the electromagnetic, the practical, and the electrostatic units of current and of quantity of electricity Name the units in each system.

7. Why is the quantity defined in terms of the current in two of the systems, and the current in terms of the quantity in the third?

8. How many statcoulombs in 1 abcoulomb? In 1 coulomb?

9. How many electrons in 1 statcoulomb? In 1 coulomb?

10. Explain why the factor 10 appears as it does in the relation $Q = 10Q'$.

11. Give and explain the algebraic relation between Q, Q', and Q'' and that between I, I', and I''.

12. Derive the expression for the intensity of the magnetic field at the center of a coil of wire of N turns when the current is I'.

13. Why is the magnetic needle of a tangent galvanometer made short compared with the diameter of the coil?

14. Why is the plane of the coil of the tangent galvanometer placed in the magnetic meridian?

15. Derive the expressions for the two torques acting on the needle of a tangent galvanometer and the expression for the current in amperes.

16. What is meant by an absolute method of measurement? Is the measurement of a current by a tangent galvanometer such a method? Explain.

17. Derive the expression for the force acting on an energized conductor in a magnetic field.

18. Derive the expression for the amount each moving electron contributes to the current in any given length of wire.

19. Derive the expression for the force acting on an electron moving in a magnetic field.

20. Give the expression for the force acting on an electron or an electric charge moving in superposed electric and magnetic fields.

Problems

1. The population of the earth is about 1.75×10^9. How many times larger would the population of the earth have to be in order that (*a*) one person would bear the same ratio to the population that 1 statcoulomb bears to 1 abcoulomb? (*b*) To 1 coulomb?

2. If 1 sec of time represents an electron, (*a*) how many years represent the number of electrons in a statcoulomb? (*b*) In a coulomb? (One year $= 31.6 \times 10^6$ sec.)

3. (*a*) How many coulombs of electricity flow through any section of a wire in 2 min when the current is 10 amp? (*b*) How many abcoulombs? (*c*) How many statcoulombs?

4. (*a*) What is the current in amperes when 20 coulombs flow through every section of a wire in 4 sec? (*b*) In abamperes? (*c*) In statamperes?

5. What is the intensity of the magnetic field at the center of a coil of 5 turns and 40-cm radius carrying a current of 20 amp?

6. What must be the radius of a coil of 2 loops of wire so that a current of 5 amp may produce a magnetic field of 0.4 oersteds at the center?

7. What is the constant of a tangent galvanometer whose coil of 8 turns and 30-cm radius is in a magnetic field whose horizontal component has an intensity of 0 16 oersteds?

8. What is the strength of the current, in amperes, when it produces a deflection of 30° on the foregoing tangent galvanometer?

9. With what force, in grams, will a wire having a length of 10 cm and carrying a current of 20 amp be acted on when it is placed at right angles to the lines of force in a magnetic field whose flux density is 16,000 gauss?

10. How much, in amperes, does an electron moving with a velocity of 0.2 cm/sec contribute to the current in a wire 20 cm in length?

11. With what force does a magnetic field whose flux density is 1,000 gauss act on an electron moving at right angles to the lines of force with a velocity of 3×10^9 cm/sec?

Experiments

1. A magnetic needle deflected by the magnetic field produced at the center of a loop of wire in which an electric current is flowing.

2. Model of electrons with their electric fields moving in a loop and showing the motions of the superposed fields at the center of the loop.

3. Tangent galvanometer.

4. A part of the iron core of an electromagnet projects some distance into a glass bulb from which the air is exhausted until an induction coil produces a luminous stream of ions parallel to the length of the core. When the electromagnet is energized, the luminous stream rotates, about the pole, showing that individual moving charges feel a force in a magnetic field.

5. A wire bent into the form of a U is inverted and suspended at its center over one pole of a magnet and has its extremities dipping into a circular trough of mercury. When a current is passed from the center down both of the prongs, the conditions of Exp. 4 are reproduced in each prong except that the moving charges are within the wire which is forced to move with them.

6. A conducting flexible strip suspended along a projecting core of an electromagnet winds about the core when the electromagnet and strip are energized.

CHAPTER IX

UNITS OF POTENTIAL DIFFERENCE, RESISTANCE, WORK, AND POWER

1. The Quantity of Electricity Transferred between Any Two Planes in an Electric Circuit.—It follows from the definition of the unit quantity of electricity, the coulomb (Art. VIII–3), that the quantity which passes any cross-sectional plane of the circuit is

$$Q = It,$$

from which

$$I = \frac{Q}{t}.$$

Imagine the electron flow to be from a to d, Fig. 1, and the quantity of electricity Q to pass through the plane a in any given time. The same quantity passes in the same time through any other plane, such as b, c, or d.

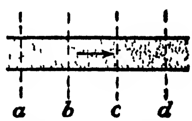


FIG. 1.—Same number of electrons pass every cross section of the wire when a steady current is flowing.

If the same quantity, Q, be imagined to be concentrated at a point and transferred from a to d, it would likewise pass through every such plane. The same number of electrons are transferred from plane to plane through the same potential difference whether the same quantity be considered as a point charge or as the quantity passing every such plane in a continuous stream. In each case the energy expended (Art. 4) is QE joules. This fact may be illustrated by a grain-conveyor belt (Fig. 2); the number of bushels of grain elevated to any height h in a given time is that which passes in the same time through every plane which is perpendicular to the belt, such as the planes represented by the lines m and n. If 1 bu of grain passes the plane m in 1 sec, 1 bu leaves the belt A and 1 bu falls upon the belt C. One bushel of grain, then, has been elevated through the height h

in 1 sec. The quantity of electricity Q, therefore, is said to have passed through (or around) any circuit if such a quantity has moved through every plane of that circuit, regardless of how small the actual displacement of the individual electrons may be.

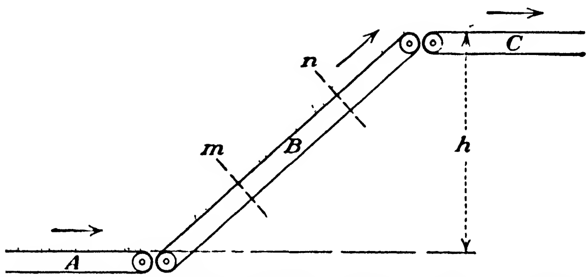


FIG. 2.—Conveyor belt B elevating grain from belt A to belt C.

Whenever a current is flowing, a quantity Q (= It) passes every plane perpendicular to the conductor in the time t. This quantity, for the purpose of calculating the expenditure of energy, may be considered the quantity that has been transferred through the potential difference between any two planes under consideration. **(1)**

2. Electron Flow and Potential Difference in an Electric Circuit.—The portion AB of the electric circuit, Fig. 3, is assumed to contain an e.m.f. (Arts. 4, XIII-4) which gives energy to the

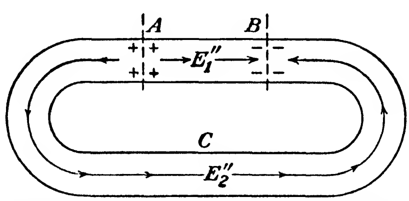


FIG. 3.—An electric circuit showing the displacement of electrons and the electric fields produced by an electromotive force.

free electrons by forcing them to the right. The result of this action is a continuous flow of electrons around the whole circuit. The free electrons may be treated like an almost incompressible fluid, so that in any given time the same number of them pass any given section of the circuit regardless of whether or not the e.m.f. exists in that section. This action in a circuit may be considered from three points of view:

1. The electrons forced to the right by the e.m.f. repel those ahead of them. A force, in this manner, is transmitted by the succession of displaced electrons around the whole circuit, much as it would be in a row of ivory balls filling a circular trough if in some section of it a force were being applied that continually pushed the balls in one direction.

2. The side of the circuit toward which the electrons are being forced becomes negatively charged because of slight compression of the crowded electrons. The side from which they are being forced becomes positively charged. This continuous displacing of the charges maintains the electric fields F_1'' and F_2'' (represented in Fig. 3 by the potential differences E_1'' and E_2''), one of which opposes the displacing of the free electrons by the e.m.f. in the section AB, and the other forces the electrons to flow through the section BCA.

3. The electric fields established by the electron displacement span the equal potential differences E_1'' and E_2''. In the section AB the e.m.f. is forcing electrons against the action of the electric field, *i.e.*, from points of higher to points of lower potential, and thereby is giving them potential energy. In the section BCA the free electrons move from points of lower to points of higher potential and thereby are giving up the potential energy they received in the section AB of the circuit.

3. Electric Energy Changed into Heat.—If an e.m.f. were supplying energy to the electrons in the section AB of the circuit, Fig. 3, and the accelerated electrons retained the energy they received, their velocity, and therefore the current, would continually increase. This condition, however, does not exist in conductors because the electrons are continually losing energy by impacts with atoms; and in the case of a steady current they give up as much energy as they receive. All the energy which is being supplied by the e.m.f. in any part of a circuit, then, is simultaneously being transformed into heat energy which is distributed around the whole of the circuit.

The blocks of Fig. 4 illustrate by means of a mechanical analogue the energy transfer through the free electrons in the circuit of Fig. 3. On the incline AB energy is being supplied to the blocks from some external source (e.m.f.), and each block is urged by a force f throughout the whole length of the incline

and is acquiring potential energy. The force of gravity (electric field) forces the blocks to slide down the curved incline BCA and thereby to give up the potential energy they received while they were being raised up the incline AB. When the blocks are moving with uniform speed, all the energy they receive on the incline AB is being transformed into heat throughout the whole length of the circuit including both of the inclines.

4. E.M.F.—Abvolt—Volt.—The amount of potential energy given to the electrons while they are being forced through the section AB, Fig. 3, depends on the magnitude of the potential

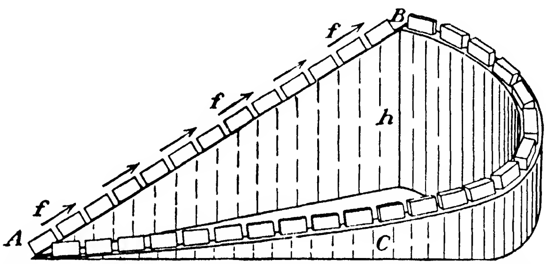


FIG. 4—Mechanical analogue of an electric circuit (applicable only to one given velocity of the blocks, if the velocity changed the height h which corresponds to E'' would have to change).

difference, $E_1'' = E_2''$, which exists between the planes A and B. The magnitude of this potential energy (Art. III-3) is

$$W_1 = E_1''Q'' = E_1''I''t \text{ ergs.}$$

While the electrons are being forced through this section AB, they collide with the atoms of the conductor and thereby heat it. Some energy, therefore, is being converted there into heat at the time potential energy is being given the electrons. These energy transformations are similar to those associated with a mass being forced up an inclined plane. The mass is receiving potential energy in being raised against the force of gravity while at the same time energy is expended in the generation of heat by friction between the mass and the plane. The energy converted into heat in the section AB may be expressed by

$$W_0 = E_0''Q'' = E_0''I''t \text{ ergs,}$$

where E_0'' represents the energy converted into heat in forcing 1 statcoulomb through that section. The term E_0'' then is so

defined that it may be treated as a virtual potential difference (usually called *RI drop*).

The term E_0 must not be confused with the term E_1; one is the measure of the energy converted into heat in forcing each statcoulomb through the section AB, and the other the measure of the potential energy given each statcoulomb in moving it through the length of that section; the magnitude of E_1 is necessarily that of E_2 but, except in an exceptional case, is not that of E_0.

Count Alessandro Volta (1745–1827), professor at Como and at Pavia, Italy; inventor of the electroscope, the electrophorus (1775), and the voltaic cell (1799). The unit of electromotive force and of potential difference, the *volt*, is named in his honor.

The total energy supplied to, and therefore also that which can be expended in, the circuit is

$$W = W_1 + W_0 = (E_1'' + E_0'')I''t = (E_2'' + E_0'')I''t = E''I''t \text{ ergs,} \tag{2}$$

where E'' represents the total energy expended on each statcoulomb through the action of the *e.m.f.* and therefore also the energy converted into heat by each statcoulomb in its motion once around the whole circuit. The potential energy given each statcoulomb in "raising" it through the potential difference E_1'' is all converted into heat when the statcoulomb "falls" through the equal potential difference E_2''. This energy, $E'' = E_2'' + E_0''$, is, by convention, the measure of the e.m.f. in the circuit. It is the measure of the total potential difference which the e.m.f. can establish on open circuit (Art. XIII–3) and also that of the

total potential difference in a closed circuit when the virtual potential difference E_0'' is treated as though it were an actual potential difference. Electromotive force and potential difference therefore can be and are measured in terms of a common unit.

In the study of electricity in motion the electrostatic unit of potential difference is almost always replaced by the electromagnetic unit or by the practical unit.

An electromagnetic unit of potential difference, the abvolt, exists between two points when 1 erg of energy is expended in the transfer of each abcoulomb of electricity from one of the points to the other.

Then

$$W = E'Q' = E'I't \text{ ergs.} \tag{3}$$

The potential difference between the electrodes of a lead storage cell is about 2×10^8 abvolts. Since this is too large a number for practical purposes, the *practical unit* of potential difference, *the volt,* is arbitrarily made equal to 10^8 abvolts. This means that 10^8 ergs of energy are expended when 1 abcoulomb of electricity moves through a potential difference of 1 volt. From this it follows, since a coulomb is equal to 0.1 abcoulomb, that 1 *joule* (10^7 *ergs*) *of energy is expended in the moving of* 1 *coulomb of electricity through a potential difference of* 1 *volt.* If the potential difference between two points is E volts, E joules of energy are expended in the moving of 1 coulomb.

Then

$$W_J = EQ = EIt \text{ joules.} \tag{4}$$

The practical unit of potential difference, the volt, is taken to be 10^8 abvolts and exists between two points when 1 joule of energy is expended in the moving of 1 coulomb of electricity from one of the points to the other.

(1 joule = 10.20 kg-cm = 0.7376 ft-lb.)

Electromotive force is that which imparts energy to an electron flow or that which displaces the flow electrons in a conductor. It is conveniently measured in the same units as potential difference.

5. Relation between the Magnitudes of the Units of Potential Difference in the Three Systems.—Since the abcoulomb is equal to 3×10^{10} statcoulombs, 3×10^{10} ergs are required to move an abcoulomb through a potential difference of 1 statvolt. There are then 3×10^{10} abvolts in 1 statvolt. Since there are 10^8 abvolts in 1 volt, 1 statvolt is equal to 300 volts.

This is conveniently shown in Fig. 5 in which the potential difference between two charged plates is represented as being 1 statvolt. The same potential difference then, when measured in the electromagnetic system. is 3×10^{10} abvolts, and in the practical system 300 volts.

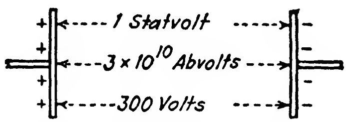


FIG. 5—Number of units of potential difference in each of the three systems for a given potential difference of 1 statvolt.

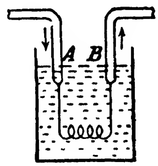


FIG. 6—Electric calorimeter.

The relation of the magnitudes of the units, then, is

$$1 \text{ volt} \equiv 10^8 \text{ abvolts} \equiv \frac{1}{300} \text{ statvolts},$$

and the relation between the *number* of units in any given potential difference is

$$E = \frac{E'}{10^8} = 300E''. \tag{5}$$

The algebraic equation reads, "The number of volts $E = 10^{-8}$ times the number of abvolts $E' = 300$ times the number of statvolts E''.

Illustrative Examples.

1. How many volts in 7 statvolts?

Solution: $E = 300E'' = 300 \times 7 = 2{,}100$ volts.

2. How many volts in 4,000 abvolts?

Solution: $E = \dfrac{E'}{10^8} = \dfrac{4{,}000}{10^8} = 0.00004$ volts.

6. Measurement of Potential Difference by the Calorimeter Method.—In the part of the circuit immersed in water, Fig. 6,

electrons flowing from A to B are falling through a potential difference E; hence the part of their potential energy which is transformed into heat in that section, Eq. (4), is

$$W_J = EQ = EIt \text{ joules.}$$

The amount of this energy can be determined from the change in the temperature of the water and the calorimeter. The expression for the amount of heat energy is

$$W_J = J_J H \text{ joules,}$$

in which J_J is the mechanical equivalent of heat in joules per calorie, and H the number of calories of heat generated

Then

$$W_J = EIt = J_J H \text{ joules,}$$

from which the magnitude of the potential difference,

$$E = \frac{J_J H}{It} \text{ volts.} \tag{6}$$

The value of J_J varies somewhat with the temperature of the water and at 15°C. = 4.185 joules ≡ 42.68 kg-cm. It should be noted that the equation expresses the converted energy in joules per coulomb, which conforms with the definition of potential difference (Art. 4), and that the method is one of the absolute methods (Art. VIII–5) applicable to such measurements.

7. Ohm's Law.—In determining the potential difference between two points on a wire for different currents, it is found experimentally that under normal conditions

$$E \propto I.$$

Then,

$$E = RI. \tag{7}$$

The constant R is called the *resistance* of that part of the circuit between the two points; and the relationship expressed by the equation is called *Ohm's law* after its discoverer. The constant is called resistance because the larger its magnitude, the smaller is the current produced by a given potential difference. It conveys an idea of resistance to the electron flow. It follows from the equation and from the fact (Art. VIII–2) that the

velocity v of the conduction electrons is proportional to the current that

$$E \propto I \propto v \propto \frac{E}{l} \propto F'',$$

where E/l is the potential gradient (Art. III–6) and F'' the intensity of the electric field within that part of the conductor in which the established electric field alone is acting on the electrons.

Ohm's law simply states that

Georg Simon Ohm (1787–1854), a German physicist, discoverer of Ohm's law (1826). The practical unit of resistance, the *ohm*, is named in his honor.

The strength of an electric current is proportional to the potential difference; from which it follows that the velocity of the electron flow is proportional to the potential gradient. **(8)**

The reciprocal of resistance is called *conductance.*

8. Units of Resistance.—The derived units, such as that of resistance, are defined in terms of the primary or fundamental units through the applicable *defining equation.* The appropriate defining equation for resistance is Ohm's law, Eq. (7); *i.e.*, $E = RI$. When E and I are both unity, R is unity; from which it follows that

An ohm is that resistance through which a potential difference of 1 volt causes a current of 1 ampere to flow.

The *abohm* and the *statohm* are similarly defined in terms of the primary units in their respective systems.

A wire, for example, has a resistance of 10 ohms when a potential difference of 10 volts forces a current of 1 amp to flow through the wire. The resistance, then, gives the potential difference required to force a current of 1 amp through the conductor under consideration.

9. Work and Power.—The expression $W_J = EQ = EIt$ joules may, for convenience, be written in other forms. Since $E = RI$, the energy exper

W es. (9)

The form W_J and the heating of a conductor l

The unit of p eration, supply, or expenditure c joule/sec.

Then from th

. (10)

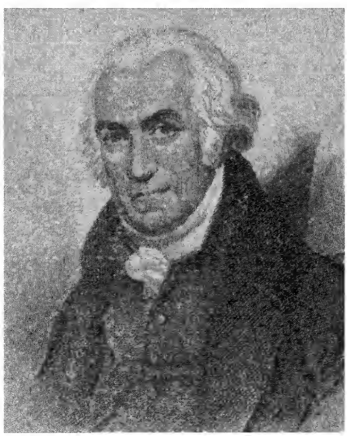
The power is eing generated, supplied, or exp s/sec; *i.e.*, it is equal to 1,000 w

$$P_{kw} = \frac{EI}{1{,}000} = \frac{RI^2}{1{,}000} = \frac{E^2}{1{,}000R} \text{ kw.}$$

A *kilowatt-hour* is the amount of energy generated, supplied, or expended in an hour when the power is 1 kw, *i.e.*, 1,000 joules/sec for 1 hr. This is 3.6×10^6 joules or 2.655×10^6 ft-lb of energy. Similarly a *watt-second* is the energy supplied or expended in a second when the power is 1 watt; *i.e.*, a watt-second is a joule of energy.

The equation $P = RI^2$ shows that the power expended in any given resistance varies as the square of the current. This is also shown from the equation $P = EI$, in which the strength of the current I is doubled only when the potential difference E is doubled.

James Prescott Joule (1818–1889), English physicist, discoverer of the law of heating in a conductor by an electric current; proved experimentally (1839–1843) the law of conservation of energy. A unit of work, the *joule*, is named in his honor



James Watt; (1736–1819), instrument maker at the University of Glasgow, Scotland; inventor of the practical steam engine (1765–1769), the heart of modern industrial progress. The electric unit of power, the *watt*, is named in his honor.

When electric energy sells at the rate r per kilowatt-hour, the total cost for t_h hr. during which the energy is being supplied is

$$\text{Cost} = P_{kw} t_h r. \tag{11}$$

Questions

1. Explain $Q = It$.

2. Explain why the work done in maintaining a current between two given points in a circuit is measured by the product of the potential difference between the points and the quantity of electricity that has passed through any plane in the circuit. Illustrate this by means of a conveyor belt.

3. Show that, if in some part of a circuit electrons are being urged in one direction along the circuit, there must necessarily be a flow of electrons along the whole circuit. Use the concepts of potential difference and electric field as well as the law of attraction and repulsion between electric charges.

4. Explain why work is done in moving electrons along a circuit, and why, in a simple circuit, all the energy expended is changed into heat.

5. Explain, illustrating by means of a mechanical analogue, the energy transformations in a simple electric circuit.

6. Define electromotive force.

7. Define an abvolt, and state how $W = E'Q' = E'I't$ ergs.

8. Show that a joule of energy is liberated or expended in the moving of 1 coulomb of electricity through a potential difference of 1 volt.

9. Define volt in terms of abvolts and in terms of energy liberated or expended.

10. Show that $W_J = EQ$ joules and that $EQ = J_J H$.

11. Show how E is measured by the calorimeter method.

12. Show how Ohm's law is proved experimentally.

13. What is a defining equation? Give the defining equation for resistance.

14. Define resistance; an ohm; a resistance of 10 ohms.

15. Explain how it follows from Ohm's law that the velocity of the conduction electrons in any given part of a circuit is proportional to the potential difference.

16. Write and explain the various forms of the equation for the energy supplied to or expended in any part of an electric circuit.

17. Define power; watt; kilowatt; kilowatt-hour; watt-second.

18. Write the various forms of the equation for the power in watts and in kilowatts expended in a circuit.

19. Show that the power expended between any two given planes in a circuit varies as the square of the current.

Problems

1. How many coulombs of electricity pass any given plane in a circuit in 1 hr when the current is 5 amperes?

2. How many coulombs of electricity may be considered as transferred in 30 min from any point *a* to any point *b* in the circuit when the current is 3 amp?

3. How much work is required (*a*) to move 20 abcoulombs of electricity through a potential difference of 7 abvolts? (*b*) 20 statcoulombs through 7 statvolts? (*c*) 20 coulombs through 7 volts?

4. How much energy is expended in maintaining a current of 10 amp for 5 min when the potential difference is 8 volts?

5. A current of 4 amp generates 10,000 calories of heat in 2 min (*a*) What is the potential difference in volts? (*b*) In abvolts? (*c*) In statvolts? (J_J = 4.18 joules per calorie).

6. What power is being expended when a current of 10 amp is maintained through a resistance of 20 ohms?

7. (*a*) What is the current flowing through a 60-watt lamp when connected to mains having a potential difference of 110 volts? (*b*) What is the resistance of the lamp?

8. When electric energy sells for 3 cts/kw-hr, how much does it cost to operate a motor for 6 hr when it is taking 12 amp from 110-volt mains?

9. A wire whose resistance is 10 ohms is immersed in 10 kg of water and is connected to 110-volt direct-current mains. When electric energy sells for 3 cts/kw-hr, (*a*) how long does it take to heat the water from 20 to 100°C? (*b*) How much does it cost?

10. When electric energy sells for 3 cts/kw-hr, how high in miles will 3 cts worth of electricity raise a man weighing 70 kg (154.3 lb)? (1 mile = 1.6093 km.)

Experiments

1. Long iron wire heated to redness by an electric current.

2. Electric calorimeter.

3. Magnitude of the energy represented by a joule and by a calorie shown by raising a kilogram weight through 10.2 and 42.7 cm.

CHAPTER X

RESISTANCE

1. Resistance—Conductivity.—If the potential difference between the same two points on a conductor when different currents are flowing through it is measured, it is found (Art. IX–7) that

$$E = RI.$$

From this, because the current is proportional to the velocity of the conduction electrons (Art. VIII–2), it follows, as already stated, that in any particular conductor the electron velocity is proportional to the potential gradient.

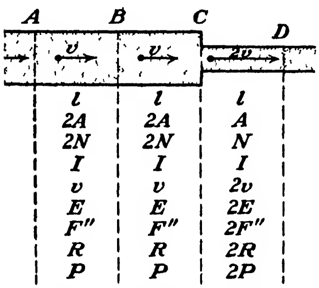


FIG. 1.—Relation of various quantities in different parts of equal length in a homogeneous conductor. The letters l, A, N, I, v, E, F'', R, and P represent length, cross-sectional area, number of conduction electrons per unit length, current, velocity of conduction electrons, potential difference, intensity of the electric field, resistance, and power expended in the respective sections.

Let Fig. 1 represent a part of an electric circuit which has the cross section of the part AC twice that of the part CD and its free-electron density the same throughout. The current everywhere is of equal intensity, for the number of electrons passing through any given plane must be that passing through any other plane (in a given time). The average velocity of the electrons in CD, then, must be twice that of those in AC. Because potential gradient (electric field) is proportional to the electron velocity in any given section, the potential difference in CD is twice that in the section of equal length BC. Since the current strength is the same in both sections, it follows from $E = RI$ (Ohm's law) that $E \propto R$. But E in CD is twice that in BC; therefore the resistance of CD is twice that of the thicker section BC of equal length. Therefore R varies inversely as the cross-sectional area A, *i.e.*,

$$R \propto \frac{1}{A}.$$

If the section AC is imagined to be composed of two parts of equal length, AB and BC, the potential difference across AC is twice that across AB. The resistance of section AC then is twice that of AB; *i.e.*,

$$R \propto l.$$

Combining,

$$R \propto \frac{l}{A}.$$

Therefore the property of a conductor for limiting the current, expressed quantitatively by the resistance, is

$$R = \frac{\rho l}{A}, \tag{1}$$

where the constant ρ is the *specific resistance* or *resistivity.* A study of the equation shows that ρ is the resistance between opposite faces of a centimeter cube of the material. The magnitude of ρ varies with the material of the conductor and depends on the number of conduction electrons and on their mean free path. The following table gives the specific resistance ρ and the temperature coefficient α (Art. 2) for the resistance of a few substances:

	ρ at 20°C	α per 1°C
Silver	$1\ 59 \times 10^{-6}$	$38. \times 10^{-4}$
Copper	$1\ 77 \times 10^{-6}$	38×10^{-4}
Gold	$2\ 44 \times 10^{-6}$	$34. \times 10^{-4}$
Aluminum	$2\ 82 \times 10^{-6}$	39×10^{-4}
Nickel	$7\ 8 \times 10^{-6}$	$60. \times 10^{-4}$
Iron	$10\ 0 \times 10^{-6}$	$50. \times 10^{-4}$
Manganin (alloy)	$44\ 5 \times 10^{-6}$	$0\ 01 \times 10^{-4}$
Constantan (alloy)	$49\ 0 \times 10^{-6}$	$0\ 04 \times 10^{-4}$
Advance (alloy)	$50\ 0 \times 10^{-6}$	$0\ 06 \times 10^{-4}$
Nichrome I (alloy)	$99\ 6 \times 10^{-6}$	$4\ 4 \times 10^{-4}$

Manganin (Art. 7) is an alloy used for resistance standards. Advance and constantan are the "same" copper-nickel alloy employed with copper in thermocouples. Nichrome is a nickel-chromium resistance alloy used in electric heating devices.

Because the resistance of the thinner section is twice that of the thicker section, Fig. 1, the power expended (converted into heat) in the thinner section is twice that expended in the thicker section. The same current is flowing through twice the potential difference.

The reciprocal of resistance is called *conductance* and that of resistivity is called *conductivity*.

2. Relation of Resistance to the Number of Conduction Electrons and to Their Mean Free Path—Temperature Coefficient.—In the conductor $ABCD$, Fig. 2, the parts CD and BC are of the same material, the cross section of BC being twice that of CD. The part AB has the same cross section as BC but only one-half the number of conduction electrons per cubic centimeter. Assume first that the electrons in both materials have the same mean free path. When a current flows through the three parts, the same quantity of electricity must cross any plane in one that passes any such plane in either of the other two. In order that this may be the case, the larger number of electrons in BC move with one-half the average velocity of the smaller number in each of the other two parts; therefore, the potential difference in the conductor BC is one-half that in either AB or CD. The resistance of BC, then, is one-half that of either AB or CD.

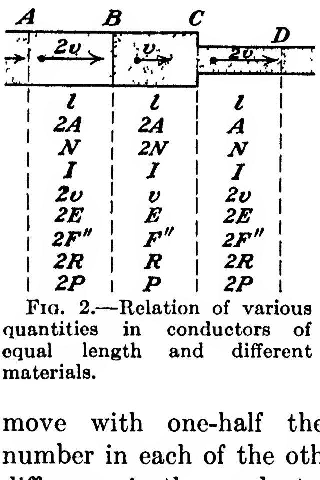


FIG. 2.—Relation of various quantities in conductors of equal length and different materials.

The part AB, having the smaller number of electrons per unit length, is electrically equivalent to the part CD because the number of conduction electrons in the two sections of equal length is the same. It follows that in different materials the resistivity varies inversely as the number of free electrons per unit volume.

It can be seen that, when the mean free path of the conduction electrons is longer, the same potential difference produces a larger electron velocity between impacts and therefore a larger current. From this and other considerations it can be shown

(not readily) that, when the mean free paths of the conduction electrons in two materials differ, the resistivity of the materials varies inversely as the length, λ, of the mean free path. The length of the mean free path is not accurately known but depends on the material and on temperature. At ordinary temperatures it is probably not much greater than the distance between adjacent atoms of the conductor. Variations in resistivity therefore depend mainly on variations in the length of the mean free path.

Restating:

Resistivity varies inversely as the number of conduction electrons and inversely as their mean free path.

Mathematically expressed:

$$\rho \propto \frac{1}{N\lambda}.$$

Substituting for ρ in Eq. (1),

$$R \propto \frac{l}{N\lambda A}. \tag{2}$$

This relationship shows the factors which determine resistance, *i.e.*, that which limits the electron flow in any given conductor.

The number of conduction electrons, and therefore their actual velocity in an electron flow, is not known with certainty, but there are reasons for believing that in good conductors, as already stated (Art. II–4), the number of such electrons is approximately that of the valence electrons, *i.e.*, the number of atoms times the valence. By assuming this number, the electron velocity in a 1-amp flow in a copper wire of 1-mm^2 cross section is 0.0074 cm/sec. The microscopic tungsten filament, usually wound in a spiral, of a 40-watt lamp has a diameter of 0.004 cm. In such a lamp, when it is energized by a direct p.d. of 110 volts, the electron velocity of the flow is 2.9 cm/sec. These progressive velocities are insignificant compared with the chaotic speeds which the electrons normally possess. From the quantum theory the average chaotic speed of the electrons is of the order of 10^8 cm/sec, regardless of temperature below 1000°K, and from the classical theory 10^5 cm/sec at room temperature and varying with absolute temperature like the speed of gas molecules.

If R_0 is the resistance of any conductor at 0°C., its resistance, R_t, at any other temperature t is

$$R_t = R_0(1 + \alpha t), \tag{3}$$

in which α is the temperature coefficient, *i.e.*, the amount by which the resistance of each ohm changes with a temperature change of 1°C. Some of these coefficients are tabulated in Art. 1. The resistance of pure metals diminishes with decrease of temperature in such a manner that it approaches zero at 0°K; *i.e.*, the temperature coefficient is about $\frac{1}{273}$ part of the resistance at 0°C. This is ascribed by the simplest theory to the tendency of the atoms of the metal to align in symmetrical order so that, when the chaotic heat motions become greatly reduced, more and more nearly unobstructed paths extend around the whole circuit. The conduction electrons then pass freely through these open lanes. The energy converted into heat is then so small that currents in such conductors, when once started, continue to flow for several hours without any further application of an e.m.f. This *superconductivity* appears in lead at 7.2°K and in mercury at 4.2°K.

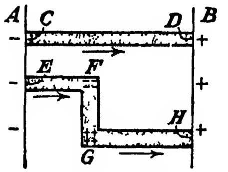


FIG. 3.—Distribution of electrons in a conductor carrying an electric current.

In a few alloys and in carbon the molecular rearrangement is such as to decrease the resistance with temperature.

3. Distribution of Electrons in a Conductor Carrying a Current.—A uniform potential gradient exists in the space between the two charged plates, A and B, Fig. 3; hence in the straight conductor CD, placed in the electric field, there is a gradual change in potential from one point to the other along the wire, and consequently electrons flow through it from C to D. A current flows in the wire as long as a potential difference is maintained.

In the bent conductor $EFGH$, however, the potential at F due to the charges on the plates A and B is that at G. Under this condition electrons are not forced from F to G; but since F is at a higher potential than E, electrons flow from E to F and lower the potential at F. Similarly, electrons flow from G to H leaving unneutralized + charges at G which raise the potential at that

point. Electrons then flow from F to G. Within any homogeneous conductor of uniform cross section into which electrons are being forced at one end and out at the other, the electrons distribute themselves in such a manner as to produce a uniform change in potential along the whole length. In any case the electron distribution becomes such that the potential gradients established by it cause the same current to flow in all sections of the circuit.

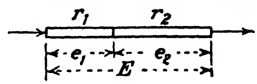


FIG. 4.—Relation of potential difference to resistance in a circuit.

4. Fall of Potential along a Wire Is Proportional to the Resistance.—Let e_1 and e_2, Fig. 4, be the potential differences across the two parts of a wire having the resistances r_1 and r_2. The same current, I, must flow in both parts. Then from Ohm's law (Eq. IX–7),

$$I = \frac{e_1}{r_1} = \frac{e_2}{r_2},$$

from which

$$\frac{e_1}{e_2} = \frac{r_1}{r_2},$$

or otherwise stated,

$$e \propto r. \tag{4}$$

Energy is expended in forcing electrons through the section in which the e.m.f. is supplying energy to the circuit (Art. IX–4); hence in that section, as already explained, the *RI drop* is equivalent to a potential difference. The term *RI drop* may be applied also to an actual potential difference between any two points in a circuit.

5. Combined Resistance of Wires in Series.—Let R be the combined resistance of the two conductors r_1 and r_2, Fig. 4, when placed as shown, *i.e.*, so that the same and the whole current passes through both. The conductors so placed are said to be *connected in series*. Also let E represent the potential difference across the two conductors in series. Since the current in all parts of a series circuit is the same,

$$I = i_1 = i_2,$$

in which I represents the current in the two conductors considered as one, i_1 the current in r_1, and i_2 the current in r_2.

Also,

$$E = e_1 + e_2.$$

Substituting for E, e_1, and e_2, in the foregoing equation, their equivalents (Ohm's law):

$$RI = r_1 i_1 + r_2 i_2.$$

From which

$$R = r_1 + r_2. \tag{5}$$

The resistance of any number of conductors in series is the sum of their individual resistances.

6. Combined Resistance of Wires in Parallel.—Two wires are said to be *connected in parallel* when the current divides between

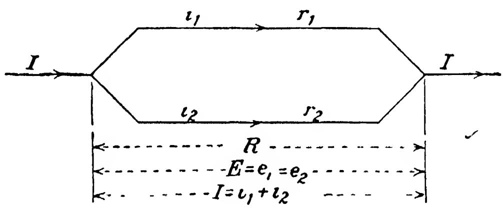


FIG. 5.—Resistance of wires in parallel

them as shown in Fig. 5. In this case the potential difference across the two wires together is that across either of them alone. Then

$$E = e_1 = e_2$$

Also

$$I = i_1 + i_2$$

Substituting for the currents their equivalents (Ohm's law),

$$\frac{E}{R} = \frac{e_1}{r_1} + \frac{e_2}{r_2},$$

from which

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}. \tag{6}$$

Hence the reciprocal of the combined resistance of two wires in parallel is equal to the sum of the reciprocals of the individual resistances. This statement holds for any number of wires

7. Resistance Standards.—*Manganin wire* has a high resistance, a low temperature coefficient, a thermoelectric power

(Art. XXIV–7) practically that of copper, and when properly tempered does not change its resistance with time. It is an alloy of Cu, Mn, Ni, and Fe in the proportion of (83.6) : (12) : (3.4) : (1).

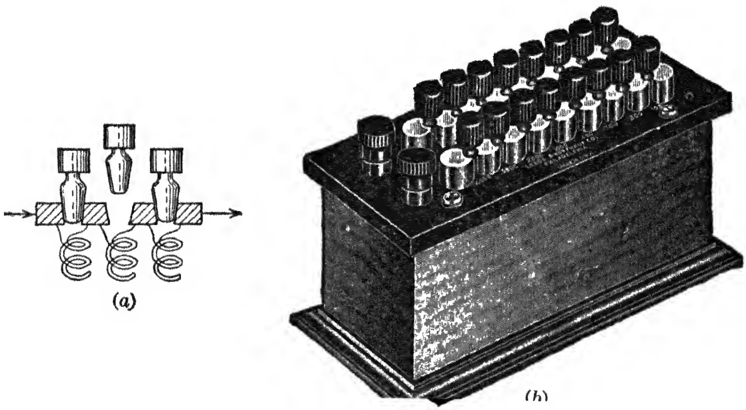


FIG. 6.—(a) Coils of plug-type resistance box. (b) Plug-type box.

Its resistance temperature coefficient varies somewhat with different samples but is usually about +0.00001 per degree

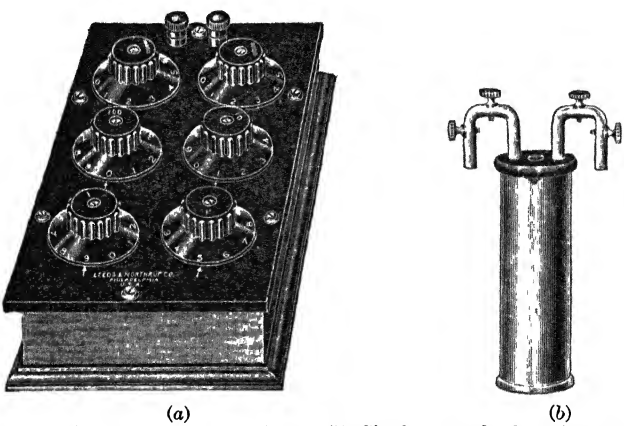


FIG. 7.—(a) Dial resistance box. (b) Single standard resistance of high precision.

centigrade; that of pure copper is +0.0038. *Therlo* is an alloy of Cu, Mn, and Al and has properties similar to those of manganin.

Because of its constancy, thermoelectric properties, and low temperature coefficient, manganin wire is generally used in the construction of resistance standards. These standards are of convenient known values and are employed for measuring other resistances by comparison methods. Several of these resistances in a box form what is called a *resistance box*. There are two types of these boxes: the plug type, Fig. 6, and the dial type, Fig. 7(a). To introduce a resistance into the circuit in using the plug-type box, the appropriate plugs, which normally short-circuit the resistances, are removed as shown in Fig. 6(a); in using the dial-type box, the dial is turned to the appropriately marked position.

8. Relation of Expended Energy to Resistance.—When the same current is forced through two unequal resistances in series, it has been shown, Eq. (4), that $e \propto r$. Since the work required to move a given quantity of electricity between two points varies as the potential difference (Art. 4), it follows that the power expended between points along a circuit varies directly as the resistance.

This fact is shown also from Eq. IX–10 where

$$P = RI^2.$$

When the current I is constant,

$$P \propto R.$$

When two conductors are connected in parallel the same p.d. forces currents through them. These currents and therefore the power expended in heating the conductors must vary inversely as the magnitudes of the individual resistances.

Questions

1. Show how it follows from Ohm's law that, when the number of conduction electrons in any circuit is assumed to be constant, the average velocity of the moving electrons is doubled if the potential difference between any two planes in the circuit is doubled.

2. In one circuit the number of free electrons per unit length is twice that of the second circuit, but their average velocity is one-half that of those in the second circuit. What are the relative magnitudes of the currents in the two circuits?

3. Show that because the current is the same in all parts of a simple circuit the p.d. between cross-sectional planes in the circuit varies as the resistance.

4. Show that the resistance of a homogeneous wire varies directly as the length and inversely as the cross-sectional area.

5. Two parts of a simple electric circuit made of the same material have equal lengths, but the cross sections have the ratio of 5:1. What are the relative velocities of the electrons in the two parts? What are the relative values of the currents? The potential differences? The resistances?

6. Show that the resistance of materials in which the electron mean free paths are equal varies inversely as the density of the conduction electrons.

7. How does the resistance vary with the mean free path?

8. Define specific resistance (resistivity) and write the defining equation.

9. Explain why bends in a wire do not affect the current in the circuit.

10. Derive the expression for the combined resistance of two wires connected in series. In parallel.

11. What is manganin wire, and what are its desirable properties? What is a resistance standard? A resistance box?

12. Write the equation for the resistance of a conductor at any temperature in terms of its resistance at 0°C.

13. How does the power expended in any part of a circuit depend on the resistance? On the current?

Problems

1. What is the resistance of a copper wire having a cross section of 2 mm^2 and a length of 40 meters?

2. The potential difference between two planes in a circuit is 18 volts, and the current is 2 amp. What is the resistance of the section of the wire between the two planes?

3. What current flows through a wire of 5 ohms resistance when the potential difference across the wire is 100 volts?

4. The resistance of two wires is 5 and 10 ohms, respectively. (*a*) What is their combined resistance when placed in series? (*b*) In parallel?

5. If the wires of Prob. 4 are energized by 110-volt direct-current mains, (*a*) what is the current in each wire when they are connected in parallel? (*b*) When connected in series? (*c*) What is the power expended in heating each of the wires when they are in parallel? (*d*) When they are in series?

6. The resistance of a spool of copper wire is 20 ohms at 0°C. What is its resistance (*a*) at +20°C and (*b*) at −20°C? ($\alpha = +0.00428$.)

7. The resistance of a spool of wire is found to be 5.045 ohms. What resistance must be placed in parallel with it to make the combined resistance 5.000 ohms?

8. What is the resistance of a heating coil which when attached to 110-volt mains heats 1 liter of water from 20 to 100°C in 5 min?

Experiments

1. Two wires of the same material but of different cross section heat unequally when they are connected in series.

2. Two wires of different materials and having the same cross section heat unequally when connected in series.

3. The potential differences between the same lengths of the wires of Exps. 1, 2 shown to be different (voltmeter used).

4. Fall of potential is proportional to the current (ammeter-voltmeter method).

5. Fall of potential along a circuit is proportional to the resistance, and along a uniform wire to the length.

6. Manganin wire, resistance standards, resistance boxes (dial and plug).

7. Three equal resistances in series have equal potential differences between their extremities, and the potential difference across either consecutive two of them is twice that across any single one. When the combination of two of the resistances in parallel is connected in series with the third, the potential difference across the two is one-half that across the third.

8. Resistance of iron wire shown to change with temperature In a low-resistance circuit a thin platinum wire is connected in series with a looped spiral of iron wire and a storage battery. The platinum wire loses its luminosity when the iron spiral is heated to redness by means of a Bunsen burner.

9. The shorter of two uniform wires connected in parallel, as a part of an electric circuit, heats more than the longer wire; shown by the smoke from the cotton insulation.

CHAPTER XI

CONDUCTION OF ELECTRICITY THROUGH LIQUIDS AND GASES

1. Ions in Liquids.—The molecules of sodium chloride (NaCl), for example, when dissolved in water are broken into their constituent parts. The sodium atom (Na) is always charged positively, and the chlorine atom (Cl) negatively. These charged atoms are called *ions* (Art. VIII–1). Atoms of the metals and of hydrogen always form positively charged or *positive ions;* and those of the nonmetals negatively charged or *negative ions.* Any solute whose molecules are dissociated in this manner is said to be *ionized.* A group of atoms, separated from a complex molecule, may form a single ion which carries one or more charges. An ion of a univalent element or group carries one elemental charge while an ion of a higher-valence element or group carries a corresponding number of elemental charges.

The separation of the molecules of a solute into ions may be explained by the diminution (Art. XII–4) of the forces acting between the oppositely charged parts of the molecules due to some grouping with the water molecules of high dielectric constant.

All inorganic acids, bases, and salts when dissolved in water ionize. Some substances dissolve and ionize in molten NaCl and in some other solvents. The following is a list of the + and − ions formed in water from a few well-known substances:

Substance	+ ions	− ions
NaCl	Na^+	Cl^-
$CuSO_4$	Cu^{++}	SO_4^{--}
$AgNO_3$	Ag^+	NO_3^-
H_2SO_4	H^+, H^+	SO_4^{--}
NH_4OH	NH_4^+	OH^-

The signs + or − and the double signs ++ or −− placed after the chemical symbol indicate that the valence of the ion is 1 or 2, and therefore that the charge on the ion is one or two elemental units of positive or of negative electricity.

2. Electrolysis.—The two plates, Fig. 1, are shown in a solution of silver nitrate and charged oppositely by means of a battery. The solution contains mobile ions but does not contain the free electrons that account for electric conduction in metals. The electric field that exists between the charged plates moves the + ions in one direction and the − ions in the other, which process is equivalent to a flow of electrons from the negative to the positive plate, as already explained (Art. VIII–1). The univalent + ion lacks an electron and therefore takes one from the negative plate; the univalent − ion on reaching the positive plate causes an equivalent − charge to appear on the plate either because it takes a + ion from the plate or through a secondary reaction with the electrolyte causes one electron to be deposited. In this manner electrons are entering the solution on one side and leaving it on the other. The number of charges entering and leaving the solution must be the same; otherwise the solution would become so charged as to cause immediate equalization of the number of charges entering and emerging.

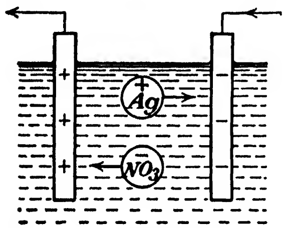


Fig. 1.—Electrolysis of $AgNO_3$.

The current or flow in an electrolyte, then, consists of the motion of oppositely charged ions in opposite directions and causes a transfer of electrons from the negative to the positive plate. The rate of the transfer must necessarily be such as to make the strength of the current in the electrolyte the same as that in the remainder of the circuit. When both kinds of ions have the same valence, the number of positive ions reaching one plate must equal the number of negative ions reaching the other. When the + and − ions do not have the same valence, the numbers of the ions carried to their respective plates are to each other inversely as the valences.

In the case of $AgNO_3$, Fig. 1, the silver ions deposit as atoms of silver on the negative plate, and the NO_3^- radicals, if the

positive plate is of silver, unite with + silver ions of the Helmholtz double layer (Art. XIV–3) to form $AgNO_3$ and thereby cause equal negative charges to appear on the plate. If the positive plate is of platinum, which under these conditions does not unite with NO_3^-, the NO_3^- ions unite with the H^+ ions of water to form HNO_3. Each two excess OH^- ions then give their − charges to the plate in forming H_2O and O_2 molecules.

When platinum electrodes are used in a solution of H_2SO_4, the H^+ ions give their charges to the negative electrode, and the resulting H atoms unite into molecules which form bubbles and rise. The SO_4^{--} ions on reaching the positive plate unite with the H^+ ions of water, cause the liberation of oxygen and the transfer of electrons to the positive electrode as in the case above. The liberation of each atom of oxygen is associated with two electrons leaving the electrolyte while each atom of hydrogen causes only one electron to enter. Since as many electrons must enter as leave the electrolyte, the volume of hydrogen liberated is twice that of oxygen. In the process the amount of H_2SO_4 in the solution does not change, but molecules of water are decomposed. This process, then, is called *decomposition of water by electrolysis.*

3. Faraday's Laws of Electrolysis.—Referring again to Fig. 1, each deposited ion of silver, which carried one elemental positive charge, has taken one electron from the negative plate. The quantity of electricity taken from that plate is then directly proportional to the number of atoms, and therefore to the mass M of the silver deposited. This quantity of electricity Q must also be that which passes every plane in the circuit and is equal to It.

Then

$$M \propto Q;$$

so that

$$M = ZQ = ZIt \text{ grams.} \tag{1}$$

This equation expresses *Faraday's first law of electrolysis.* The constant Z is called the *electrochemical equivalent* of the silver or of the element, whatever it may be, that is deposited. The constant Z is numerically equal to the mass in grams of the element deposited by 1 coulomb of electricity.

If several electrolytic cells are connected in series, Fig. 2, so that the same current passes through all of them, the same quantity of electricity passes every plane in the circuit; and therefore the same number of electrons at each of the negative plates passes from the plate to neutralize the positive ions that are deposited from the liquid.

In the cells A and B both the H^+ and the Ag^+ ions are univalent, each taking one elemental charge from the negative plate. The number of the two kinds of ions then is the same, and the masses deposited in a given time are to each other as the atomic weights; *i.e.*, $Z \propto$ atomic weight.

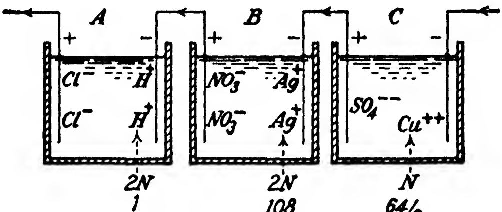


FIG. 2.—Electrolytic cells illustrating Faraday's second law of electrolysis.

In cell C the copper ion is bivalent and takes 2 electrons from the negative plate; therefore, the number of copper atoms deposited in taking the same quantity of electricity is only one-half that of the univalent ions. The mass of the copper deposited then is only one-half of what it would be if its atoms were univalent. The mass of an element deposited by 1 coulomb, therefore, varies inversely as the valence; *i.e.*, $Z \propto 1/\text{valence}$. Then

$$Z = K \cdot \frac{\text{atomic weight}}{\text{valence}}. \tag{2}$$

The term *atomic weight/valence* is known as the *chemical equivalent* of the element and Eq. (2) as *Faraday's second law of electrolysis.*

The weight in grams equal to the atomic weight of a univalent substance is called the *gram equivalent* of that substance. For substances of higher valence, the gram equivalent is the weight in grams equal to the atomic weight of the substance divided by the valence. For example, the gram equivalent of silver is 107.88 grams, and that of copper is 63.57/2 = 31.785 grams.

TABLE OF ATOMIC WEIGHT AND ELECTROCHEMICAL EQUIVALENTS

Element	Density	Atomic weight	Valence	Electrochemical equivalent = Z
Ag	10 5	107 88	1	0 0011180
Cl	...	35 46	1	0 0003676
Cu	8 93	63 57	2	0 0003295
H	...	1 008	1	0 00001045
O.	...	16 00	2	0 00008293
Zn	7 1	65.57	2	0 0003388

From the definitions of gram equivalent and electrochemical equivalent it follows that the quantity of electricity required to deposit a gram equivalent of a substance is

$$q = \frac{\text{gram equivalent}}{\text{electrochemical equivalent}} = 96{,}494 \text{ coulombs.}$$

This quantity of electricity deposits a gram equivalent of any substance and for that reason makes a convenient unit for some purposes. This unit of quantity is called a *faraday.*

A faraday is the quantity of electricity which deposits electrolytically 1 gram equivalent of any substance and is found to be 96,494 coulombs.

Electrolysis is the greatest decomposing agent known to chemistry. It is used for the separation of metals from their compounds, for the separation and building up of organic compounds, for the purification of metals, for electroplating, electrotyping, etc.

The number of atoms n in 1 gram of any element may be calculated from Eq. (1), the known magnitude e'' of the elemental charge and the known valence z of the element:

$$M = ZQ = \frac{ZQ''}{3 \times 10^9} = \frac{Znze''}{3 \times 10^9},$$

from which, when $M = 1$,

$$n = \frac{3 \times 10^9}{Zze''} \text{ atoms/gram.}$$

For copper, this equation gives $n = 9.54 \times 10^{21}$ atoms/gram.

4. Silver Voltameter.—Electric current may be measured by the mass of an element deposited in a given time from an

electrolytic solution. A solution of silver nitrate is the most desirable for this purpose. Although the + electrode must be of silver, the − electrode may be of either platinum or silver and is usually in the form of a cup which contains the silver nitrate solution. The instrument containing the electrodes is called *silver voltameter.*

From Eq. (1),

$$I = \frac{M}{Zt} \text{ amp.} \tag{3}$$

5. International Ampere, Ohm, and Volt.—It has been found convenient, for practical reasons, to use secondary definitions for the ampere, ohm, and volt. These are:

The *international ampere* is the unvarying electric current which when passed through a solution of silver nitrate in water deposits silver at the rate of 0.00111800 grams/sec.

The *international ohm* is the resistance at 0°C of a column of mercury 14.4521 grams in mass, 106.300 cm in length, and of a constant cross-sectional area (which is 1 mm^2).

The *international volt* is the potential difference which when steadily applied to a conductor whose resistance is 1 international ohm produces a current of 1 international ampere. For practical purposes it is equal to the 1/1.01830 part of the e.m.f. of a standard cadmium cell (Art. XIV–7).

These units are intended to be identical, within experimental limits, with the fundamental practical units as defined in Arts. VIII–2, IX–4 and are now the legal definitions of the practical units in civilized countries.

6. Resistance of Electrolytes.—When a current is flowing through an electrolyte, a potential difference necessarily exists between the plates. The ratio E/I of applied voltage to current flowing is the apparent resistance of the electrolytic cell. This apparent resistance is due in part to two effects which together are called *polarization:* (1) A gas liberated by the electrolysis usually collects on one of the plates, diminishing the effective area of the current paths in the electrolyte and thereby increasing the normal resistance R by an amount r. (2) The ions moving in the direction of the affected plate collect on or in the region of this gas and repel other ions moving toward that plate. This

is equivalent to an e.m.f. e (opposing the impressed e.m.f.), called the *counterelectromotive force of polarization,* which varies from 0.7 to 2.35 volts depending on the electrolyte. Then, applying Ohm's law,

$$E - e = I(R + r),$$

or

$$I = \frac{E - e}{R + r}. \tag{4}$$

7. Ions in Gases.—A few ions are always present in the atmosphere because the air is continually being ionized by helium nuclei (α-particles) and electrons (β-particles) ejected into space in the natural disintegration of radioactive substances

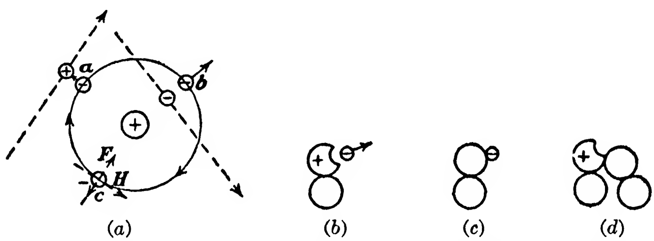


FIG. 3.—(*a*) The orbital electrons *a* and *b* being ejected from their orbits or energy levels by moving charges, and electron *c* by the electromagnetic pulse *FH*. (*b*) Representation of an electron being ejected from a molecule. (*c*) and (*d*) Representation of normal − and + ions in air.

always present in the earth and in the atmosphere, by the ultraviolet rays of sunlight, and by a penetrating radiation of cosmic origin.

The ionization consists of the ejection of an orbital electron from an atom by one of two methods:

1. The + α- and the − β-particles are projected from radioactive substances with what is called an *ionizing velocity* (greater than 10^8 cm/sec), *i.e.,* with a velocity large enough to produce ionization. These ionizing particles occasionally pass through the atoms in such a manner as to move long enough near to and parallel with the orbital electrons *a* and *b*, Fig. 3(*a*), to impart sufficient energy to force these orbital electrons out of the atoms.

2. Electromagnetic waves (Arts. XVIII–4, XXVIII–4) consist of alternating, associated nonconservative-electric and magnetic fields. When such a wave of sufficiently concentrated

intensity moves through an orbital electron, as shown at *c*, Fig. 3(*a*), it occasionally gives the electron sufficient energy and a velocity in the proper direction to force it out of the atom. The γ-rays of radioactive substances and ultraviolet rays of sunlight have sufficiently concentrated fields (see photons, Art. XXVII–1) to produce ionization in this manner. The nature of the cosmic rays is not definitely known, but they are now supposed to be particles (Art. XXVII–15).

The electron ejected by either of these causes, Fig. 3(*b*), attaches itself in an instant to a neutral molecule to form a negative ion, Fig. 3(*c*), and the molecule which has lost an electron likewise attaches itself to a neutral molecule to form the heavier positive ion, Fig. 3(*d*). In this manner each ionized molecule produces two ions.

There are approximately 750 ions/cm^3 (Art. XXX–2) of each kind ordinarily in the atmosphere. The rate at which ions are formed (about 10 of each kind per second) equals that at which they *recombine*. If the ions are removed from any space, new ions form at a greater rate than that with which they recombine. The number of ions per cubic centimeter then increases until the condition of equilibrium is established.

The approximate number of pairs of ions formed per second in a cubic centimeter of air at the surface of the earth is

3 from radioactive substances in the earth.

6 from radioactive substances in the earth's atmosphere and the penetrating radiation from its upper conducting layer.

1.4 from the penetrating radiation of cosmic origin.

The total number of molecules, 27.09×10^{18}/cm^3 in the atmosphere (N.P.T.) is large compared with the 1,500 ions. Since there is one ion to every 18×10^{15} molecules, and the population of the earth is about 1.75×10^9, the atmosphere contains one ion to a number of neutral molecules which is greater than 10^7 times the human population of the earth.

An electroscope, even though imagined to be perfectly insulated, loses 3 per cent of its charge per minute by attracting to itself the ions of the opposite sign. Since each ion contains a charge of only 4.770×10^{-10} statcoulombs and their number and rate of formation and velocity are small, the loss of charge due to this cause is scarcely appreciable unless large numbers of ions

are produced artificially. The space charge due to each kind of ions normally in the atmosphere near the surface of the earth is 0.36 statcoulombs/meter^3.

Air is ionized artificially by means of x-rays, radium, and ultraviolet light, and by incandescent bodies and large potential gradients. The electric fields ionize by giving the electrons liberated by natural ionization an ionizing velocity before they have time to unite with neutral atoms to form ions. Each electron then passes through many atoms and ionizes some of them.

8. Corona—Ionizing Potential Gradient.—In any electric field the two kinds of ions of the air are given velocities in opposite directions. In fields whose potential gradient is 1 volt/cm, newly formed positive ions, in moderately dry air, are given a velocity of 1.36 cm/sec, and the lighter negative ions a velocity of 1.87 cm/sec. The velocity in each case does not increase beyond that amount in such a field because at greater velocities the opposing resistance due to the induced charges on the neighboring molecules and to the viscosity of the air exceeds the force with which that field acts on the ion. The velocity of the ions decreases somewhat with their age and with the moisture content of the air.

The drift of the ions and of the electrons freed in the process of ionization increases directly as the potential gradient, and finally the velocity of the *electrons* (not of the comparatively heavy ions) becomes sufficiently large to cause ionization. When this *ionizing potential gradient* (about 30,000 volts/cm) exists in the immediate neighborhood of highly charged conductors, each electron liberated in the natural process of ionization is given an ionizing velocity by the electric field. The electron then ionizes many molecules in its path before, if it is repelled, its velocity is diminished below that which ionizes, and, if attracted, before it reaches the charged conductor. Each electron from each of these many ionized molecules repeats the process of ionizing many other molecules. This action is repeated again and again until at least a large share of the molecules of the air become ions. The charged conductor is discharged by the attracted electrons and ions whose charges are opposite (in sign) to that of its own. In this ionized condition the air is a conduc-

tor, and the flow of ions in it to and from the charged conductor is called a *corona*. If the potential gradient is sufficient, a luminosity accompanies the flow and may be seen in the dark. The discharge in this case is called *visual corona.*

If a fine wire is stretched down through the center of a chimney and charged to a potential high enough for the corona to form, the repelled ions travel from the center to the walls of the chimney. These ions attach themselves to the particles of smoke and carry them to the wall. This is called the *Cottrell process of smoke removal.*

9. Discharge from Points on Conductors—Brush Discharge.—It was shown (Art. IV–3) that the density of electricity (quantity per unit area) is greatest at the pointed end of a conductor. The intensity of the electric field (potential gradient), therefore, is the greatest at the pointed end, where then, when the conductor is being charged, the ionizing potential gradient is established first. The air that has been made conducting by this ionization furnishes the ions which are attracted to the pointed end and discharge the conductor. The repelled ions produce a wind of electrified air which carries away as much electricity as is lost by the conductor through its attraction of the opposite kind of ions. This process is called a *brush discharge.*

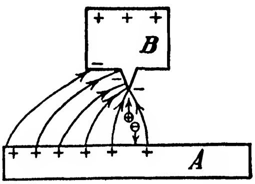


FIG 4.—The pointed end of a conductor "collecting" a charge from a plate.

10. How Points "Collect" Electricity from a Charged Plate.—The charged plate A, Fig. 4, induces a negative charge on the point of the conductor B. The density of this charge becomes sufficient to produce an ionizing potential gradient. The ions that are formed neutralize both the + charge on the plate A and the − charge on the conductor B. The positive charge disappears from A and appears on B. The point is therefore said to collect electricity from the plate.

11. Disruptive Discharge.—If the potential gradient between two oppositely charged conductors is increased beyond that necessary to produce visual corona, some value is reached at which the whole distance between the conductors becomes

ionized. At this potential gradient (critical disruptive voltage) a disruptive discharge takes place between the conductors.

The oppositely charged ions of the conducting air, moving in opposite directions, generate heat; the resulting sudden expansion of the air produces the familiar sharp sound, and the intense ionization the brilliant light.

Disruptive discharge is made use of in measuring large potential differences. The potential difference to be determined is impressed across two separated spheres (sphere gap), and the maximum distance between the spheres at which discharge takes place is determined. For spheres 2.0 cm in diameter the critical disruptive voltage is about 30,000 volts/cm.

Questions

1. Define ion and state how ions are formed in a liquid.

2. What are univalent ions? Bivalent ions?

3. What part does the high dielectric constant of water play in its great ionizing power?

4. What are the + and the − ions of NaCl, $CuSO_4$, $AgNO_3$, and H_2SO_4?

5. Make a general statement concerning the signs of the ions of the different elements and radicals.

6. Describe how the ions are separated from a solution by electrolysis, and show that the motion of the ions causes a "transfer" of electrons from one plate to the other and therefore constitutes an electric current.

7. Describe in detail all that takes place in an electrolytic cell composed of silver electrodes in a solution of silver nitrate ($AgNO_3$).

8. Describe in detail all that takes place, including the secondary reaction, when water is being decomposed in a platinum-electrode cell containing dilute sulphuric acid (H_2SO_4).

9. Give Faraday's first law of electrolysis, and show how this law necessarily follows from the known facts of electrolysis.

10. Give Faraday's second law of electrolysis and show how this follows from the known facts.

11. Define electrochemical equivalent and chemical equivalent.

12. Give some of the uses of electrolysis.

13. Show how a current may be measured by the amount of silver deposited in a given time from silver nitrate.

14. Define the international ampere, ohm, and volt. Why are these definitions given?

15. Explain the variations in the apparent resistance of an electrolyte. What is the counter e m.f. of polarization?

16. Explain the two ways in which ions are normally produced in the atmosphere.

17. What is the number of pairs of ions per cubic centimeter in the atmosphere? How many pairs of them are formed each second from each

of the three sources? Is this number large compared with the total number of molecules? How can that number be greatly increased?

18. What is meant by the recombination of ions?

19. Explain how the ions of the atmosphere gradually discharge an electroscope.

20. Explain how the air is made conducting in the neighborhood of highly charged conductors. What is a corona?

21. Explain why sharp points on a conductor discharge it.

22. What is a brush discharge? How is the electrified wind blowing from a point of a charged conductor produced?

23. Explain in detail the process by which sharp points "collect" electricity from a charged plate moving by them.

24. Explain how a disruptive discharge is produced, and state what potential gradient is required. What causes the sharp sound?

Problems

1. How much silver is deposited from a silver nitrate solution in 1 hr by a current of 2 amp?

2. How much copper is deposited from a solution of copper sulphate by the same current in the same time?

3. What is the average current when 0 5 grams of silver is deposited from a silver nitrate solution in 30 min?

4. Calculate the number of atoms in 1 cc of silver.

5. When the potential difference between two plates 3 cm apart is 1,800 volts, (*a*) what is the potential gradient in volts per centimeter in the space between the plates? (*b*) What is the intensity of the electric field in electrostatic units?

Experiments

1. Decomposition of water in large U tube; also in a small cell (projected).

2. Lead tree formed and then dissolved, in lead acetate (projected)

3. A cell containing horizontal electrodes, one of copper and the other of platinum, in copper sulphate solution, projected, showing changes in the density of the solution, the deposition and dissolving of copper, and the formation of hydrogen.

4. Silver voltameter.

5. Electroscope discharged by air ionized by radium, by an arc light, and by a hot wire.

6. Wind from the pointed end of a charged conductor blowing out a candle flame and rotating a reaction wheel.

7. Luminosity of a brush discharge.

8. Corona about a wire charged by means of a Tesla coil.

9. Cottrell process of smoke removal.

10. A sharp point placed on the plate of an electroscope "collects" electricity from a charged body near it and charges the electroscope.

11. Disruptive discharge.

CHAPTER XII

CAPACITANCE

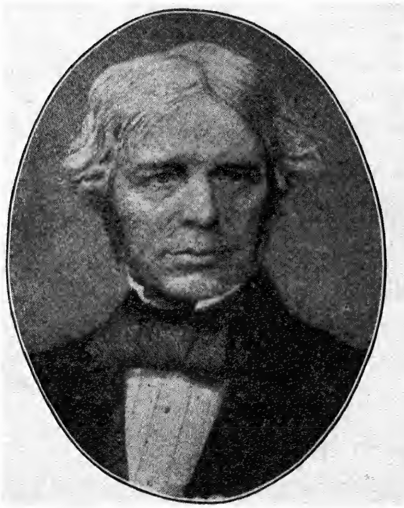
1. Capacitance.—The equation, $V'' = Q''/R$, for the potential of a sphere shows that for the same sphere

$$Q'' \propto V''.$$

Then

$$Q'' = C''V''.$$

The constant C'' is numerically equal to the charge per unit of potential, and, since its magnitude increases with the charge



Michael Faraday (1791–1867), director of the laboratory of the Royal Institution, London, England, universally conceded the title of "greatest experimental physicist"; discoverer of the laws of electrolysis (1833) and of the fact that inducing and induced charges are equal, made the first dynamo; and discovered (independently of Joseph Henry) electromagnetic induction (1831). The practical unit of capacitance, the *farad*, and a unit of quantity of electricity, the *faraday* are named in his honor.

required to produce any given potential, it is called the *capacitance* or the *capacity* of the sphere. Similar relationships hold

between the quantity and the potential irrespective of the form of the conductor or of the system of units used.

A conductor has unit capacitance in any system of units when the unit charge of that system causes the potential to change one unit. These units in the three systems are called *statfarad*, *abfarad*, and *farad*. The capacitance, *i.e.*, the number of units of capacitance, in each of the three systems is

$$\begin{aligned} C'' &= \frac{Q''}{V''} \text{ statfarads,} \\ C' &= \frac{Q'}{V'} \text{ abfarads,} \\ C &= \frac{Q}{V} \text{ farads.} \end{aligned} \tag{1}$$

The units of capacitance are derived units whose magnitudes depend on those of the primary units of quantity and potential. Equations (1) are the defining equations for capacitance.

2. Relation between the Units of Capacitance.—The relation between the units is obtained from Eqs. (1) and the known relation between the units of quantity and of potential.

$$C = \frac{Q}{V} = \frac{\dfrac{Q''}{3 \times 10^9}}{300V''} = \frac{1}{9 \times 10^{11}} \frac{Q''}{V''} = \frac{C''}{9 \times 10^{11}}.$$

In a similar manner it can be shown that

$$C = 10^9 C'.$$

Then

$$C = 10^9 C' = \frac{C''}{9 \times 10^{11}}. \tag{2}$$

From which, if it is remembered that in the equation C, C', and C'' represent the *numbers* of the three kinds of units which are a measure of an identical capacitance, it is seen that

$$1 \text{ farad} \equiv \frac{1}{10^9} \text{ abfarads} \equiv 9 \times 10^{11} \text{ statfarads.}$$

A *microfarad* is a millionth of a farad, and a *micromicrofarad* is a millionth of a microfarad. A micromicrofarad is then equal to 0.9 statfarads. Although the microfarad is generally used as a

unit of capacitance because of its more practical magnitude, the letter C in equations represents the capacitance in farads. The micromicrofarad is used as the unit for such small capacitances as are usually employed in radio. A statfarad is often called a *capacitance of* 1 *cm* because it is the capacitance of a sphere of 1 cm radius.

3. Dependence of Capacitance on Dimensions and on Neighboring Charges—Electric Condenser.—It was shown (Arts. III–9, 10) that, since $V'' = \Sigma q''/r$, the potential of a conductor is dependent not only on the charge it contains but also on its dimensions and on all the neighboring charges, even though these

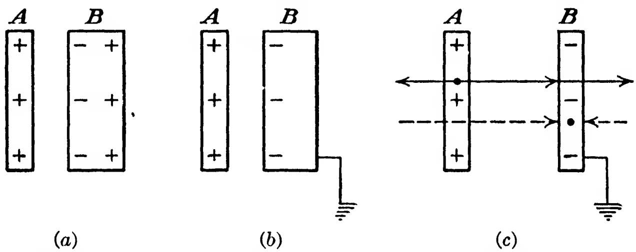


FIG. 1.—The capacitance of the plate A increased by the presence of the conductor B in its neighborhood.

charges may be induced by the charge on the conductor in question. An increase in dimensions, therefore, diminishes the potential due to any charge on the conductor and therefore, since $C = Q/V$, increases the conductor's capacitance. A neighboring $-$ charge lowers the potential of a positively charged conductor, while a neighboring $+$ charge increases it. Neighboring charges of *unlike* signs, therefore, increase capacitance, while neighboring charges of *like* signs decrease it.

The plate A, Fig. 1(a), is charged with positive electricity and because of this charge has a positive potential. An uncharged conductor B, brought into the neighborhood, has two induced charges, of which the nearer $-$ charge lowers the potential of A more than the more distant $+$ charge raises it. The combined effect of the induced charges, therefore, is the lowering of the potential of conductor A without changing the magnitude of its charge Q. Since V in the equation $C = Q/V$ is reduced, while Q remains unchanged, the capacitance C of plate A is increased by the presence of plate B. The increase in the capacitance

depends on the distance between the plates and the distance between the two induced charges. The thicker the plate B, the greater is the separation of the induced charges and the greater the increase in the capacitance of plate A. If the plate B is grounded, Fig. 1(b), the earth and the plate together become one conductor and thereby the induced + charge, which now is on the opposite side of the earth, is too far removed to affect the potential of the plate A. The disappearance of the induced + charge lowers the potential of A still more and therefore increases its capacitance proportionately.

The lowering of the potential of A by the oppositely charged plate B may be considered from several points of view:

1. The bound charge on B lowers the potential on A as described above. The leaves of an electroscope attached to A, Fig. 1(b), collapse as the plate B approaches A, experimentally showing the lowering of the potential.

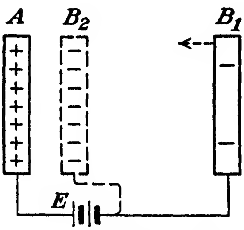


FIG. 2.—The approach of one plate toward the other increases the magnitude of the charges on both plates.

2. The potential on A is measured by the work required to move a unit + charge of electricity from the earth, or from an infinite distance, to the plate. The − charge of the plate B attracts this unit + charge; hence less work is done in moving the given charge to A when the grounded plate B is in the neighborhood.

3. The positive and negative electric fields of the two charges on the plates may be imagined to be superposed and almost to neutralize each other except in the space between the plates, Fig. 1(c). The weakened field now acts with less force on a unit charge placed in it, and therefore less work is done in moving the conventional unit + charge from the earth to the plate.

4. When an electric battery, E, is permanently connected by wires to two plates A and B which are far apart, Fig. 2, the battery forces electrons from one plate to the other until the potential difference which it can maintain is established between the plates. If now the plates are moved toward each other, the magnitude of the potential of each plate is diminished by the approach of the charge of opposite sign; the battery then can transfer more and more electrons until finally, when the plates

are very near each other, the charge on each plate may be many hundred times the original. From another point of view, the + charges on the positive plate attract − charges to the negative plate, and the − charges on the negative plate repel − charges from the positive plate. This interaction aids the battery in transferring electrons and thereby increases the charges on both plates.

If the two plates are very near each other, the quantity of − electricity on B is practically that of the + electricity on A. When the plate B, Fig. 1(b), is disconnected from the ground or the plates A and B, Fig. 2, from the battery, the two plates in each case have practically equal charges of opposite sign. If now the plates are connected together by a conductor, the conduction electrons on plate B flow toward plate A until the potentials are equalized and the charge on each of the plates is practically zero. The quantity of electricity which flows through every plane of the connecting wire during the discharge is practically that on one of the plates. The capacitance of the plate A, or of the two parallel plates A and B, then, is measured by the charge on one of the plates and in either case is said to be the *capacitance of two parallel plates.* Two such parallel plates, for obvious reasons, are called an *electric condenser.*

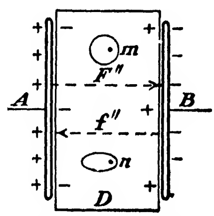


FIG. 3.—Increase of capacitance due to a material dielectric.

4. Effect of the Dielectric on Capacitance—Dielectric Constant.—The capacitance of two parallel plates, Fig. 3, is increased by placing an insulating solid or liquid, D, between them. The insulating material when so placed exhibits electric charges on the sides facing the plates, and, since such an insulating material (dielectric) has practically no conduction electrons, these charges cannot be explained in terms of them.

Each atom of the dielectric consists (Art. II–2) of a positive nucleus and orbital electrons. The electric field due to the charges on the plates either displaces the positive nucleus toward the negative plate B and the orbital electrons toward the positive plate A, as shown in the magnified atom m, or turns the axis of an elliptical orbit as indicated in the magnified atom n. This displacement causes the orbital electrons to be nearer the positive

plate more than one-half of the time. Each atom then behaves as though it had two stationary electric charges: a + charge on one side and a − charge on the other. The atom then is said to be *polarized* or to be an *electric doublet* (Art. VII–8). All atoms of the dielectric become such doublets with their positive sides facing the negative plate B. This polarization is equivalent to having the dielectric charged oppositely on the two sides.

The electric field f'' due to the polarized dielectric is opposed to the field F'' due to the charges on the plates. The product of the resultant field $(F'' - f'')$ and the distance d between the plates measures the potential difference (Eq. III–8).
Then

$$V_2'' = (F'' - f'')d = \frac{1}{K}F''d,$$

which is less by the factor $1/K$ than the potential difference, $V_1'' = F''d$, which existed between the plates before the introduction of the dielectric. This decrease in the potential difference without a change in the charges on the plates constitutes an increase in the capacitance of the condenser by the factor K.

From another point of view, the potential of the plate A may be considered as due to all the charges in the neighborhood (Art. III–9); *i.e.*, $V'' = \Sigma q''/r$. The charges of the electric doublets of the dielectric as well as those on the plates contribute to this potential, and, since the negative charges of the doublets lower the potential of plate A more than the more distant equal positive charges increase it, the potential of plate A is lowered by the electric doublets. In a similar manner the potential of the negative plate is raised. Therefore the p.d. between the plates is diminished and the capacitance increased.

From still another point of view, the electric doublets of the dielectric bind (Art. IV–5) the greater part of the charges on the plates. These bound charges are parts of the charges on the plates but they and the doublets together contribute nothing to the potential difference between the plates. Therefore the bound charges and also the capacitance of the condenser are increased because of the presence of the dielectric.

Any dielectric always increases the capacitance of such plates by some constant K whose magnitude depends on the nature of

the dielectric. This constant is called the *dielectric constant* or the *specific inductive capacity* of the material.

The following table gives the dielectric constants of a few substances:

Substance	Dielectric Constant K
Gases	1 00007 to 1 009
Air at 0°C, N.P	1 000586
Air at 20°C, N.P.	1 000576
Olive oil	3 1
Crown glass	5 to 7
Mica	5 7 to 7
Dry paper	2 to 2 5
Paraffin wax	2 to 2 3
Methyl alcohol	35 4
Water	81

It might appear from the table that alcohol and water should be used as dielectrics to increase capacitance; their comparatively low resistance, however, prevents their general use for such purposes.

When a conductor is placed between the two plates, without touching them, the displacement of the conduction electrons in it produces the same effect as the distortion of the atoms in a dielectric.

When the electrons in a dielectric are being displaced with reference to the atomic nuclei, their combined motion is equivalent to an electric current in the dielectric. This *displacement current*, however, lasts only during the interval the condenser is charging or discharging.

5. Capacitance of a Sphere.—The potential of an isolated sphere (Art. III-8) is

$$V'' = \frac{Q''}{R},$$

from which

$$Q'' = RV''.$$

But

$$Q'' = C''V''.$$

Therefore,

$$C''V'' = RV'',$$

and

$$\mathbf{C'' = R \text{ statfarads.}} \qquad \mathbf{(3)}$$

The capacitance of a sphere in statfarads is equal to its radius in centimeters. This fact, of course, is true only when there are no other charges near enough to affect the potential appreciably.

6. Capacitance of Two Concentric Spheres.—Imagine the inner sphere A, Fig. 4, to be charged with $+Q_1''$ statcoulombs, and the outer sphere B to be grounded. The potential given to the outer sphere by the charge $+Q_1''$ of the inner sphere (Art. III–8) is

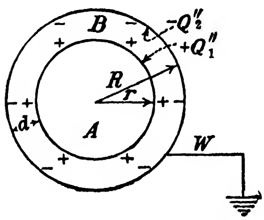


FIG. 4.—Capacitance of two concentric spheres.

$$V_1'' = +\frac{Q_1''}{R}.$$

Electrons then move from the zero potential of the earth toward this higher potential through the grounding wire W until the charge $-Q''$ on the outer sphere is such that it alone would give this sphere a negative potential V_2'' equal in magnitude to the positive potential V_1''. This potential

$$V_2'' = -\frac{Q_2''}{R}.$$

Therefore

$$V_1'' + V_2'' = \frac{Q_1''}{R} - \frac{Q_2''}{R} = 0,$$

and the induced charge, Q_2'', is equal to the inducing charge, Q_1''. This method of reasoning leads to the same conclusion as Faraday's ice-pail experiment (Art. IV–7).

Since the potential inside any charged surface is that on the surface (Art. IV–3), the potential at any point on or within the inner sphere is the algebraic sum of the potentials due to the charges on the two spheres. The potential of the inner sphere, then, is

$$V'' = \frac{Q_1''}{r} - \frac{Q_2''}{R} = Q_1''\left(\frac{1}{r} - \frac{1}{R}\right) = Q_1''\frac{R - r}{Rr}.$$

Then (Art. 1),

$$Q_1'' = \frac{Rr}{R - r}V'' = C''V''.$$

From which

$$C'' = \frac{Rr}{R - r} \text{ statfarads,}$$

where C'' is the capacitance of the inner sphere, but, since the outer sphere contributes to this capacitance, as in the case of two parallel plates, C'' is called the capacitance of the two concentric spheres.

7. Capacitance of Parallel Plates.—Imagine the two spheres, Fig. 4, to have radii which are large compared with the distance d between them. Then r and R are practically equal when their product is considered. Hence

$$C'' = \frac{Rr}{R - r} \cong \frac{R^2}{d} = \frac{A}{4\pi d}.$$

(The area A of a sphere equals $4\pi R^2$; from which $R^2 = A/4\pi$.) Since any small section of the concentric spheres, if the radii are large, is in reality two parallel plates, the capacitance of two parallel plates in a vacuum is

$$C'' = \frac{A}{4\pi d},$$

in which A is now the area of one of the plates, and d is the distance between them.

If a material dielectric is placed between the plates, the capacitance is increased by the factor K, the magnitude of the dielectric constant (Art. 4). The capacitance of two parallel plates then is

$$C'' = \frac{KA}{4\pi d} \text{ statfarads.} \tag{4a}$$

When two parallel plates are considered as a small section of the two large concentric spheres, the electric field between them has the same intensity at all points. This is not the case with ordinary parallel plates, for, as seen in Fig. 5(a), the field is fringed out at the edges. This distortion affects the capacitance of the plates; but for plates very close together, the change in the capacitance is small and may be neglected.

The field between two parallel plates may be made practically uniform by surrounding the plates with guard rings, Figs. 5(b) (c),

which are at the same potential as the plates. Equation (4*a*) then applies accurately to the parallel plates P.

If several parallel plates are connected as shown in Fig. 6 the number of two-plate condensers formed is $(N - 1)$, in which N is the number of individual plates. The capacitance of such an arrangement of plates then is

$$C'' = \frac{KA(N-1)}{4\pi d} \text{ statfarads.} \tag{4b}$$

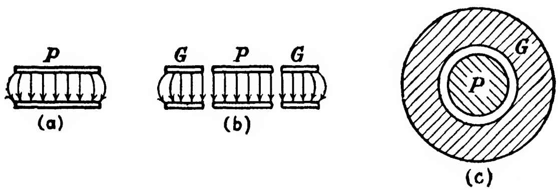


FIG. 5.—The electric field between parallel plates P; (a) without guard rings; (b) with guard rings G, (c) top view of Fig (b).

The area A is that of one of the plates only, and d is the distance, in centimeters, between two adjacent plates.

From the relationship between the different systems of units of capacitance, Eq. (2), it can be shown that the capacitance of a condenser of N plates is

$$C = \frac{KA(N-1)}{11\ 31 \times 10^{12} d} \text{ farads.}$$

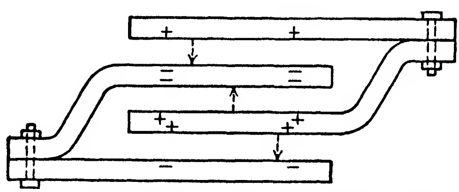


FIG. 6.—Capacitance of several parallel plates.

The capacitance of a thin circular disk is

$$C = 0{\cdot}354 d \times 10^{-12} \text{ farads}$$

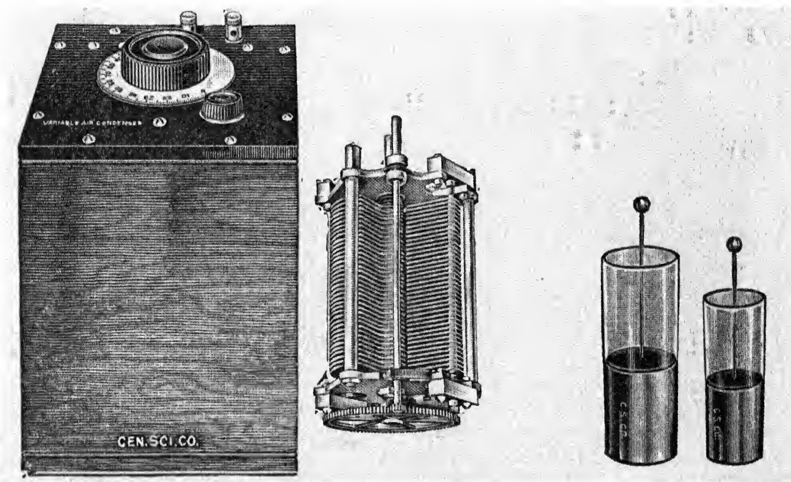
and that of two horizontal wires, when d and D are small compared with l, is

$$C = \frac{0.1208 l}{\log_{10} \frac{2D}{d} - \frac{D^2}{8h^2}} \times 10^{-12} \text{ farads,}$$

where d = diameter of disk or wire, D = distance between wires, l = length, and h = average height above ground, all being expressed in centimeters.

8. Practical Forms of the Electric Condenser.—The capacitance of two parallel plates placed close together may be several hundred times that of a single plate. Such an arrangement of parallel plates, as already noted, is called an *electric condenser*.

Condensers are of various forms adaptable to the purpose for which they are to be used. The *air condenser*, in which air is the dielectric, usually consists of two sets of metallically con-

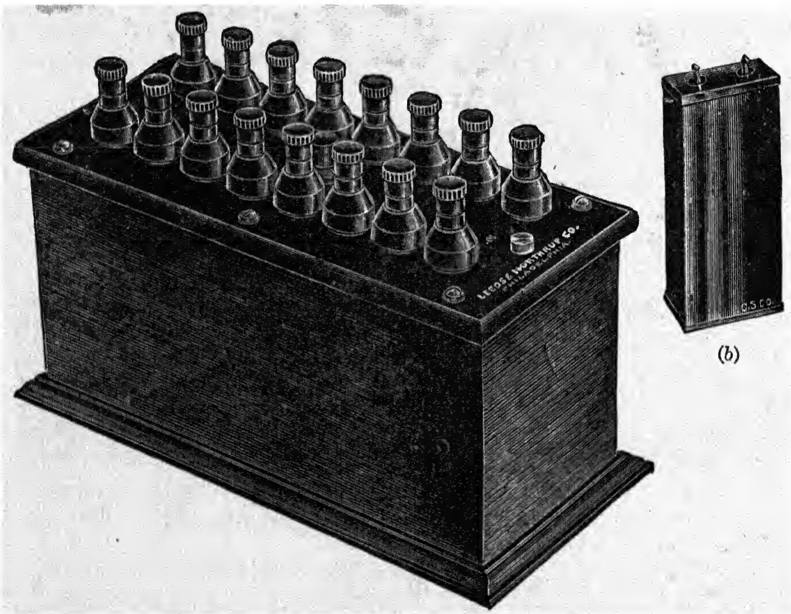


(*a*) (*b*)

FIG. 7.—(*a*). Air condenser. (*b*). Leyden jars.

nected parallel plates, Fig. 7(*a*); one set, being insulated from the other, is movable and may be inserted, more or less, between the alternately placed stationary plates of the other set. This form of condenser, whose capacitance may be varied at will, is commonly used in radio. The *Leyden jar*, Fig. 7(*b*), consists of a glass jar, the lower half of which serves as the dielectric for two tinfoil coatings (one on the inside and the other on the outside of the jar) which serve as the plates of a condenser. This form is in common use with electrostatic machines. The *mica condenser*, Fig. 8(*a*), consists of sheets of tinfoil separated by thin pieces of mica. Alternate sheets of the tinfoil are connected together, as in Fig. 6, to form a condenser of many parallel plates. The best condensers are baked in a vacuum to remove moisture and then, while subjected to great mechanical pressure in a vacuum, are sealed by an insulating compound. Mica

condensers are used as standards for the comparison of electric capacitances, for the measurement of quantities of electricity, and wherever a high grade condenser is required. The *paper condenser*, Fig. 8(*b*), consists of two long strips of tinfoil separated by two sheets of paper. This combination is covered with other strips of paper, rolled into a compact form, boiled under vacuum in paraffin or some insulating wax, pressed and then after being



(*a*) (*b*)

FIG. 8.—(*a*) Mica condenser (*b*) Paper condenser.

placed into an appropriate container is sealed by a moisture-proof insulating compound. These condensers are used mainly in telephone service and with radio rectifiers.

Any two parallel conductors, such as telephone lines or a transmission cable and its lead sheath, are equivalent to two parallel plates and form a condenser. Such wires or cables are said to have *distributed capacitance.* This distributed capacitance plays an important part in alternating-current circuits and in telephone transmission.

9. Charging and Discharging a Condenser.—The condenser *AB*, Fig. 9(*a*), is charged by means of an electrostatic machine

(Art. XV–5) represented by the arrowed circle. This machine forces electrons from the earth into the plate A of the condenser; the plate B is charged by induction (Arts. 3, IV–5), and, if the plates A and B are close together, the charges on them are practically equal.

Since the earth acts here merely as a very thick conductor, the condenser may be and usually is charged by connecting its plates directly to the two electrodes of the electrostatic machine.

The condenser may also be charged by connecting its plates, Fig. 9(b), to any two points on an electric circuit between which there is a potential difference. The plate A, for example, is

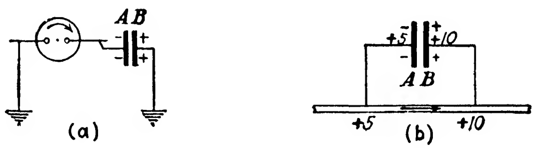


FIG. 9 —(a) A condenser being charged by means of an electrostatic machine. (b) A condenser being charged when connected to two points on an electric circuit. The numbers represent potentials

connected to a point whose potential is $+5$, and the plate B to a point whose potential is $+10$. Imagine the charge on B to correspond to the potential $+10$. This charge gives the plate A a potential of nearly $+10$. Since the plate A must have the potential $+5$ of the point to which it is connected, electrons flow from the circuit to the plate until they lower its potential to $+5$. This negative charge on A lowers the potential on B and thereby causes more electrons to move from that plate to the circuit. In this manner the two plates become oppositely charged, even though they are connected to potentials of the same sign. The charge on a condenser is determined solely by the potential difference between its plates and not by the actual potential of either plate.

When the oppositely charged plates of a condenser are connected by means of a conductor, the electrons of the negative plate are displaced toward the positive until the potentials are equalized. In this *discharge of the condenser* only the electricity on the negative plate "moves" through the discharging conductor. Hence, if the usually negligibly small free charge is disregarded, the quantity of electricity in a condenser discharge

electricity, Q, which measures the capacitance of the two condensers together is that on both negative plates because that quantity passes through every section of the wire during discharge. Then

$$Q = CE = q_1 + q_2 = C_1e_1 + C_2e_2,$$

and, since

$$E = e_1 = e_2,$$

then

$$C = C_1 + C_2, \tag{5}$$

in which C is the combined capacitance of the two condensers in parallel.

The combined capacitance of any number of condensers joined in parallel may similarly be shown to be equal to the sum of the several individual capacitances.

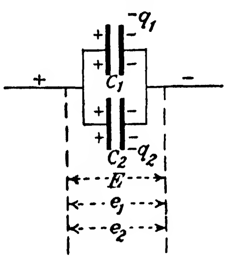


FIG. 10.—Condensers connected in parallel.

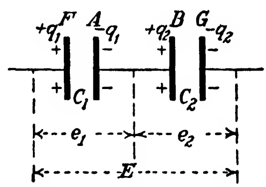


FIG. 11.—Condensers connected in series.

11. Capacitance of Condensers in Series.—The condensers C_1 and C_2, Fig. 11, are connected in series and have the end plates charged with equal charges of opposite sign. In this arrangement each condenser is charged by only a part of the total charging potential difference E, where

$$E = e_1 + e_2.$$

The potential of the plates A and B must be the same, and their charges, $-q_1$ and $+q_2$, must be equal because one is due to the displacement of the other. From this statement, and since opposite plates on the same condenser have practically equal, opposite charges, $-q_2$ must equal $+q_1$; hence when condensers which are connected in series are discharged, the quantity that passes through the discharging wire is the quantity $-q_2$ (the quantity on one of the condenser plates only). The charge $-q_1$

electricity, Q, which measures the capacitance of the two condensers together is that on both negative plates because that quantity passes through every section of the wire during discharge. Then

$$Q = CE = q_1 + q_2 = C_1e_1 + C_2e_2,$$

and, since

$$E = e_1 = e_2,$$

then

$$C = C_1 + C_2, \tag{5}$$

in which C is the combined capacitance of the two condensers in parallel.

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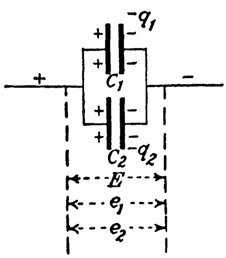


FIG. 10.—Condensers connected in parallel.

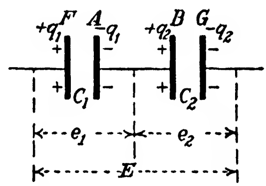


FIG. 11.—Condensers connected in series.

11. Capacitance of Condensers in Series.—The condensers C_1 and C_2, Fig. 11, are connected in series and have the end plates charged with equal charges of opposite sign. In this arrangement each condenser is charged by only a part of the total charging potential difference E, where

$$E = e_1 + e_2.$$

The potential of the plates A and B must be the same, and their charges, $-q_1$ and $+q_2$, must be equal because one is due to the displacement of the other. From this statement, and since opposite plates on the same condenser have practically equal, opposite charges, $-q_2$ must equal $+q_1$; hence when condensers which are connected in series are discharged, the quantity that passes through the discharging wire is the quantity $-q_2$ (the quantity on one of the condenser plates only). The charge $-q_1$

on the plate A neutralizes $+q_2$ on the plate B and does not pass through the discharging wire.

When Q represents the quantity of electricity discharged from condensers connected in series,

$$Q = q_1 = q_2.$$

But

$$E = \frac{Q}{C}, \qquad e_1 = \frac{q_1}{C_1}, \qquad \text{and} \qquad e_2 = \frac{q_2}{C_2}.$$

Substituting these values in the equation

$$E = e_1 + e_2$$

gives

$$\frac{Q}{C} = \frac{q_1}{C_1} + \frac{q_2}{C_2};$$

from which

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}. \tag{6}$$

In the same manner a similar relationship may be shown to express the combined capacitance of any number of condensers in series. The reciprocal of the combined capacitance of condensers in series is equal to the sum of the reciprocals of the capacitances of the individual condensers. It follows that the capacitance of a number of like condensers in series is equal to the capacitance of one of the condensers divided by their number.

12. Energy of a Charged Conductor or Condenser.—When electrons are being transferred from the earth to a conductor or from one plate to the other in a condenser, the potential difference at first is zero and then increases uniformly to the final value V''. On the average the electrons are moved through a potential difference of $V''/2$, and the energy expended in moving them through this potential difference appears as the potential energy of the charge, which is assumed to be entirely in the electric field made effective by the separation of the charges. The energy expended,

$$W = Q'' \times \frac{V''}{2} = \frac{1}{2}Q''V'' = \frac{1}{2}C''(V'')^2 \text{ ergs},$$

or

$$W_J = Q \times \frac{V}{2} = \frac{1}{2}QV = \frac{1}{2}CV^2 \text{ joules}. \tag{7}$$

When a conductor or a condenser is being charged, the energy is supplied to it by the charging apparatus; and when it is being discharged, its potential energy is changed into the energy of the current and thereby into heat.

Since in charging or discharging conductors or condensers the potential changes from 0 to V or from V to 0, the electricity that is moved passes through the average potential difference $V/2$; however, when electricity is moved in the electric field between two charged plates or between any two planes in a wire while a uniform current is flowing, the whole quantity of electricity moves through a constant potential difference V. In one case, then, the energy expended is $\frac{1}{2}QV$, and in the other, QV joules.

13. Energy of the Electric Field.—Since the energy stored in a condenser is considered to be in the form of potential energy within the electric field, Eq. (7) for the energy of a condenser is also the equation for the energy of the electric field between the plates. The energy density, w_1 ergs/cm³ of the electric field between the plates, is the total energy, W, divided by the volume, Ad, where A is the area of one of the plates and d the distance between them.

Then, since

$$W = \tfrac{1}{2}C''(V'')^2,$$

$$w_1 = \frac{W}{Ad} = \frac{C''(V'')^2}{2Ad} \text{ ergs/cm.}^3$$

To make this equation applicable to any electric field in space, it is necessary to replace C'' and V'' by their equivalents:

$$C'' = \frac{KA}{4\pi d}, \qquad \text{and} \qquad V'' = F''d.$$

Then

$$w_1 = \frac{KA}{4\pi d} \times \frac{(F''d)^2}{2Ad} = \frac{K(F'')^2}{8\pi} \text{ ergs/cm.}^3 \tag{8}$$

Questions

1. Give the defining equation for capacitance in each of the three systems of units. Define capacitance.

2. Define statfarad, abfarad, and farad; and derive the expressions from which the relation between their magnitudes can be determined.

3. Show that 1 micromicrofarad is equal to 0.9 statfarads.

4. Show why an increase in the dimensions of a conductor increases its capacitance.

5. Show how a neighboring charge of like sign lowers the capacitance of a conductor and how a neighboring unlike charge increases it.

6. What effect has a neighboring uncharged, insulated body on the potential and the capacitance of a conductor? What effect has a neighboring grounded body? Explain from three points of view.

7. Explain from three points of view why a material dielectric increases the capacitance of two parallel plates.

8. Define dielectric constant.

9. Derive the expression for the capacitance of an isolated sphere. Why is a statfarad sometimes called a capacitance of 1 cm?

10. Develop the equation for the capacitance of two concentric spheres; of two parallel plates; of any number of parallel plates with a material dielectric.

11. What is the function of guard rings in connection with the capacitance of parallel plates?

12. What is an electric condenser? Are the charges on the two plates equal in magnitude? Why is the capacitance measured by the charge on only one of the plates?

13. What are air condensers, Leyden jars, paper condensers, and mica condensers?

14. Name some other forms of condensers.

15. Explain how a condenser is charged when one plate is grounded; when the plates are connected to the opposite electrodes of the charging machine without grounding.

16. Explain how it is possible for the plates of a condenser to become oppositely charged even though they are connected to points on an electric circuit that have potentials of like sign

17. What is taking place in a condenser when it is charging or discharging?

18. Distinguish between the free and absorbed charges of a condenser, and explain how each is produced.

19. Explain why the charges on each of two condensers connected in series must be the same regardless of their capacitances.

20. Derive the expression for the capacitance of condensers in parallel. In series.

21. When condensers are connected in series, what is the relation between the potential differences of the individual condensers and their capacitances? Why is the combined capacitance less than that of one of the condensers alone?

22. Derive the expression for the energy of a charged condenser.

Problems

1. What is the capacitance of a sphere whose radius is 5 cm (*a*) in statfarads? (*b*) In abfarads? (*c*) In farads? (*d*) In microfarads? (*e*) In micro-microfarads?

2. Calculate the capacitance, in statfarads, of two concentric spheres whose radii are 5 and 5.05 cm.

3. (*a*) What is the potential, in volts, of a sphere of 2-cm radius when charged with 30 statcoulombs of electricity? (*b*) With 1 coulomb?

4. What is the capacitance, in statfarads, of a 10-plate air condenser at 20°C when the area of each plate is 30 cm^2 and the distance between the plates is 0.5 mm, neglecting the end corrections but not the dielectric constant of air?

5. What is the capacitance, in statfarads, of the foregoing condenser when the dielectric is mica ($K = 7$) and the distance between the plates is 0.1 mm?

6. The capacitances of three condensers are respectively 1, 2, and 4 microfarads. (*a*) What is the combined capacitance of the three when they are connected in parallel? (*b*) When connected in series?

7. Each of 900 condensers has a capacitance of 1 microfarad. (*a*) What is their combined capacitance when connected in parallel? (*b*) When connected in series?

8. A condenser whose capacitance is 1,000 statfarads is charged to a potential difference of 6,000 volts. What is the potential energy of the charge?

9. When a condenser whose capacitance is 2 microfarads is charged with a potential difference of 100 volts, what is the potential energy of the charge?

10. A sphere of 10-cm radius is charged with 3,000 statcoulombs of electricity and then is connected to a distant uncharged sphere of 15-cm radius by a thin wire. (*a*) What is the resulting potential? (*b*) How much of the charge is transferred to the other sphere? (*c*) What was the energy (in joules) of the original charge? (*d*) The sum of the energies of the distributed charges? (*e*) How much energy is converted into heat by the transfer?

11. What is the energy per cubic centimeter, in air, of an electric field whose intensity is 4 e.s.u.?

12. What is the energy of the electric field per cubic centimeter in the mica dielectric ($K = 7$) between the plates of a mica condenser when the thickness of the mica is 0.1 mm and the condenser is charged with a potential difference of 600 volts?

Experiments

1. A charged, insulated sheet of tinfoil wound on a Chinese pulley is unwound without changing the amount of the charge. An attached electroscope indicates that the potential decreases and therefore that the capacitance increases with an increase in the dimensions of the charged conductor.

2. The change in potential of a charged plate is observed when another grounded plate is moved back and forth in its neighborhood.

3. The change in potential of a charged electroscope is observed on bringing the hand near to the plate.

4. Change of potential is observed when a plate of metal or of hard rubber is placed between the two plates of a condenser.

5. A sphere and the inner coating of a grounded Leyden jar are charged to the same potential. The charges are then compared by observing the sparks to a grounded conductor.

6. Removable-coat Leyden jar shown to have a charge on the glass.

7. The energy of the condenser charge shown by ringing a bell. An insulated metal ball swings between the knob of a Leyden jar and the bell which is metallically connected to the outer coating of the jar.

8. Capacitances of equal condensers in parallel and in series compared with the capacitance of one of the condensers by observing the throw produced by their discharge through a galvanometer.

9. A condenser charged by connecting its plates to any two points on an electric circuit.

10. Air condenser, Leyden jar, mica condenser, paper condenser, telephone cable, and lead-encased transmission cable, shown.

11. A burglar alarm operated by change in capacitance.

CHAPTER XIII

ELECTROMOTIVE FORCE PRODUCED BY RELATIVE MOTION BETWEEN A CONDUCTOR AND A MAGNETIC FIELD

1. Displacement of Free Electrons and Production of Electron Flow by the Cutting of Magnetic Flux.—When a conductor is moving through a magnetic field at right angles to the lines of force or when it is at rest and a magnetic field is moving through it, the conductor is said to be *cutting magnetic lines of force* or being *cut by them*, as the case may be.

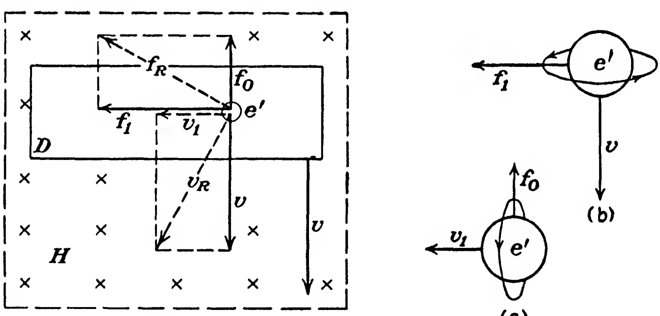


FIG. 1.—(*a*), (*b*), (*c*) Magnetic forces acting on a free electron of a conductor when the conductor is moving in a stationary magnetic field.

Imagine the conductor *D*, Fig. 1(*a*), to be moving downward with a velocity v at right angles to the lines of magnetic flux. The free electrons in the conductor necessarily move with it through the flux with the same velocity v. Each electron, e', because of this velocity, is surrounded by a magnetic field (Law A_1) as shown in Fig. 1(*b*). This field strengthens the magnetic field being cut on the right side of the electron. The electron then, through "electromagnetic reaction" (Law A_2), has exerted upon it a *magnetic force* f_1 from right to left, and, since it is mobile, it is displaced to the left. All the free electrons are displaced in the same manner and thereby cause the two ends

of the conductor to become charged oppositely. An electrostatic field established by this displacement throughout the length of the conductor urges the electrons in the reverse direction. The displacement of the electrons therefore continues only until the opposing electric and magnetic forces to which the electrons are subjected are in equilibrium.

When the conductor of Fig. 1 is the part $D(=ADB)$ of a closed electric circuit, Fig. 2, and is moving downward in a magnetic field H, the free electrons are displaced and establish a potential difference $E_1 = E_2$, and therefore electric fields, between A and B in the manner just described. In this case, however, the electric

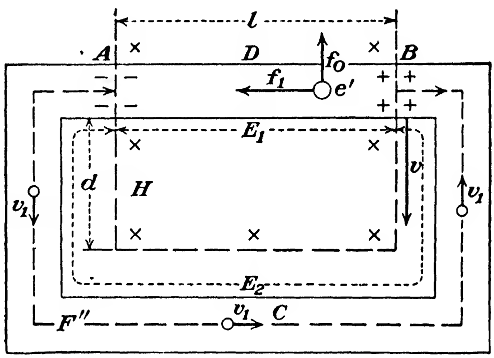


FIG. 2.—A closed electric circuit showing a potential difference, $E_1 = E_2$, maintained by the motion of the part D through a magnetic field H.

fields urge the free electrons from A to B in opposite directions in the two parts D and C of the circuit; but in the part D, where the magnetic forces exist, the magnetic forces now are never completely balanced by the electric forces because the electrons can be, and are, maintained in continuous motion around the circuit (Art. IX–2).

2. Mechanical Energy Transformed into Electric Energy.—When the electrons in the part D, Figs. 1, 2, of the circuit acquire a velocity, v_1, along the circuit, each electron, owing to this velocity, has the additional magnetic field whose direction is indicated in Fig. 1(c). The electromagnetic reaction (Law A_2) due to this field urges each electron, such as e', upward with a force f_0. The magnitude of the mechanical power, which must come from some external source and which is being expended

in moving the conductor against this opposing force with the velocity v of the conductor is

$$P_0 = f_0 v \text{ ergs/sec.}$$

The power expended in forcing the same electron along the circuit with a velocity v_1 at right angles to the downward velocity of the conductor is

$$P_1 = f_1 v_1 \text{ ergs/sec.}$$

The forces f_1 and f_0 exist only because some external force is causing the downward motion of the electron. The mechanical power P_0 expended against the opposing force f_0 appears only as the power P_1 expended through the force f_1 in maintaining the progressive motion of the electron and in giving it potential energy. The power given to the circuit and expended in maintaining the velocity of each electron, therefore, is

$$P = f_0 v = f_1 v_1 \text{ ergs/sec.} \tag{1}$$

The total mechanical energy expended in moving the conductor downward the distance d against the forces opposing the downward motion of the N electrons in the moving section D of length l can be derived by either of the following two methods:

1. The force opposing the downward motion of the conductor as a whole due to the electron flow generated in it by the motion (Art. VIII–6) is $f = BlI'$ dynes. The energy expended in moving this conductor the distance d against this force then is

$$w = fd = BlI'd = \phi I' \text{ ergs.} \tag{2}$$

2. The same conclusion follows also from Eqs. VIII–5, 6, 7; Art. VII–4. The total energy expended in terms of that, w_1, expended on individual electrons is

$$w = Nw_1 = Nf_0 d = NBe'v_1 d \frac{l}{l} = BldI' = \phi I' \text{ ergs.}$$

This expression states the fact that the total mechanical energy expended in maintaining the current for any given time is equal to the current strength (in abamperes) times the magnetic flux cut in that time by the conductor.

3. E.M.F.—That which is forcing electrons through the electric circuit, as already stated in Art. IX–2, is called *electromotive*

force. It, however, is not measured in terms of force, as the name might imply, but in terms of the energy the motive force expends in forcing a unit charge once around the circuit (Art. IX–1). It will be shown that this e.m.f. may also be measured by the electrostatic potential difference it can establish in an open circuit.

In the case of a steady current, where the kinetic energy of the moving electrons is not changing, all the energy received by the electrons in their passage through the length l of the moving part D of the circuit is expended in their passage around the whole circuit. The energy received from some external source by each electron while it is moving in the part D of length l is

$$w_1 = f_1 l \text{ ergs.}$$

The energy expended in moving the electron e' around the whole circuit (all of which in the given case is converted into heat) is

$$w = E'e' \text{ ergs,}$$

in which E' represents the work required to move a unit charge once around the circuit (Arts. IX–2, 3).

Since the energy expended is equal to the energy received,

$$w = E'e' = f_1 l \text{ ergs,}$$

from which

$$E' = \frac{f_1 l}{e'} \text{ abvolts.} \tag{3}$$

Since e' is a constant, the equation shows that in any moving length l of the conductor the force f_1 which causes the electron flow through the circuit is proportional to the work, E', required to move a unit charge once around the whole circuit. In any case $E' \propto f_1 l$. That which is supplying energy and causing the current to flow, *i.e.*, the e.m.f., therefore, can be, and is, measured in terms of the work required to move a unit charge once around the circuit. It then is measured in the same units as potential difference.

If the section D is a part of an open circuit, Fig. 3(a), the electric force f_E acting on each electron due to the potential difference V'' established by the electron displacement just balances the magnetic force f_1 due to the e.m.f. E'.

Then since from Eq. (3)

$$f_1 = \frac{E'e'}{l} = \frac{E''e''}{l},$$

and (Arts. III–2, 3)

$$f_E = F''e'' = \frac{V''e''}{l},$$

$$\frac{E''e''}{l} = \frac{V''e''}{l},$$

from which

$$E'' = V'' = F''l \textbf{ statvolts.} \quad E = 300F''l \textbf{ volts.} \qquad (4)$$

The e.m.f. then can also be measured by the electrostatic potential difference it establishes in an open circuit.

The direction of an e.m.f., by the conventions of this text, is the direction of the electron flow it produces; but it is convenient to give it the reverse direction whenever the conventional direction of the current in place of that of the electron flow is employed.

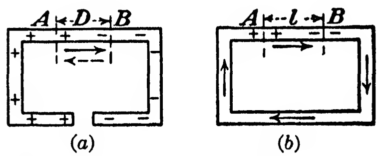


FIG. 3.—(*a*) An open circuit. (*b*) A closed circuit.

Restating:

Electromotive force is that which supplies energy to an electron flow. It is conveniently measured by the energy it imparts to each unit charge in forcing that charge once around the circuit, or by the electrostatic potential difference it establishes in an open circuit.

4. E.M.F. Induced in a Moving Conductor When the Conductor Is Cutting Stationary Magnetic Flux.—The practical expressions for the e.m.f. induced in a circuit when some part of the circuit is cutting magnetic flux are derived by either of the following two methods:

1. The work required to move the length l of the circuit, Fig. 4, against the opposing force evoked by the motion of the electrons through that length is due entirely, as already explained, to the fact that the electrons are being forced to move along the circuit. The mechanical energy expended (Art. 2 and Eq. VIII–5) in pushing the wire through the distance d is

$$w_1 = fd = BlI'd \text{ ergs.}$$

The electric energy produced by this and expended in the circuit (Art. IX–4) is

$$w_2 = E'Q' \text{ ergs.}$$

Then

$$E'Q' = BlI'd,$$

from which

$$E' = \frac{BlI'd}{Q'} = \frac{BlI'd}{I't} = Blv = \frac{Bld}{t} = \frac{\phi}{t} \text{ abvolts.} \qquad (5)$$

2. The energy expended in forcing one electron of charge e', Figs. 2, 4, through the length l of the moving part of the circuit is

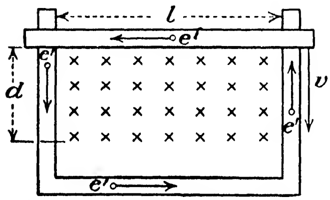


FIG. 4.—An electric circuit with the part l moving downward and cutting magnetic flux at right angles to the lines of force.

$$w_1 = f_1 l = Be'vl \text{ ergs.}$$

This energy is converted into heat by the electron in its motion around the complete circuit, but this energy (Art. IX–4) expressed in electric terms is

$$w_2 = E'e' \text{ ergs.}$$

Then

$$w = E'e' = Be'vl,$$

from which

$$E' = Blv = \frac{Bld}{t} = \frac{\phi}{t} \text{ abvolts.}$$

From which, in both derivations, the e.m.f.

$$E = \frac{E'}{10^8} = \frac{Blv}{10^8} = \frac{\phi}{10^8 t} \text{ volts.}$$

The e.m.f. may be calculated by using either of the forms $Blv/10^8$ or $\phi/10^8 t$.

The general expression for the instantaneous magnitude of a variable e.m.f. must necessarily be expressed in terms of the symbols of calculus and is

$$E = -\frac{1}{10^8}\frac{d\phi}{dt}.$$

The − sign placed in the last term of this equation is the conventional manner of representing that the direction of the induced e.m.f. is such as to cause an electron flow whose electromagnetic reaction opposes the motion or change of that which produces the e.m.f. The equation also shows that *a volt may be defined as the electromotive force induced when the conductor is cutting or is being cut by* 10^8 *maxwells/sec.* The directional − sign is omitted in this text except when the symbols of calculus are used.

When N wires of a coil are cutting a magnetic field,

$$E = \frac{N \cdot Blv}{10^8} = \frac{N\phi}{10^8 t} = -\frac{Nd\phi}{10^8 dt} \text{ volts.} \tag{6}$$

This is a basic equation expressing quantitatively what is known as *Faraday's law of electromagnetic induction.*

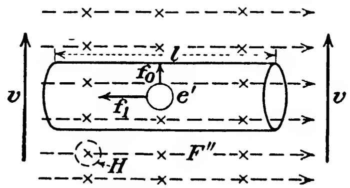


FIG. 5.—Forces acting on a free electron in a conductor when it is being cut by moving magnetic flux. The moving flux H is represented by crosses and the evoked electric field F'' by arrowed broken lines.

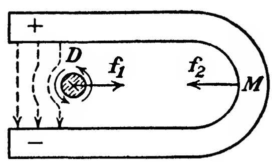


FIG. 6.—Forces ($f_1 = f_2$) acting between the conductor D (shown in cross section) and the horseshoe magnet M when the electron flow in the conductor is as represented.

5. E.M.F. Induced in a Stationary Conductor When the Conductor Is Being Cut by Moving Magnetic Flux—Basic Phenomenon 3—Nonconservative Electric Field.—When a stationary conductor is being cut by a moving magnetic field, as represented by the crosses and long vertical arrows in Fig. 5, an e.m.f. is induced in the conductor which in direction and magnitude is that induced when the conductor is moving in the reverse direction and cutting a like stationary field at the same rate. The Faraday equation, Eq. (6), then gives the magnitude of the induced e.m.f. in either case; and the velocity v represents, in general, the relative velocity between the conductor and the magnetic flux, and ϕ/t or $d\phi/dt$, the rate with which the conductor is cutting or is being cut by the flux.

Let D, Fig. 6, be the cross section of the part of an electric circuit immersed in a magnetic field which, in the illustrated case,

is that of a horseshoe magnet. Experiment shows, as noted above, that the same e.m.f. is induced in the conductor D and therefore the same current, regardless of whether the conductor is moved to the left and is cutting the field or the magnet is moved to the right so that its flux cuts the conductor at the same rate.

The electromagnetic reaction (Law A_2) between the conductor, so energized, and the magnetic field causes the conductor to be urged toward the right with a force f_1; but since the magnetic field is a conventional aspect of or inseparable from the magnet itself and the forces of action and reaction are equal (Newton's third law of motion), the conductor and the magnet are urged in opposite directions, *i.e.*, in the given case, attract each other with a force $f_1 = f_2$. The moving of the conductor to the left against this force of attraction or of the magnet the same distance to the right requires the expenditure of equal amounts of energy. This energy in each case is expended only because a current is being evoked and maintained in the conductor by the motion.

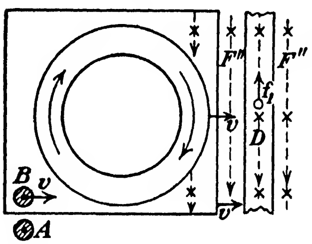


FIG. 7—Electromotive force in conductor D which to observer A appears to be caused by electric forces, and to observer B, by magnetic forces. The crosses represent the direction of the magnetic field due to the flow and F'' the electric field in terms of which observer A explains the induced e.m.f.

The mechanism by which an e.m.f. is induced when the *conductor is doing the cutting* was explained in Arts. 1, 2, 3. Although the energy relationships given above explain the magnitude of the evoked e.m.f. when the *field is doing the cutting*, they reveal nothing concerning the "mechanism" by which the e.m.f. is impressed. This e.m.f. cannot be ascribed directly to the magnetic flux because stationary electrons are not affected by it (Art. V–1). Even after the electrons are given a velocity along the conductor, they are urged (Law A_2) at right angles to this velocity and therefore the electromagnetic reaction can contribute nothing to the electron motion.

Let the ring of Fig. 7 represent a magnetic loop (a magnet) on a moving train, and A and B two observers, one of whom is

stationary and the other in motion with the train. Let D be a stationary conductor which is a part of an electric circuit. Since all motions are relative, and all ordinary laws of action are formulated to hold from the point of view of an observer who assumes himself to be at rest, the observer B on the moving train considers the loop and himself to be at rest and the conductor D to be moving toward him. He therefore explains the presence of the induced e.m.f. in terms of magnetic forces as in Arts. 1, 2, 3.

The observer A, however, sees the loop and its magnetic field moving toward a stationary conductor D in which the flow electrons appear to be at rest before they are acted on by the moving field. He cannot explain the presence of the flow induced by the field motion in terms of magnetic forces (B.P.2) as was done by the moving observer B. He knows also that the (conservative) electrostatic field which the relative motion, from his point of view, produces about the moving loop or magnet (Appendix V–8) can induce no e.m.f. whatsoever in the circuit, because such a field can displace electrons only until a potential equilibrium is established throughout the circuit (Art. IV–3). The observed flow, so easily explained by the moving observer B, therefore is inexplicable to the observer A in terms of either B.P.1 or B.P.2.

Since the observed force acts on the electrons regardless of their state of motion or rest, the moving magnetic field acts on them in the same manner as an electrostatic field would act if that field had no electrostatic potential gradient along its lines of force. In such a quasi-electric field the displaced electrons, although the forces acting on them balance, would establish an actual unbalanced electrostatic potential difference between the extremities of any conductor placed within that field. This unbalanced p.d., if the conductor were a part of an electric circuit would urge electrons, as observed, around the remaining part which may or may not be within the field.

We, who are in the position of observer A, find it more convenient to attribute the observed action on conductor D to such a quasi-electric field than directly to the motional action of the moving magnetic field itself. This assumed or conventional field is called a *non-conservative electric field*, because the work done in moving a charge between two points depends on the path taken. In such a field, energy, in general, is expended in

moving a charge completely around any closed loop, while in the conservative electrostatic field, never.

The existence of such an action on electric charges by a moving magnetic field is to observer A, and therefore to us, a basic fact not explainable in terms of the other two basic phenomena. The action which is attributed to the nonconservative or motional electric field therefore, in this text, is called *basic phenomenon* 3 (B.P.3). This may be stated as follows:

Every moving magnetic field is assumed to have associated with it an effective nonconservative electric field whose direction and intensity are such as to induce in any conductor an electromotive force which can be measured in terms of the rate with which the magnetic field is cutting as well as it can be measured in terms of the intensity of the electric field itself ($E'' = F''l$). The directional relationship of the two kinds of fields is identical with that formulated in Law B. (B.P.3) (7)

It should be noted that one observer ascribes the induced e.m.f. in the circuit to magnetic forces and the other to nonconservative electric. However, one and the same force, whatever may be the attributes or names different observers are obliged to give it, is urging the electrons along the circuit.

6. Magnitude Relationship between Associated Electric and Magnetic Fluxes.—When a magnetic flux is cutting a conductor, Fig. 5, the nonconservative electric field, F'', associated with the moving magnetic field, displaces the free electrons to the left side of the conductor, in the illustrated case, until an equal opposing conservative electric field, $F''_E = F''$, produces equilibrium. The potential difference V'' which exists across the electric field F_E is a measure of the e.m.f. E'', Eq. (4), and therefore (Arts. III–3, IX–5, and Art. 4), since $F''_E = F''$,

$$E = 300E'' = 300V'' = 300F''l = \frac{\phi}{10^8 t}.$$

From which the intensity of the nonconservative electric field associated with the moving magnetic field is

$$F'' = \frac{\phi}{3 \times 10^{10} lt} = \frac{Bld}{clt} = \frac{[vB]}{c} \text{ e.s.u.} \tag{8}$$

The square brackets inclosing $[vB]$ indicate that the component of v at right angles to B must be taken and that the two terms cannot be separated by transposition in the equation. The equation gives the magnitude F'' when the magnetic field B has the velocity v and gives nothing concerning what the magnitude B would be if a conservative electric field of strength F''_E had the velocity v.

Law 7 can be tested by Eqs. (6), (8). Assume a wire 10 cm in length to be cut by a magnetic field of 2,000 gauss moving with a velocity of 100 meters/sec. The strength of the nonconservative electric field which is the actual cause of the induced e.m.f. is

$$F'' = \frac{[vB]}{c} = \frac{10{,}000 \times 2{,}000}{3 \times 10^{10}} = 6.66\dot{6} \times 10^{-4} \text{ e s.u.}$$

Then

$$E = 300V'' = 300F''l = 300 \times F'' \times 10 = 2 \text{ volts.}$$

But the same magnitude is obtained more conveniently by Eq. (6) which gives

$$E = \frac{Blv}{10^8} = \frac{2{,}000 \times 10 \times 10{,}000}{10^8} = 2 \text{ volts.}$$

The intensity of the magnetic field associated with any moving electric field is derived as follows. Since electric lines of force always move parallel to themselves (Art. I–3) inspection of Fig. 8 shows (1) that the elemental fields pass through the center of the loop with the velocity v of their associated electrons which are moving in the loop circuit, and (2) that all moving electrons in the loop contribute equally to the intensity of the magnetic field at the center. The electron flow in the loop of circumference l causes the quantity $Q'' = I't$ to pass every plane of the loop in the time $t = l/v$. This distributed quantity is, for the purpose here considered, equivalent to the same quantity (Art. IX–1), concentrated at a point and moving once around the loop in the given time t. The intensity of the moving electron field of this charge at the center of the loop is

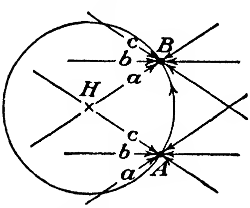


FIG. 8.—The center of an energized loop shown to be cut by the elemental electric fields of moving electrons A and B and the fields to be contributing equally to the magnetic field H at that point.

$$F''_E = \frac{Q''}{r^2},$$

and therefore the intensity of the magnetic field (Art. VIII–2) at the center is

$$\mathcal{H} = \frac{lI'}{r^2} = \frac{l}{r^2}\cdot\frac{Q'}{t} = \frac{l}{r^2}\cdot\frac{Q''}{ct} = \frac{l}{r^2}\cdot\frac{F''_E r^2}{ct} = \frac{l}{r^2}\cdot\frac{F''_E r^2}{c}\cdot\frac{v}{l} = \frac{[vF''_E]}{c}\cdot$$

oersteds. (9)

Summarizing the facts shown in Eqs. (8), (9):

A moving conservative electric field evokes an associated magnetic field and a moving magnetic field evokes a nonconservative electric field; in either case the intensity of the evoked field varies as the velocity and as the intensity of the primary moving field, and its direction with respect to the moving field is that given by Law B. **(10)**

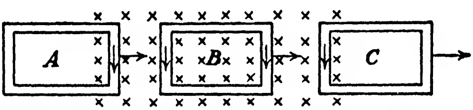


FIG. 9.—The direction of the e.m f. induced in a loop moving into, through, and out of a magnetic field.

7. Induced E.M.F. in Terms of Rate of Change of Line Linkage.—When a loop of wire is moving or turning in a magnetic field, the induced e.m.f. established in the circuit may be expressed in terms of the rate with which the magnetic flux is changing within the loop.

Imagine the rectangular loop, Fig. 9, to be moving from left to right through a uniform magnetic field. In position A, only the front part of the loop is cutting the flux. The number of lines cut by it is the same as the number of lines which are entering the space within the loop. The induced e.m.f. in abvolts then is equal to the rate at which the flux is changing within the loop.

In position B the two opposite sides are cutting lines of force at the same rate; hence there is no change of flux within the loop; but equal e.m.fs., which are both downward, are induced in the two sides and tend to force electrons in opposite directions around the circuit. The resultant of these e.m.fs., therefore, is zero; and the induced e.m.f. is said to be zero.

In position C the loop is emerging from the magnetic field, and only the rear side is cutting the flux. The induced e.m.f. then is the rate at which the flux within the loop is diminishing.

In a similar manner it can be shown that when a loop is rotating in a magnetic field the e.m.f. (in abvolts) induced in it is likewise equal to the rate of change of the linking magnetic flux.

When the flux is changing in a coil of N loops, the induced e.m.fs. in the individual loops are in the same direction; then

$$E = NE_1 = \frac{N\phi}{10^8 t} = \frac{\text{rate of change of line turns}}{10^8}. \qquad (11)$$

When the flux changes within coils of wire, the induced e.m.f. depends on the rate of change of $N\Phi$, as expressed by Eq. (11). The product $N\Phi$ represents the number of loops times the flux linking the loops and for convenience is called *line turns, flux turns*, or *flux* or *line linkage*.

8. Lenz's Law.—Article 1, together with an inspection of Figs. 1, 5, shows that the magnetic field of the induced electron flow always strengthens the inducing magnetic field on the cutting side. A study of Fig. 9 shows that the induced e.m.f. within a loop also has a direction such as to tend to produce a flow whose magnetic field increases the flux within the loop when that flux is being diminished by the loop's motion and decreases it when the loop's motion tends to increase that flux. These two sets of facts are stated in what is known as *Lenz's law:*

The direction of an induced electromotive force is such as to tend to produce a flow whose magnetic flux (1) in the case of a conductor strengthens the inducing flux on the cutting side and thereby evokes a force acting in opposition to that which produces the flow, and (2) in the case of a loop also tends to oppose any change in the flux linking the loop. (12)

Either of these statements of Lenz's law may be used conveniently for determining the direction of the induced e.m.f. in place of the more direct Law A_2 and in place of the nonconservative electric field which a moving magnetic field is assumed to evoke.

Questions

1. Show that, when a conductor is moving at right angles to the lines of force in a magnetic field, the free electrons in the conductor are pushed to

one side and establish a potential difference between its two ends; and, also, that if the conductor is a part of an electric circuit, the free electrons in the whole circuit are set into motion.

2. Explain why, when the electrons in the foregoing case are set in motion along the conductor, a force is exerted on them opposing that which is moving the conductor of which the electrons are a part. Show that all the work done in pushing the conductor against this opposing force is used to move the electrons, *i.e.*, is converted into electric energy and finally into heat.

3. Give the direction of the resultant of the two electron velocities in the foregoing conductor and of the resultant of the two forces acting on each electron.

4. Derive the expression for the energy expended when a part of a closed circuit has cut a definite amount of magnetic flux. Show from the equation that energy is expended whenever a current is induced.

5. Define e m.f. and show why it can be measured in terms of work. Also why it can be measured by the electrostatic potential difference it establishes in an open circuit. Distinguish between e.m.f. and p.d.

6. Derive by two methods the expression for the induced e.m f. in volts. (*a*) In terms of the velocity of the conductor. (*b*) In terms of the rate of cutting magnetic flux.

7. Give the experiment which demonstrates that the e.m.f. induced in a stationary conductor by a moving magnetic field is the same as that induced in a moving conductor cutting a stationary magnetic field at the same rate. Explain why the moving magnetic field cannot be the direct cause of the induced e.m.f ; also why no electrostatic field can be.

8. What is a nonconservative electric field?

9. State B.P.3.

10. Derive the expression for the intensity of the nonconservative electric field evoked by a moving magnetic field. For the intensity of the magnetic field evoked by a moving electrostatic field.

11. Show that when a current is induced in any circuit its direction is such that its magnetic field tends to oppose any change of magnetic flux linking the circuit. State Lenz's law.

12. What is the induced e.m.f. in terms of the rate of change of flux within a loop? Why?

13. Define line turns, flux turns, line linkage, and flux linkage.

Problems

1. A wire in which the current is 20 amp cuts 200,000 maxwells in a given time. What is the amount of work, in joules, expended in moving the wire against the evoked force?

2. What is the induced e.m.f., in volts, when a wire 20 cm long is cutting magnetic flux at the rate of 40,000,000 maxwells/sec?

3. What is the induced e.m.f. when the same wire is moving at right angles to a field of 10,000 oersteds with a velocity of 10 meters/sec?

4. A loop of wire has the magnetic flux which links it changing at the rate of 40,000 maxwells/sec. What is the induced e.m.f. in volts?

5. The flux linking a coil of 8 turns is changing at the rate of 20,000 maxwells/sec. What is the induced e.m.f. in volts?

6. A magnetic field of 10,000 gauss is moving with a velocity of 100 meters/sec. What is (*a*) the intensity of the evoked nonconservative electric field and (*b*) the potential difference, in volts, that this electric field establishes in a conductor 10 cm in length?

7. An electric field of 100 e.s.u. intensity is moving with a velocity of 100 meters/sec. What is the intensity of the evoked magnetic field?

8. A magnetic field whose flux density is 5,000 gauss is cutting through a conductor of 20 cm length with a velocity of 30 meters/sec. What are the magnitude and direction of the induced e.m.f. when the flux points from the observer and the field is moving upward? (Assume direction of e.m.f. to be that in which electrons are urged.)

9. What is the induced e.m.f. in volts when the flux turns linking a coil are changing at the rate of 500×10^6 per 0.04 sec?

Experiments

1. An e.m.f. is induced in a wire, which is part of a circuit, by moving a magnet or a solenoid in its neighborhood.

2. An e.m.f. is induced in a circuit when any part of it is cutting magnetic flux in such a manner that the amount of flux within the circuit changes.

3. An e.m.f. is induced in a circuit by relative motion between a magnet and a coil which is part of the circuit.

4. A copper ring swung into a strong magnetic field is checked by the forces evoked by the induced flow (Lenz's law).

5. A copper ring with a slit does not show this effect.

CHAPTER XIV

ELECTROMOTIVE FORCE BETWEEN DISSIMILAR SUBSTANCES IN CONTACT

1. Potential Difference between Dissimilar Solids in Contact.—When two dissimilar metals are in contact, one of them is always at a higher potential than the other (Art. I–1). This fact may be attributed in part to the differences in the chaotic velocities and in the densities of the free electrons within the materials. More free electrons pass through the boundary in one direction than in the other until a potential difference between the metals is established which equalizes the tendency to transfer. That which causes the electron transfer is the e.m.f. and must be distinguished from the electrostatic potential difference it establishes between the metals in contact. The e.m.f. between metals in contact is always small; that between zinc and copper at room temperature is 0.0007 volts.

In the following list each metal when in contact with one lower on the list acquires the higher potential: aluminum, zinc, iron, nickel, copper, bismuth, antimony, silver, platinum, gold.

The above-described *e.m.f. between metals in contact* must be distinguished from the conventional term *contact electromotive force.* A potential difference exists between each metal and the air in contact with it whose magnitude depends on the nature of the metal. When two metals are in contact, a p.d., therefore, exists between points just outside the metals. That which causes this p.d. is called the contact e.m.f. It is larger than the e.m.f. between the metals and for zinc and copper in contact has a resultant magnitude of 0.7 volts.

2. Potential Difference between Dissimilar Liquids in Contact.—A potential difference exists between two dissimilar liquids, or between two liquid solutions of the same substance but of different concentrations, when they are placed in contact. In dilute H_2SO_4, for example, the H^+ ions have a greater average

velocity than the heavier SO_4^{--} ions. If dilute H_2SO_4 then be placed in contact with pure water, more of the faster positive ions than of the slower negative enter the water. This action charges the water positively and the dilute H_2SO_4 negatively. The H^+ and OH^- ions of the water are too few to produce an appreciable transfer of H^+ ions in the reverse direction.

3. Potential Difference between a Solid and a Liquid in Contact.—The forces of cohesion between the atoms of a solid are especially great at the surface because here the atoms are brought nearer one another by surface tension. This condition diminishes the tendency, common to all substances, for passing into a gaseous state.

When a metal is immersed in an electrolyte, the forces of adhesion between it and the liquid decrease the surface tension of the metal, thereby enabling the + ions of such brittle substances as zinc and cadmium, in which the forces of cohesion are comparatively small, to pass out of the metal into the solution. This departure of + ions leaves the metal charged negatively and the liquid positively. The departure continues until an electric field is established between the metal and the liquid whose magnitude is such that in conjunction with the "osmotic pressure" (see below) it just prevents further passage of ions into the liquid. The metal and the liquid form a condenser whose dielectric is the intermolecular space between the two charged contact surfaces (the *Helmholtz double layer*), between which an electric field and a potential difference exist. The charges on the plates of a condenser are almost entirely on the sides of the plates facing each other. In the Helmholtz double layer, then, the Zn^{++} ions that have left the metal are for the most part only slightly displaced to form the liquid side of the double layer. Only a trace of the metal, too small to be detected by chemical means, passes into the solution. That Zn^{++} ions are actually in solution can be detected only by biological methods, *e.g.*, the toxicity of zinc toward the root cells of growing plants.

The tendency of all metals to pass into solution as + ions and to form the Helmholtz double layer is referred to as *electrolytic solution pressure.*

When the metal immersed in an electrolyte is one in which the cohesive forces are very great, such as is the case in copper,

the contact of the liquid does not diminish the cohesion sufficiently for the metal ions to pass into solution.[1] The forces of adhesion between metals (including hydrogen) are greater than those between a metal and a nonmetal. The + metal ions of the solution, therefore, are forced nearer the immersed metal than the normal intermolecular distance of the solution. These ions thereby deposit on or become a part of the immersed metal and charge it positively. They continue to deposit until a potential difference is established between the metal and the electrolyte which prevents further deposition of the + ions. The excess − ions are drawn toward the now positively charged plate but are prevented from being pulled out of the solution by greater molecular forces which hold them within and at practically normal intermolecular distance from the charged plate. The intermolecular space between the metal and the liquid then becomes the dielectric of an electric condenser and the two charged contact surfaces are a Helmholtz double layer as in the case of zinc and the electrolyte. In this case, however, the metal is normally positively charged and always has a higher potential than the liquid. This displacement of the ions is usually referred to by the statements that "+ ions tend to pass out of solution" or that "+ ions are forced out of solution by *osmotic pressure.*"

The + ions of metals tend to pass into solution, and the + ions of solutions tend to pass out of solution. **(1)**

Which of these actions takes place depends on which of the forces urging the ions is the greater, and therefore upon the particular metal and liquid in contact. Any solid immersed in a liquid, including the colloidal particle, becomes more or less positively or negatively charged.

4. Simple Voltaic Cell.—When a copper and a zinc plate are immersed in a dilute solution of H_2SO_4, one of the plates acquires a higher and the other a lower potential than that of the liquid, as explained in Art. 3. If a copper wire is connected to each of the plates, Fig. 1, the component parts contributing to the potential difference E between the two copper wires are illustrated in the potential diagram of the figure. The magnitudes of the potential differences that make up the total depend

[1] However, copper appears to dissolve slightly, perhaps as CuO, for algae and ferns are killed when copper is placed in water with them.

somewhat on the density of the solution. If the solution contains 1 gram equivalent of H_2SO_4 per liter, the potential difference between the two copper wires is 1.08 volts, of which probably 0.62 volts is between the zinc and the electrolyte, and 0.46 volts between the electrolyte and the copper. The potential difference of 0.0007 volts due to the contact of one of the copper wires with the zinc contributes only a negligible amount to the total potential difference.

If the local action which may occur on the zinc plate, when it contains impurities, is disregarded, all action stops as soon as the potential differences illustrated by the diagram are established.

If the two copper wires are now connected together by another copper wire, not shown, an electric circuit is completed and no

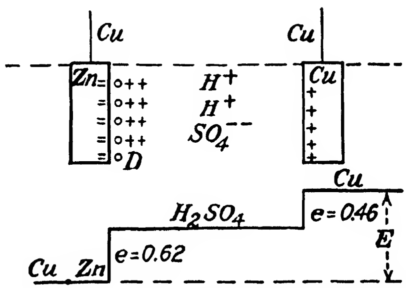


FIG. 1.—Simple voltaic cell and its potential diagram.

additional contact potential differences are introduced. Electrons then flow from the zinc through the connecting wire to the copper plate, *i.e.*, from the lower to the higher potential. The departure of these electrons liberates the zinc ions that were bound by them in the Helmholtz double layer of the zinc plate, and the transfer of the electrons to the copper plate lowers the potential of that plate. The departure of the electrons lowers the potential difference between the two sides of the double layer, enabling the solution pressure to force more Zn^{++} ions out of the metal. The lowering of the potential by the arrival of the electrons at the copper plate enables the "osmotic pressure" to force more H^+ ions out of the solution. The Zn^{++} ions entering the solution repel the H^+ ions toward the copper plate and form $ZnSO_4$ with the appropriate number of the SO_4^{--} ions of the solution.

The H^+ ions on reaching the copper plate are discharged by electrons from that plate In this manner electrons are continually leaving the copper plate and therefore may be considered as entering the solution. At the zinc side the Zn^{++} ions, going into the solution, leave − charges on the zinc plate. This action is equivalent to − charges entering the plate from the solution. Just as many electrons must enter the solution on the copper side as "leave it" on the zinc side; otherwise the solution would become charged and stop further action.

The total potential difference established by the various contacts in the circuit is the measure of the e m.f. of the cell.

The hydrogen gas collecting on the copper plate diminishes the area through which the H^+ ions can be discharged and in addition forms an insulating layer upon which charged ions can collect and repel others that are moving toward the copper. Both of these actions reduce the current: the one increases the resistance, and the other introduces a counter e.m f., as already noted in Art. XI–6. Any cell whose resistance is increased in this manner and in which a counter e m f. exists is said to be *polarized*. This polarization is best eliminated by use of two solutions as in the Daniell cell (Art 5).

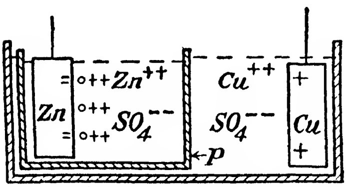


FIG 2 —Diagram of Daniell cell

Any two dissimilar metals immersed in an electrolyte produce a current when they are connected externally by a conductor. The e m.f. depends only on the electrostatic contact potential differences (Art. XIII 3) between the plates and the electrolyte. But the magnitude of the current also depends on the resistance of the solution, the degree of polarization, and the external resistance.

5. Daniell Cell.—The Daniell cell (Figs. 2, 3) consists of a zinc plate in a porous cup P which is filled with a weak solution of $ZnSO_4$ or H_2SO_4. The porous cup with its zinc plate and the copper plate are immersed in a saturated solution of $CuSO_4$. The ions present are as shown in the figure. Potential differences are established between the metal plates and the solutions as in the case of the simple voltaic cell. At the copper side,

however, the saturated $CuSO_4$ solution now furnishes Cu^{++} ions, which are forced out of solution and charge the copper plate positively.

When the plates are connected together by a wire, electrons, as in the simple voltaic cell, flow through it from the zinc to the copper plate. In the liquid the Zn^{++} ions that are released from the double layer go into the solution and raise its potential near the zinc plate. The Zn^{++} ions of the solution are then forced into the pores of the vessel, where they attract and with-

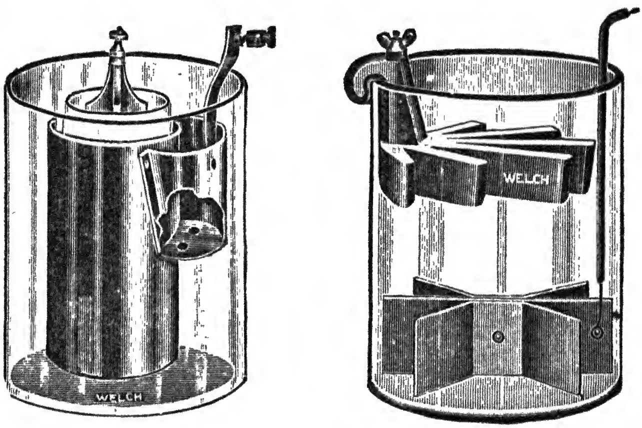


FIG. 3.—Two commercial types of the Daniell cell. In one the zinc sulphate solution is in a porous cup, and, in the other, it floats on the denser copper sulphate solution.

draw the SO_4^{--} ions from the $CuSO_4$ solution. The electrons which enter the copper plate through the connecting wire diminish the plate potential and thereby enable more Cu^{++} ions to be forced out of the $CuSO_4$ solution. The electrons which neutralize these copper ions may be considered as entering the solution. At the zinc side the departing Zn^{++} ions leave $-$ charges on the plate. These two transfers are equivalent to a transfer of electrons through the electrolyte from the copper to the zinc, whereby the electron flow in the circuit is completed as already described for the case of the simple voltaic cell.

The potential diagram of the Daniell cell is similar to that of the simple voltaic cell, Fig. 1, except that a small potential difference exists at the contact of the two liquids.

The Daniell cell does not polarize because copper depositing on copper does not change the conditions at the electrode. The e.m.f. of this cell is 1.10 volts.

6. Leclanché Cell.—Any two dissimilar elements in an electrolyte or in two electrolytes form a voltaic cell. The Leclanché cell, Fig. 4(*a*), consists of zinc and carbon electrodes immersed in a solution of ammonium chloride. The depolarizing or second liquid is replaced by manganese dioxide mixed with granular coke and bound around the carbon plate. The MnO_2 unites with the + ions which otherwise would collect on the carbon and polarize

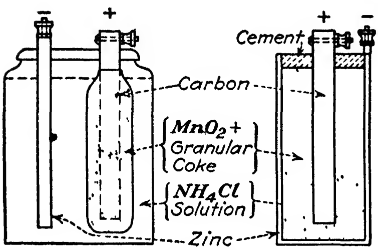


FIG. 4.—(*a*) Leclanché cell; (*b*) dry cell; absorbent layer holding the solution is not shown.

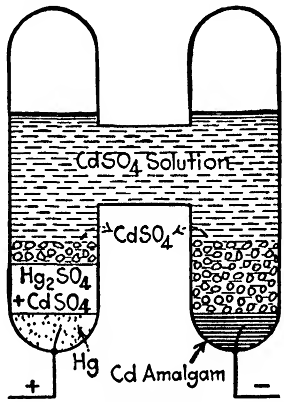


FIG. 5.—Cadmium standard cell.

the cell. These cells are sold under various names. The dry cell, Fig. 4(*b*), is a sealed Leclanché cell in which all the solution is within the pores of some absorbent material next to the zinc. This type of cell depolarizes slowly and has an e.m.f. of 1.4 volts.

7. Cadmium Standard Cell.—The cadmium standard cell, constructed as shown in Fig. 5, has a very low temperature coefficient and, when made according to certain specifications from pure chemicals, has an accurately known e.m.f. Therefore it is employed to furnish a known e.m.f. for comparison purposes. Its e.m.f. at 20°C is 1.01830 international volts. The cell polarizes and is not used for the production of an electric current.

8. Storage Cell (Secondary Cell).—The storage cell, or, as it is sometimes called, the secondary cell or accumulator, consists of two plates or two sets of plates in a single electrolyte. It

therefore does not differ essentially from a simple voltaic cell except that it does not polarize and that the chemical changes which take place during the discharge of the cell may be reversed and the active material restored to its original condition by passing an electric current through the cell in a direction opposite that of the discharge. No electricity is actually stored in the cell. Energy is expended in charging because ions are being forced against the e.m.f. (Art. XVII-8) of the cell. This

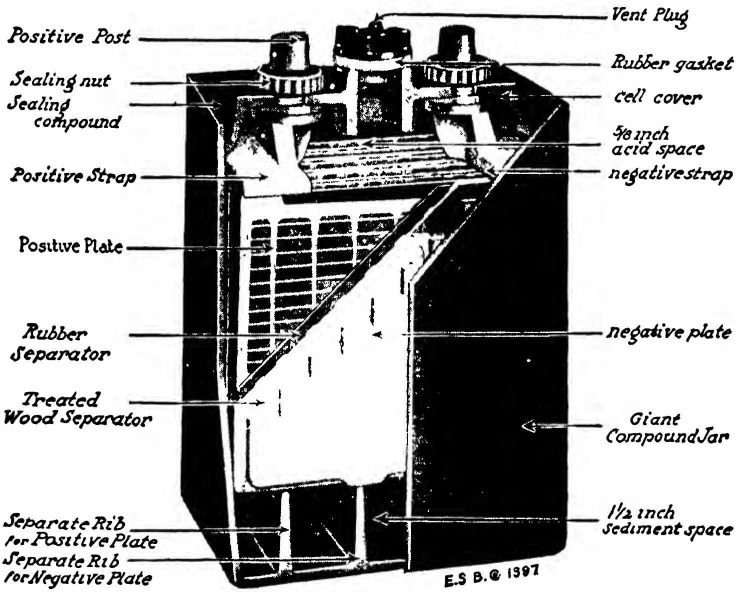


FIG. 6—Lead storage cell. (*The Electric Storage Battery Company*)

expended energy becomes the potential energy of the ions or of their products in their new chemical positions. This chemical potential energy is reconverted into electric energy when the cell discharges.

Lead Storage Cell.—The lead storage cell, Fig. 6, consists of two or more grids of lead or lead-antimony alloy which support the active materials in dilute sulphuric acid. In the charged condition the active material of the positive plate is lead peroxide (PbO_2), and of the negative plate spongy lead (Pb). During discharge the SO_4^{--} ions of the sulphuric acid in the electrolyte

combine with the active materials of both plates and form lead sulphate ($PbSO_4$).

The following formula shows the composition of the positive and the negative plates when charged and when discharged:

$$\underbrace{\underset{\text{Positive}}{PbO_2} + 2H_2SO_4 + \underset{\text{Negative}}{Pb}}_{\text{Charged}} \rightarrow \underbrace{\underset{\text{Positive}}{PbSO_4} + 2H_2O + \underset{\text{Negative}}{PbSO_4}}_{\text{Discharged}}$$

It will be noted that during the discharge H_2SO_4 is withdrawn from the electrolyte and combines with the active material of the plates. This withdrawal and the formation of an additional quantity of water cause the density of the electrolyte to diminish; hence the state of charge can be measured by the

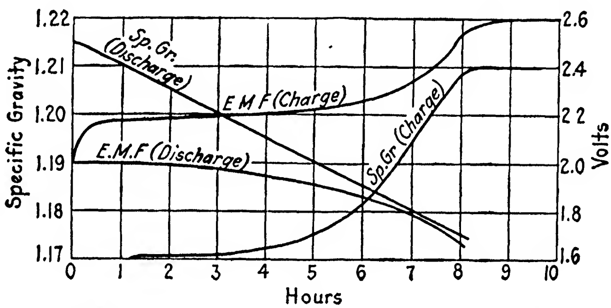


FIG. 7.—Charge, discharge, and density curves for a lead storage cell.

density of the solution. The plates when charged are dissimilar and on being connected into an electric circuit function like the plates of a simple voltaic cell.

The relation between the state of charge and the density of the electrolyte is shown in Fig. 7. This figure also gives curves showing the voltage of the cell during charge and discharge at normal rates.

Edison Storage Cell.—The Edison storage cell, Fig. 8, consists of a set of positive plates containing the positive active material, nickel oxide[1] (NiO_2), and a set of negative plates containing the negative active material, finely divided iron (Fe). The nickel oxide, packed in alternate layers with nickel flakes, is contained in perforated tubes; the finely divided iron, in perforated pockets.

[1] When the cell is manufactured, the positive active material is nickel hydrate ($Ni(OH)_3$) which after formation becomes NiO_2.

Tubes, pockets, grids, pole pieces, containers, etc., are made of nickel-plated steel. The electrolyte is potassium hydroxide (KOH).

The chemical changes within the cell are not definitely known, but during discharge the NiO_2 changes to Ni_3O_4, and the Fe to

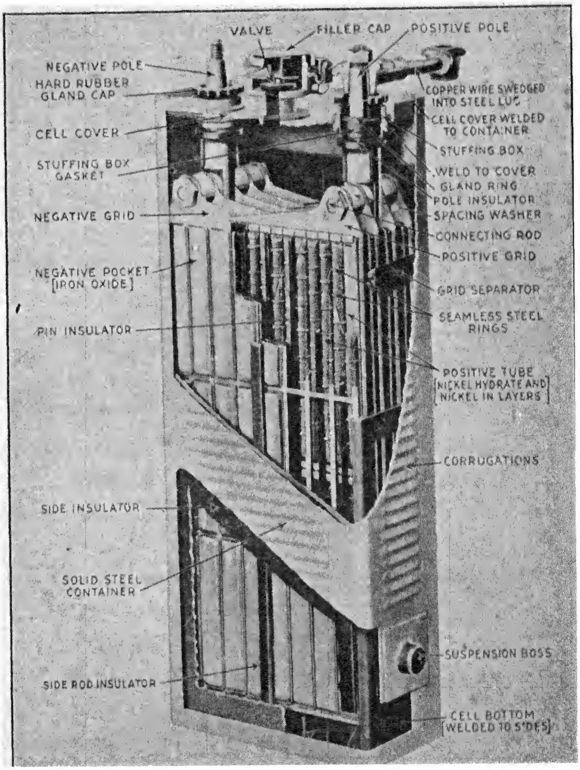


FIG. 8.—Edison storage cell. (*Edison Storage Battery Company.*)

Fe_3O_4. The density (1.20) of the solution does not change appreciably during the cycle of charge and discharge. The state of charge of the cell is determined by the voltage.

The curves of Fig. 9 give the voltage of the cell when charging and discharging at normal rates.

9. Source of Energy of the Voltaic Cell.—When a metal goes into solution, heat is liberated; and when it goes out of solution,

heat is absorbed. This means that atoms of metals in the solid state have a greater potential energy than in solution.

In the Daniell cell, for example, zinc is going into and copper out of solution. The amounts of energy absorbed and liberated

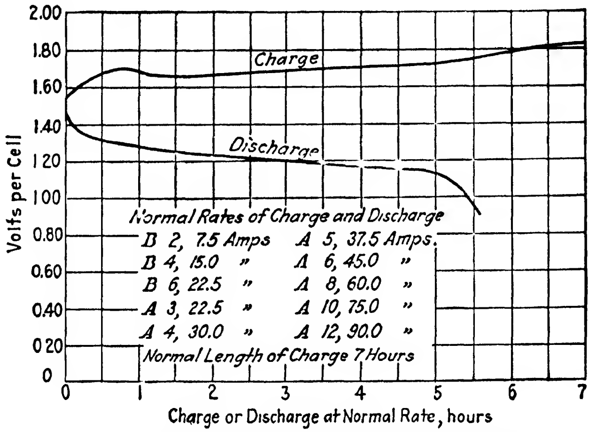


FIG. 9.—Charge and discharge curves for Edison storage cell.

by the zinc being dissolved and the copper deposited give a difference in favor of the energy liberated. This difference is calculable; and if, as in the case of the Daniell cell, no heating other than that due to the current is produced, all this excess energy is expended in producing the electric current. The amount of this excess energy, calculated from the mass of the zinc dissolved and of the copper deposited per coulomb, gives the datum for the calculation of the e.m.f. of the cell.

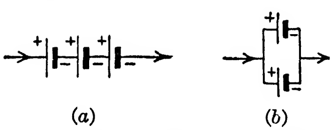


FIG. 10.—(a) Cells in series; (b) cells in parallel.

A voltaic cell transforms chemical energy into electric energy. In a perfect voltaic cell, the excess of chemical energy in joules per coulomb liberated at one plate over that absorbed at the other gives the electromotive force of the cell in volts. (2)

10. E.M.F. of Cells in Series and in Parallel.—The e.m.f. of cells connected in series, Fig. 10(a), is the sum of the e.m.fs. of the individual cells. When similar cells are connected in parallel, Fig. 10(b), they are equivalent to one larger cell; hence there is no increase in the e.m.f. The only advantage of this arrangement

is to diminish the internal resistance of the battery and to increase the current-carrying capacity.

An *electric battery* consists of two or more primary or storage cells connected either in series or in parallel. The term electric battery is also being erroneously used to designate a single voltaic cell.

Questions

1. Explain the potential difference between two dissimilar metals in contact. Between two dissimilar liquids in contact.

2. Explain why zinc acquires a negative charge and copper a positive charge when immersed in an electrolyte.

3. Plant products immersed in a water solution acquire a − charge. What colloidal dyes, with respect to the charges on their particles, should therefore be used in coloring them?

4. Explain the action of a simple voltaic cell, and draw its potential diagram.

5. Explain the polarization of the simple voltaic cell.

6. Explain the action of the Daniell cell, and show why it does not polarize.

7. What is the Leclanché cell? The cadmium standard cell?

8. What is the principle of the action in a storage cell?

9. Describe the lead and the Edison storage cells. How is the degree of charge of each type of cell determined?

10. What is the source of the energy supplied by a voltaic cell? How can the e m f of a cell be calculated from the amount of chemical energy which is liberated?

11. What are the combined e m.f. and combined resistance of several cells connected in series? In parallel?

Problems

1. A Daniell cell whose e.m.f. is 1 10 volts and whose internal resistance is 0.5 ohms is connected to an external resistance of 7.5 ohms. (*a*) What is the magnitude of the current? (*b*) The potential difference between the binding posts? (*c*) The *RI* drop within the cell? (*d*) How much power is being expended in the external resistance? (*e*) How much within the cell?

2. Six of the preceding Daniell cells are first connected in series and then in parallel. (*a*) What are the e.m.f. and internal resistance of each combination? (*b*) What current flows when the external resistance in each case is 0.1 ohms? (*c*) When the external resistance is 10 ohms? (*d*) Make a statement as to when cells should be connected in series and when in parallel to obtain the maximum current.

Experiments

1. The contact potential difference between metals and an electrolyte tested by showing that an electrostatic p.d. exists between the poles of a

battery of voltaic cells. The charge given the testing electroscope may be increased by making its plate one of the plates of a mica condenser. This charge becomes free when the second plate of the condenser is removed.

2. Copper goes out of a solution of copper sulphate and deposits on an immersed strip of iron, illustrating the forcing of positive ions out of a solution and the charging of a metal positively by contact with an electrolyte.

3. Simple voltaic cell in action. Polarization.

4. The charge and the discharge of a storage cell illustrated by two lead plates in a dilute solution of sulphuric acid.

5. Various types of primary and secondary cells shown.

6. Cells in series with a high and with a low external resistance. Cells in parallel.

CHAPTER XV

ELECTROMOTIVE FORCE PRODUCED BY ELECTROSTATIC INDUCTION, HEAT, AND LIGHT, AND BY MISCELLANEOUS PROCESSES

1. Electrophorus.—The principle of electrostatic induction (Art. IV–5) is generally employed whenever the main object is to attain large, steady potential differences. The electrophorus, embodying this principle, consists of a hard rubber or rosin plate A, Fig. 1, which when rubbed with cat's fur becomes charged negatively. Since hard rubber and rosin are dielectrics, the charges on them do not flow freely and therefore remain wherever they are produced. A conducting metal plate B when placed on top of the charged plate A touches it only at a few points, as shown in an exaggerated form in the figure. The plate B has induced on it a + bound charge on the lower surface and a − free charge on the upper. When the upper surface is grounded for an instant, as shown at D, the free charge escapes. The bound + charge, which remains on the plate, has zero potential and therefore is without potential energy. If the metal plate now is raised by means of the insulating handle C, the bound + charge becomes free and the plate acquires a positive potential and potential energy.

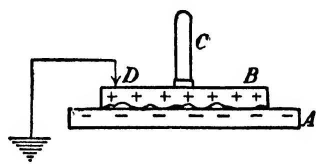


FIG. 1.—Electrophorus.

This energy of the free charge is necessarily equal to the work expended in separating the plates against the attraction of the electric charges and is $W_J = \frac{1}{2}QV$ joules (Art. XII–12). On account of the small capacitance of the plate the quantity of electricity on it is small even though the potential may be more than 50,000 volts.

2. Water Dropper.—The water dropper, Fig. 2, consists of two insulated hollow cylinders, the lower one of which contains

a funnel. Drops of water coming from a grounded vessel or the city mains fall through the cylinders into this funnel. If the upper cylinder is charged positively, the drops, while they are being formed, are charged negatively by induction. These charged drops fall into the funnel where, on striking the metal, their charges flow to the outer surface (Art. IV–3) of the cylinder. The charging of the lower cylinder continues until the charge on it becomes such that it repels the falling drops with a force equal to that of gravity. The drops then are forced outward as a spray.

3. Electric Doubler.—The electric doubler, Fig. 3, consists of two insulated metal beakers and two conducting spheres supported on insulating handles. The spheres are brought into contact in the position shown. The beaker *A* is given a small initial charge, which charges the spheres oppositely by induction as represented. The two spheres, carried by means of the insulating handles, then are touched to the inside of the beakers, sphere *D* to beaker *A* and sphere *C* to beaker *B*. Opposite charges are given to the beakers and pass to the outer surfaces. Because of the successive increases in the charges on the beakers, each repetition increases the induced charges on the spheres. Finally the potentials of the beakers become such that as much electricity leaks off as can be supplied. Only that part of the total mechanical work done which is expended against the action of electric forces is converted into electric energy of the charges on the beakers.

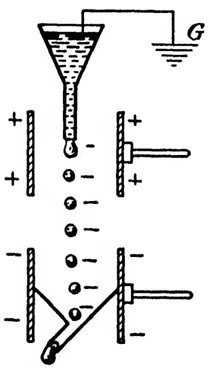


FIG. 2—Water dropper.

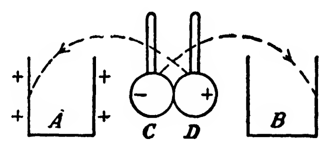


FIG. 3.—Electric doubler.

4. Simple Electrostatic Machine (Kelvin Replenisher).—The Kelvin replenisher is the simplest type of the electrostatic machine. It applies rotation to the principle of the electric doubler. The semicircular conductors, *A* and *B*, Fig. 4, are

insulated from each other, and each has a contact brush m on its inner surface. The conducting carriers C and D revolve on an insulating rod and, when in the position shown, are in metallic contact by means of brushes n_1 and n_2 through the conducting rod R. The conductor A is initially charged either by the friction of the carrier against the brush m_1 or by some other means. If the conductor has a $+$ charge, the carrier D is charged negatively by induction when in the position shown, and the carrier C positively. When the carriers leave the brushes, they carry these induced charges with them. When D touches the brush m_2, the greater part of its charge flows (Art. IV–3) to the outer

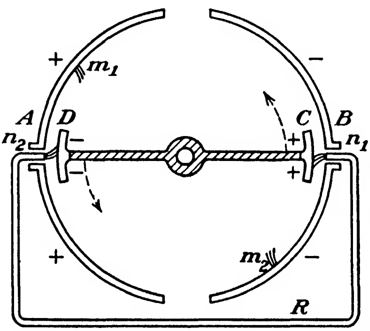


FIG. 4.—Simple electrostatic machine (Kelvin replenisher)

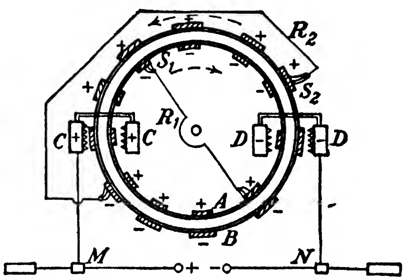


FIG. 5.—Diagram of Wimshurst machine (condensers not shown).

surface of the curved conductor B. The carrier C in a similar manner gives most of its $+$ charge to A. The increased $+$ charge on A and the $-$ charge on B induce greater charges on the carriers when they again come in contact with the brushes of rod R. These charges are again given to the conductors A and B. In this manner the charges and the potentials of the conductors increase until as much electricity escapes over the insulators or from the sharper edges of the conductors as is being supplied. If a spark gap is provided between the conductors A and B, a discharge may take place through the gap before this limiting condition is reached. The machine may be considered as accomplishing a transfer of electrons from one curved conductor to the other at the expense of external work.

5. Electrostatic Machine.—*Electrostatic* or *induction machines* are more efficient modifications of the Kelvin replenisher and

produce greater potential differences. The *Wimshurst machine*, Figs. 5, 6, is one of these modifications. It consists of two insulating disks which rotate in opposite directions on the same axis. These disks have equal diameters but for purposes of illustration are drawn as concentric cylinders, A and B, Fig. 5. Tinfoil sectors, such as those at S_1 and S_2, are spaced uniformly around the outer side of each of the disks or around the cylinders as shown.

When the cylinders begin to rotate, the upper sectors on the inner cylinder A, for example, acquire a slight $-$ charge through

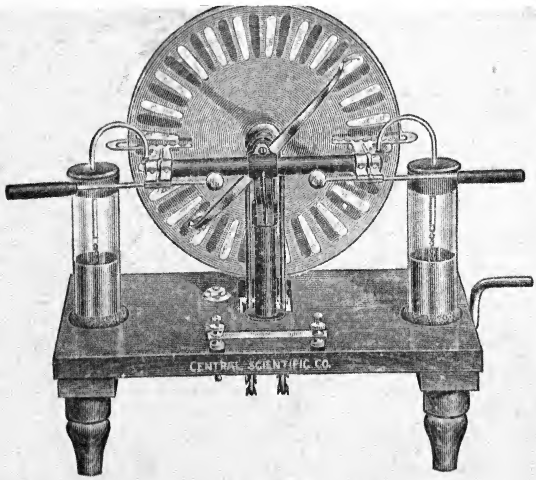


FIG. 6.—Wimshurst machine.

friction with the upper brush of the conducting rod R_1. These charged sectors, assumed to be moving clockwise, induce a $+$ charge in succession on corresponding sectors of the outer cylinder as these sectors pass by the upper brush of the conducting rod R_2. These charges induced on the sectors of the outer cylinder pass by the upper brush of the rod R_1 and induce $-$ charges on the sectors of the inner cylinder. These augmented charges induce increased charges on the sectors of the outer cylinder when they pass by the brush S_2. In this manner the charges on all the sectors continue to increase until the potential becomes such that as much electricity is lost by brush discharges and by conduction over the insulating surfaces as is separated by induction.

When induced charges flow from the rod R_1 onto the upper segments of the inner cylinder, charges of opposite sign flow onto the lower segments. Similar transfers take place on the outer cylinder through the connecting rod R_2.

On both the cylinders the + charges are carried toward the collector CC and the − charges toward the collector DD. Before the potentials are reached at which the segments lose as much electricity as they gain, their charges begin to be collected (Art. XI–10) by the sharp points on the collectors CC and DD.

The charges on the collectors continue to increase until the potential difference between them is sufficient to produce a brush (Art. XI–9) or a disruptive (Art. XI–11) discharge across

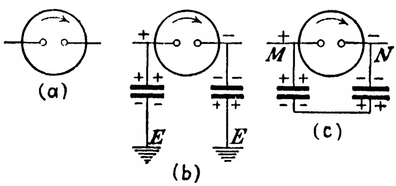


FIG 7.—Representation of an electrostatic machine and its condensers.

the sphere gap. These discharges take place when the potential gradient in the air space becomes about 30,000 volts/cm or when the field intensity $F'' = 100$ e.s.u.

The electrostatic machine transfers electrons from one conductor or collector to the other. The quantity of electricity on such collectors is very small even at the high potentials. To increase this quantity, electric condensers are attached to the collectors as shown in Figs. 6, 7.

Figure 7(*a*) represents an electrostatic machine without the details. Figure 7(*b*) shows two Leyden jars attached to the collectors. It is observed that one plate of each jar is grounded; hence the two plates are connected together because the earth is a conductor; therefore the same result is accomplished by joining these two plates by means of a copper wire as shown in Fig. 7(*c*). This connection places the jars *in series* (Art. XII–11).

If the potential difference maintained between the collectors M and N of the machine is 60,000 volts, the collectors have potentials of +30,000 and −30,000 volts, respectively. An

object brought in contact with either of them acquires a potential of 30,000 volts. If, however, one of the collectors is grounded so that its potential is zero, the potential difference of 60,000 volts maintained by the machine (disregarding the possible greater losses by leakage) appears as the potential of the other collector. An object connected to this collector now acquires a potential of 60,000 volts, positive or negative, depending on the direction in which the electrons are being transferred.

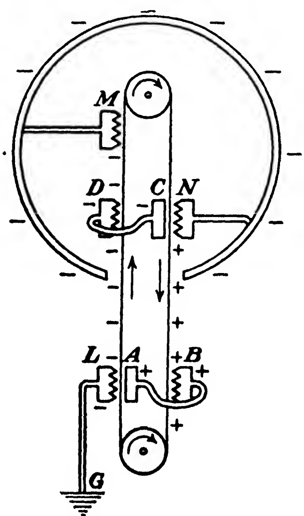


FIG. 8.—Van de Graaff electrostatic generator.

Discharges from electrostatic machines are comparatively harmless, notwithstanding the high potentials. The capacitance of the two Leyden jars in series is of the order of 0.0001 microfarads, giving, with the potential difference of 300,000 volts (10-cm spark) a charge of the order of 0.00003 coulombs with a potential energy of 4.5 joules.

The *Van de Graaff electrostatic generator* not only employs electrostatic induction but makes use of the principle that electric charges carried to the inner surface of a hollow sphere flow to the outer surface (Art. IV–3) regardless of the magnitude of the charge on that surface. The charge and potential of such a sphere can be raised in this manner until an ionizing potential gradient (Art. XI–8) is established at the outer surface. The surrounding air then becomes conducting and spark discharges take place.

Figure 8 shows in detail one method of charging such a sphere. The many-pointed bars *BDLMN* and the polished rods *A* and *C* are shown at right angles to their actual positions with respect to the motor-driven belt. The polished rod *A* is connected to the pointed rod *B* to form one conductor and, similarly, the rod *C* is connected to the rod *D*. The generator usually becomes primed automatically owing to a charge on the belt which may be produced by friction; but, for simplicity, assume the conductor *AB* to have been primed by charging it to a + potential of about

10,000 volts. The conductor L then becomes charged negatively, by induction, to a potential high enough to cause brush discharges (Art. XI–9) to spray the belt with negative ions. The belt, made of a nonconducting material, carries the charges by the points of the conductor D, where again, owing to induction, the + spray from the points discharges part of the − charge on the belt and causes the conductor C to acquire a − charge. The belt carries the balance of the − charge toward the conductor M, where a + spray neutralizes the − charge on the belt and causes the surface of the sphere to receive an equal − charge.

A similar process takes place on the right-hand side of the belt. The − charge on C causes the conductor N to spray + charges to the belt and thereby to add − charges to the surface of the sphere. Part of the + charge on the belt is neutralized in passing the conductor B, and the remainder when it reaches the conductor L. The + charge on A with that on the belt causes the spray from L to be sufficient both to neutralize the + charge and to charge the belt negatively as at the beginning of the process.

When the generator is in full operation, the conductors AB and CD spray the belt only to an extent sufficient to replenish any charge the conductors may have lost by leakage. The operation of the generator is equivalent to carrying − charges from the earth to the sphere and + charges from the sphere to the earth.

The generator usually consists of two such spheres which become charged oppositely by proper priming. Experiments then can be conducted in the strong electric field between the spheres.

It can be shown that the limiting potential to which a sphere can be raised is

$$V = 30{,}000R \text{ volts},$$

where R is the radius of the sphere in centimeters. It is found, however, that a negatively charged sphere actually can be raised to a potential about 1.3 times that of a positively charged sphere.

Electrostatic machines convert mechanical into electric energy. **(1)**

6. Peltier E.M.F.—The e.m.fs. which exist at the contact planes of dissimilar metals (Art. XIV–1) cause the electrons in

the illustrative case of Fig. 9 to drift from the iron into the copper. This electron transfer continues only until a definite potential difference is established between the metals as represented by the potential diagram V. The existence of the potential difference infers the existence of electric fields of intermolecular-space length across the contact planes a and b. That these potential differences exist is shown by forcing an electron flow through these contact planes as represented by the arrows I. This flow heats the junction a abnormally and cools the junction b. This cooling, however, often only partly counteracts the normal heating of the junction by the flow. This heating and cooling

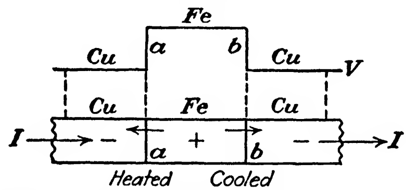


FIG. 9—Peltier effect. The electron flow is represented by the longer arrows I, and the direction of the electric field at the contacts, by the shorter arrows The potential diagram V (above) represents the potential differences at the junctions a and b.

phenomenon, which is reversible, is known as the *Peltier effect* and the e.m.fs. at the junctions are called *Peltier electromotive forces*.

When the electrons flow through the planes a and b, the electric field at a increases their speed and that at b decreases it. These changes in electron speed alter the kinetic energy of the electrons for the next impacts with atoms and therefore alter the heat generated at the junctions. These heating and cooling effects may also be explained by referring to the potential diagram V. Since electrons lose potential energy in moving from points of lower to points of higher potential (Law III–7) and gain it in being moved from points of higher to points of lower potential, the flowing electrons lose potential energy at a and gain an equal amount of it at b. The decrease in potential energy makes that much additional energy available for heating the junction a, and conversely, the increase in potential energy at b is at the expense of energy available for the production of heat.

The Peltier e.m.f. has the same order of magnitude in all metals and depends upon the nature of the metals in contact and upon

the temperature. It is determined experimentally by observing the amount of heating and cooling at the junctions and can be calculated by means of the parabolic equation

$$e_P = (A + Bt)T,$$

where t and T are the temperatures of the junction on the centigrade and Kelvin scales and A and B are constants whose magnitudes are determined by experiment as explained in Art. 8. The Peltier e m.f. at the illustrative copper-iron junction is as given in the following table:

TABLE I

Temperature, Degrees Centigrade	Copper-iron Junction Peltier E M F, Volts
0	+0 00432
100	+0 00375
200	+0 00203
274 5	0 00000
400	−0 00487
600	−0 01637

The + sign indicates that the e.m.f. forces electrons from iron to copper and the − sign that it forces them from copper to iron.

7. Thomson E.M.F.—A small e.m.f. also exists in all portions of a homogeneous conductor in which there is a temperature gradient. This is known as the *Thomson electromotive force.* A temperature difference causes the free electrons in one of any two adjacent sections to drift into the other section until a definite potential gradient (electric field) is established along the conductor. In some metals, including Cu, Ag, Zn, Cd, and Sb, this e.m.f. urges electrons from the warmer toward the colder portions, while in other metals; including Fe, Pt, Co, Ni, and Bi, it urges them from the colder toward the warmer portions.

When a copper rod (representing one of the two classes of metals) is heated as shown in Fig. 10(a), the Thomson e.m.f. forces the free electrons from the heated portion until a condition of electric equilibrium is established. The charges within the conductor then become distributed as shown and thereby produce the Thomson electric fields and their potential gradients represented by the arrows F'' or by the potential diagram V.

When electrons now are made to flow through the copper conductor, as represented by the arrows I, the Thomson electric field F_1'' in the left portion gives these flow electrons an additional velocity (Law III–7) between impacts with atoms. This increase in the kinetic energy of the electrons increases the heating on that side of the conductor. On the right side, however, the Thomson field F_2'' exerts a force opposing the motion of the flow and thereby causes the electrons to lose kinetic energy between impacts and to heat the conductor less.

When iron (representing the other class of metals) is heated, the free electrons in it are forced by the Thomson e.m.f. to flow

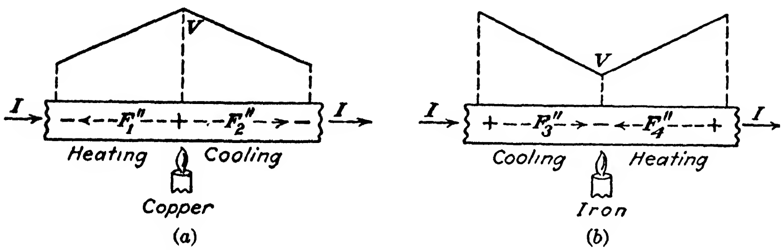


FIG. 10—Thomson effect. Heating a part of a conductor causes the free electrons to be displaced (a) in copper toward the cooler regions and (b) in iron toward the hotter. The broken-line arrows represent the directions of the Thomson electric fields established in the metals and the lines V their potential diagrams.

toward the heated portion. This action is represented in Fig. 10(b), as is also the condition when the conductor is being energized by an electron flow. In this case the right and left portions are heated and cooled in the reverse order to the heating and cooling in copper.

The heating and cooling effects in such unequally heated homogeneous conductors may also be explained by means of the potential diagrams V in a manner employed in Art. 6.

This heating and cooling due to the Thomson potential gradients is superposed in each section on the normal heating of the conductor by the current. Such Thomson potential gradients exist in all unequally heated conductors with the exception of lead.

The magnitude of the Thomson e.m.f. has not been measured but may be calculated for any given length of a conductor from the parabolic equation

$$e_T = -B_1 T_0 t - \frac{B_1}{2} t^2,$$

where T_0 is the absolute temperature at the lower temperature end and t the total temperature difference between the ends. The magnitude of the constant (Art. 8) is
For iron,

$$B_1 = -0.0482 \times 10^{-6}.$$

For copper,

$$B_2 = +0.0094 \times 10^{-6}.$$

The magnitudes of the Thomson e.m.f. between the extremities of a section of any length when one end of the section is at 0°C and the other at a temperature t are given in Table II.

TABLE II

t, degrees centigrade	Thomson e.m.f. (0 to t°C)	
	Iron, volts	Copper, volts
100	+0 00156	−0 00030
200	+0 00360	−0.00070
274 5	+0 00542	−0 00106
400	+0 00912	−0 00178
600	+0 01657	−0 00323

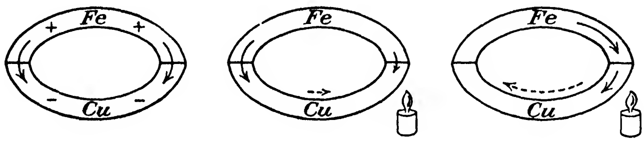
The + signs before the voltages indicate that flow electrons are urged toward the warmer side and the − signs that they are urged in the reverse direction.

8. Seebeck E.M.F.—Thermocouple.—When two wires of dissimilar metals form an electric circuit, as shown in Fig. 11, and one of their two junctions is heated to a higher temperature than the other, the resultant of the Peltier and Thomson e.m.fs. which then exist in the circuit is called the *Seebeck electromotive force,* or simply the *thermoelectromotive force.* Such a circuit, either open or closed, is called a *thermocouple.*

1. How the Peltier e.m.f. contributes to the e.m.f. in a thermocouple circuit is represented in Figs. 11(a, b). The Peltier e.m.f. (Arts. 6, XIV–1) at the contact of dissimilar metals, as already stated, causes a migration of free electrons from one of

the metals into the other and, at room temperatures, forces them from the iron into the copper. This migration causes the metals to become charged oppositely as represented in Fig. 11(a) where the Peltier e.m.f. and the electric field across the contact plane at each junction are represented by the same arrow. When the junction temperatures are equal, the e.m.fs. at the junctions also are equal and their resultant is zero.

When one of the junctions is heated, the e.m.f. and the electric field at the warm junction, in the illustrative case, are diminished as represented by the shortened arrow in Fig. 11(b). The e.m.f. at the hot junction diminishes uniformly with increase in tem-



(a) (b) (c)

FIG. 11.—The two kinds of e m fs. which together produce the e m f in a thermocouple, *i.e.*, (a, b) the Peltier e m f. and (c) the Thomson e m f. Full-line arrows represent the directions in which electrons are urged at ordinary temperatures by the two kinds of electromotive forces, and the broken-line arrows the direction in which the resultant of each of the two tends to move the electrons around the circuit.

perature until it becomes zero and then reverses in direction. It follows that at all temperatures the resultant of the two junction e.m.fs. called the Peltier e.m.f. (in the circuit) tends to force electrons around the circuit in a direction which, represented by the broken-line arrow, is from copper to iron at the hot junction. The e.m.f. in this direction is arbitrarily said to be negative and is represented by the $-$ sign.

2. The contribution of the Thomson e.m.fs. is shown in Fig. 11(c). These cause the free electrons in the iron to be forced toward the warmer and in the copper toward the colder junction (Art. 7), *i.e.*, in the same direction around the circuit as represented by the full-line arrows. The resultant of these e.m.fs. (called the Thomson e.m.f. in the circuit) then is equal to the algebraic sum of the two and is represented in the figure by the broken-line arrow. It tends to force electrons through the hot junction from iron to copper. This direction is arbitrarily called the *positive* or $+$ *direction.*

The active or *Seebeck e.m.f.* in a thermocouple circuit, then, is the resultant of the Peltier and Thomson e.m.fs. and, when one junction is kept at 0°C and the temperature t of the other is varied, is calculable from the parabolic equation

$$e_s = At + \left(\frac{B}{2}\right)t^2, \tag{2}$$

where A and B are constants whose magnitudes are determined by measuring e_s, in volts, for two different known temperatures. The magnitudes of these constants for a copper-iron thermocouple are

$$A = +15.81 \times 10^{-6},$$
$$B = -0.0576 \times 10^{-6},$$

where $B = (B_1 - B_2)$ of Art. 7 and is also the constant B used in calculating the total Thomson e.m.f. in the thermocouple circuit and the Peltier e.m.f. at a copper-iron junction. Since lead has no measurable Thomson e.m.f., the determination of B for any metal with lead gives the B_1 or B_2 for that metal alone and thereby enables the calculation of the Thomson e.m.f. in that metal.

Table III and the curves of Fig. 12 give the magnitudes of the Thomson, Peltier, and Seebeck e.m.fs., in volts, in a copper-iron circuit when one junction is kept at 0°C and the temperature t of the other junction is varied.

TABLE III
Resultant Voltage in Circuit (0° − t°)

t°, degrees centigrade	Peltier	Thomson	Seebeck
100	−0 00057	+0 00186	+0 00129
200	−0 00229	+0 00430	+0 00201
274 5	−0 00432	+0 00648	+0 00216
400	−0 00918	+0 01090	+0 00172
600	−0 02069	+0 01980	−0 00089

It should be noted that the resultant or Seebeck e.m.f. in this copper-iron thermocouple has its maximum value when the temperature of the warm junction is 274.5°C and that the direction

of the e.m.f. reverses at 549°C. All thermocouples have similar properties, except that in many cases the reversal point is at unattainable temperatures. The Peltier e.m.f. at a junction may increase with temperature and may at its beginning have a greater magnitude than the Thomson e.m.f., and the resultant Thomson and Peltier e.m.fs. are not always opposed to each other.

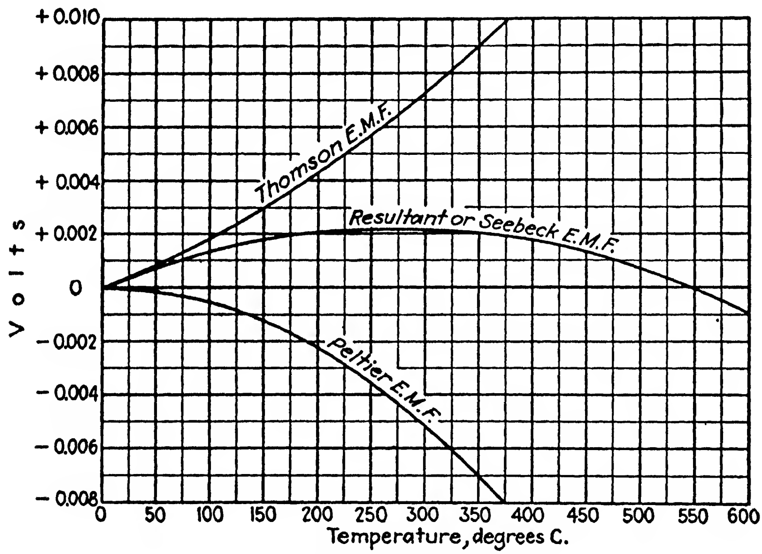


FIG. 12.—The magnitudes and directions of the e.m.fs. in a copper-iron thermocouple circuit when the temperature of the cold junction is kept at 0°C. and that of the hot junction is varied.

It follows from the parabolic equations of the Peltier, Thomson, and Seebeck e.m.fs. that the e.m.f. per 1° difference in the temperatures between the junctions, called the *thermoelectric power*, either increases or decreases uniformly with temperature and that it can be calculated from the following equations, where T_0 is the Kelvin temperature at 0°C and T that of the temperature point in question:

For Peltier thermoelectric power,

$$P_P = A + BT_0 + 2BT.$$

For Thomson thermoelectric power,

$$P_T = -BT_0 - BT.$$

For Seebeck thermoelectric power,

$$P_S = A + BT. \tag{3}$$

These are equations of straight lines which, for the copper-iron thermocouple, are shown in Fig. 13. The thermoelectric diagram of Fig. XXIV–4 shows the Seebeck thermoelectric power for vari-

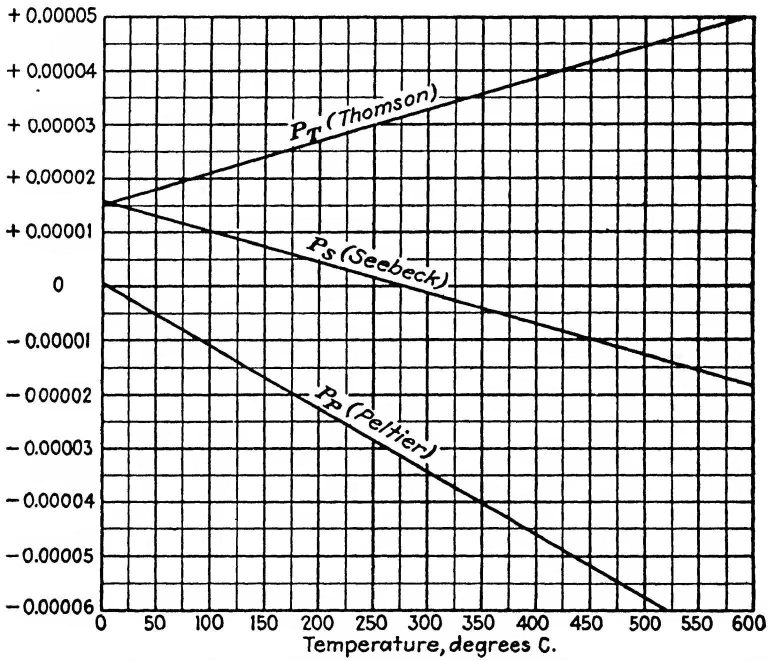


FIG 13 —Thermoelectric powers for the e.m.fs. in a copper-iron thermocouple.

ous metals coupled with lead. In the usual diagrams the term thermoelectric power, unless otherwise stated, refers to the Seebeck effect.

The energy for the maintenance of the current is supplied from heat energy. In the copper-iron thermocouple, at ordinary temperatures, electrons are urged around the circuit at and in the neighborhood of the hot junction in the direction of the established Thomson and Peltier electric fields. They, therefore, are being moved against opposing forces and thereby are being given potential energy. The energy required for this action can come

only from the heat energy within the heated parts of the conductors. The chaotic heat motions are being transformed into the more orderly velocity of the electron flow. The heat energy being absorbed in generating the electron flow is simultaneously transformed into heat in all parts of the circuit by the electron flow (Joule effect) and in passing through the cold junction by the accelerating action of the Peltier electric field. After the Peltier e.m.f. at the hot junction reverses (274.5°C), heat is generated through the action of the Peltier electric field at that junction, but only until the total Peltier e.m.f. in the circuit becomes greater than the total Thomson e.m.f. (549°C) and the electron flow reverses. The heat energy after that is transformed into potential energy in both Peltier fields and is liberated in the regions of the Thomson fields.

The thermocouple is employed mainly for the measurement of temperatures. The e.m.f. per 1°C of temperature difference between the junctions at room temperatures in a good commercial thermocouple (copper and "advance" alloy) is 0.000039 volts at 20°C and is ample for the production of currents of sufficient magnitude to measure temperatures with accuracy. The thermocouple is considered again in Arts. XXIV–6*ff*.

Thermocouples transform heat energy directly into electric energy.

9. Photovoltaic Cell.—When certain crystals which form a part of an electric circuit are illuminated, the free electrons are thereby forced to flow; therefore the light is said to impress an e.m.f. on the circuit. Such crystals and their supports form a *photovoltaic cell.* One form of such a cell consists of a plate of copper which has had its surface oxidized at a high temperature. Another copper plate, *G*, is welded to the upper surface of the cuprous oxide crystals as shown in Fig. 14(*a*). When light falls on the crystals in the immediate neighborhood of the plate *G* as represented by the arrow, electrons are forced to flow across the illuminated boundary from the crystals to the mother metal, the strength of the evoked flow being proportional to the intensity of illumination. The resistance of the cell is high and the e.m.f. small. The photovoltaic cell is used to measure intensity of illumination, to operate relays, to count passing objects, etc. It is used also as a rectifier in another manner and under another

name (Art. XXV–8). Figure 14(*b*) shows a commercial form of the cell.

Photovoltaic cells transform light energy directly into electric energy. **(5)**

10. Miscellaneous Sources of E.M.F.—1. Pressure of two electrodes against the faces of a quartz-crystal plate, cut in one of several directions with respect to the axes, causes the two pressure electrodes to become oppositely charged. The electricity produced in this manner is called *piezoelectricity*. This effect is reversible When the foregoing electrodes are charged oppositely, the charges compress the crystal plate; and when an

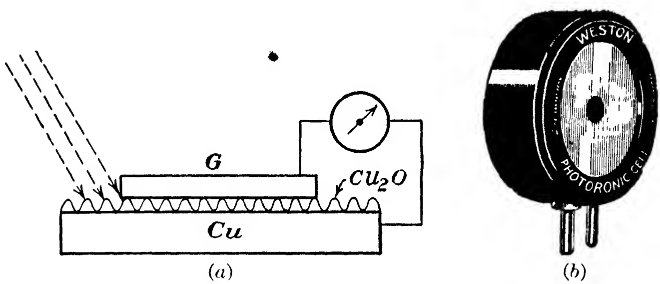


FIG. 14—(*a*) Photovoltaic cell. (*b*) A commercial form of a photovoltaic cell.

alternating e.m.f. is applied to the plates, the crystal vibrates in unison. Higher than audible frequencies produce the so-called *supersonic* waves. Similar effects are produced with plates from other doubly refracting crystals.

2. Distant parts of crystals of tourmaline, quartz, etc., become charged oppositely when the crystals are heated. The electricity produced in this manner is called *pyroelectricity*.

3. A drop of water, when breaking, forms one large positively charged and two small negatively charged drops. In this manner large potential differences may be established between the upper and lower parts of a cloud (Art. XXX–9).

4. Certain fish can give an electric shock. Potential differences exist between different parts of the human body; and the beating of the heart produces systematic potential differences from which information is obtained concerning the condition of the heart.

11. Identity of Electrostatic and Voltaic Electricity.—That voltaic and static electricities are identical is shown by the following experiments:

1. The static electricity from an electrostatic machine, when discharged through a galvanometer, deflects the coil in the same manner as the current from a generator or a voltaic cell.

2. Static electricity discharged through an electrolyte produces electrolysis.

3. When either terminal of a direct-current generator or of a voltaic cell is connected to an electroscope while the other terminal is grounded, the electroscope becomes charged with the appropriate kind of static electricity.

4. An electric condenser becomes charged with opposite kinds of static electricity when it is connected to any two points of unequal potential in an electric circuit.

5. Rowland demonstrated by direct experiment that when static electricity is set in motion a magnetic field surrounds the moving charge.

Questions

1. Explain the action of the electrophorus, the water dropper, the electric doubler, the hollow spherical collector, and the Kelvin replenisher.

2. What is the principle underlying the action of most electrostatic machines?

3. State why and how Leyden jars are used with electrostatic machines. Are they connected in series or in parallel?

4. Why are the high potentials of electrostatic machines comparatively harmless?

5. Give the principle of the Van de Graaff generator.

6. What is the Thomson effect? The Peltier effect? The Seebeck effect?

7. Describe the action of the thermocouple, explaining why heat is absorbed at the hot junction and in the portions having temperature gradients, and why heat is liberated at the cold junction and in other parts of the circuit. From what source is the energy supplied?

8. State how light energy may be transformed directly into electric energy.

9. Give five reasons for believing that static and voltaic electricities are identical.

Problems

1. The capacitance of each of the condensers (including the attached conductors) of an electrostatic machine is 0.001 microfarads. (*a*) What quantity of electricity passes in each spark when the potential difference

between the electrodes of the machine is 200,000 volts? (*b*) What amount of energy is expended in the spark?

2. What is the diameter of the smallest isolated sphere which can be charged in the atmosphere to a + potential of 3,000,000 volts?

3. The temperature difference between the two junctions of a thermocouple, in which the average e.m.f. generated per degree difference of temperature is 0.000040 volts, is 100°C. (*a*) What is the current when the resistance of the circuit is 20 ohms? (*b*) How much energy is being supplied to maintain the current?

Experiments

1. Electrophorus, water dropper, electric doubler, principle of the Van de Graaff generator.

2. Electrostatic machine operated with and without condensers.

3. Peltier effect shown. Thomson effect.

4. A thermocouple and several of them in series shown in action.

5. The current and e.m.f generated in a thermopile measured, and then used to ring a bell.

6. A simple thermocouple made to generate 135 amp in one loop and to energize an electromagnet capable of lifting 400 lb.

7. Photovoltaic generator made to operate a relay.

8. Crystal oscillator.

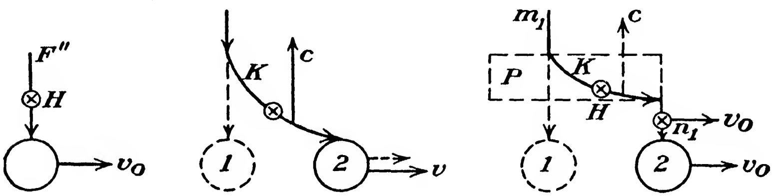
9. A current from an electrostatic machine and from a charged condenser deflects a galvanometer coil (Too high p d. will injure insulation on galvanometer coil)

10. Electroscope charged by a generator and by a voltaic cell.

CHAPTER XVI

ELECTROMAGNETIC PULSE

1. Electric and Magnetic Fields about an Accelerating Electron.—Let F'', Fig. 1(a), represent an electric line of force at right angles to the direction of motion of an electron. Then the cross H represents the direction of the associated magnetic field (Law B) at a point directly above the moving electron. When the motion is uniform, the electric line of force is a straight line and moves at all points with the same velocity as the electron.



(a) (b) (c)

FIG 1.—Electric and magnetic fields (a) about an electron having uniform velocity, (b) about an accelerating electron, (c) about an accelerating electron an instant after it acquired a uniform velocity The distortions are drawn greatly out of proportion, for the electron acceleration and velocity are small compared with the velocity of light

This assumption gives to the line the property of a weighted string whose point of support is moving with uniform speed in a vacuum.

If an electron is accelerated and thereby displaced from its stationary position 1 toward the position 2, Fig. 1(b), what happens to the electric field represented by the electric line of force? Has the field inertia? Or will all parts of the field be displaced simultaneously with the electron? The phenomena of induction (Chap. XVII) and electromagnetic waves can be explained only on the assumption that the electric field has what corresponds to inertia and therefore that the lines of force which represent it may be pictured as possessing inertia. Each electron therefore carries with it only the attached end of each

line of force, while the resulting distortion in a line is transmitted with a finite velocity to the more distant portions. The velocity with which this distortion or pulse is transmitted is found to be that of light. This velocity, represented by c, is 3×10^{10} cm/sec.

Figure 1(c) shows the distorted portion of such a line as it appears an instant after the electron ceased accelerating and therefore had moved a short distance with the uniform velocity v_0. The part m_1 of the line is still unaffected and the whole of the completely displaced part n_1 is moving with the uniform velocity of the electron. The distorted portion K is moving outward with the velocity of light, c, and in so doing is displacing the stationary line m_1 to the right, giving it kinetic energy and placing it in line as a part of the moving and ever-elongating portion n_1.

On account of the great velocity of the pulse along the line of force, the distortion of the line in the direction of the electron acceleration is of necessity drawn on a greatly exaggerated scale compared with the length of the pulse in the direction of propagation; also the properties of the distorted portion are not in every respect those of an ordinary line, because, in addition to being the result of an electric charge, they also depend on the velocity of propagation and on the changing rate of displacement of the electron.

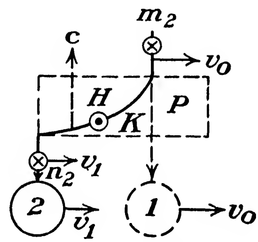


FIG. 2—Electric and magnetic fields about an electron an instant after it ceased to be negatively accelerated, its velocity having diminished during the acceleration from v_0 to v_1.

It should be noted that in the foregoing case, illustrating positive acceleration, the magnetic field associated with the moving electron has the same direction in the distorted portion as in the completely displaced part n_1 adjacent to and moving with the electron.

If an electron in uniform motion has imposed upon it a negative acceleration, the distorted portion K, Fig. 2, is similar to that formed by positive acceleration except that its direction is reversed. The unaffected part of the line m_2 continues to move with its original velocity until it becomes a part of the continually extending line n_2. The distorted portion now carries negative energy and in its motion outward removes energy from the part

m_2 of the line. It should be noted that in this case the magnetic field in the distorted portion has a direction which is the reverse of that in the parts m_2 and n_2. This reversal of the field necessarily follows from the directional relationship given in Law B.

During a positive acceleration the pulse transfers energy to the electrostatic line of force beyond and, in a negative acceleration, takes energy from it. In one case the velocity and therefore the kinetic energy of the moving field is being increased, and in the other case decreased.

2. Electric and Magnetic Fields about a Varying Current—Electromagnetic Pulse.—It was shown (Art. I–3) that surrounding every neutral material body the resultant electric field due to the fields of all the individual protons has its lines of force perpendicular to the body and pointing outward; also that the resultant electric field due to the equal number of electrons is a duplicate of this except that the lines point in the reverse direction. The two electric fields, therefore, neutralize each other's action on electric charges. They are called in this text, as already noted, the electron and proton fields.

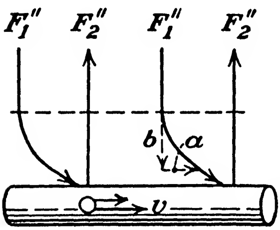


Fig. 3.—Distorted electron field about a conductor in which the intensity (velocity) of the electron flow is increasing.

When a steady current is flowing in a conductor, it was shown (Arts. V–1,4) that the electron field moving with the free electrons exhibits what is called a magnetic field about the conductor. The electron and proton fields still neutralize; and a "stationary" magnetic field alone appears about the current. This magnetic field is cylindrical and its lines of force are concentric with the conductor.

When the intensity of the flow in a conductor is increasing, the flow electrons are being accelerated in the direction of the flow and distortions (Art. 1) are formed in the electron field as represented in Fig. 3. Figure 4(a) shows a section of the distorted field an instant after the electrons have acquired a uniform velocity. In the region outside the pulse the electron field F_1 is neutralized by the proton field F_2. The distance d is equal to that traveled by the pulse while the electrons were accelerating

and is usually many kilometers in length, while l, the distance traveled by the electrons during and after the acceleration, is at the same time less than 0.01 cm.

Figure 4(b) shows in the region U the electron field moving with the uniform velocity of the flow. The superposed fields appear to the observer as the magnetic field which the pulse has just established in that region. The pulse P, represented by several distorted lines, is carrying energy to the still unaffected region beyond and is about to establish there a magnetic field in the manner explained in Art.1. Because a magnetic field is an aspect of a moving electric field, *the electric-field motion of the pulse carries all the magnetic flux which finally is to appear in the*

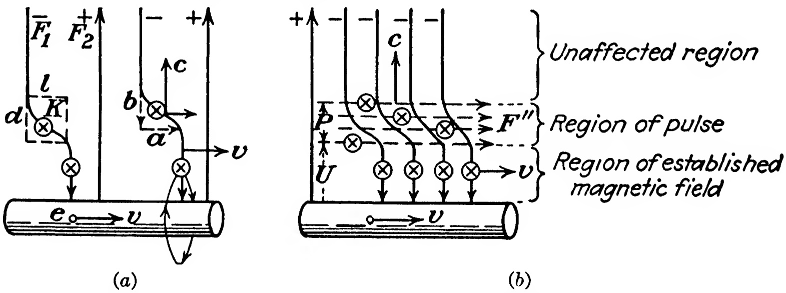


FIG. 4.—(a) Electron and proton fields about a wire an instant after the electrons have acquired a uniform velocity. (b) The same condition as in (a), but several lines of force are shown.

established magnetic field about the conductor. This moving pulse therefore, because of the complexity of its electric field, can be and is more conveniently and with greater clarity treated as a moving magnetic field. This moving magnetic field, then, evokes by its motion a nonconservative electric field as formulated (B.P.3) in Law XIII–7. The direction of this field is that indicated by the distortion and also is that of the electron acceleration, as represented by the broken-arrowed lines F'' in Fig. 4(b). This nonconservative electric field has the power to induce e.m.fs. in electric circuits in the same manner as does the nonconservative electric field evoked by the motion of any magnetic field. The e.m.f. induced in any neighboring circuit, because of field relationship, can be calculated either in terms of the rate of change in the linking magnetic flux or in terms of the rate of change in the spanning nonconservative electric flux.

The pulse is seen to consist of two effective components, the magnetic and the (nonconservative) electric, at right angles to each other and to the direction of pulse propagation. The pulse for this reason is called an *electromagnetic pulse*. The intensities of these components are numerically equal as is shown by substituting in Eq. XIII–8 the velocity c of the magnetic field in the pulse for the ordinary velocity v; *i.e.*,

$$F'' = \frac{[vB]}{c} = \frac{[cB]}{c} = B = \mu_0 H \equiv H.$$

The component b, Fig. 4(a), of the distorted electron field in the direction of the pulse propagation has the intensity of the undistorted field. This is inferred from the observation that charges at rest feel no forces urging them in that direction and is also seen from the fact that the intensity of the electron field is due to the electrons in the conductor and these, on the whole, are at the same distance from any point regardless of their acceleration along the conductor. The intensity of the nonconservative electric field superposed on the electron field by the slow motion of the magnetic field of the pulse in the direction of the electron flow is too small to be taken into consideration.

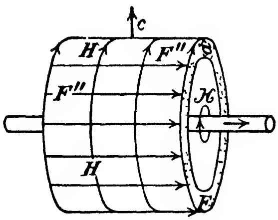


FIG 5.—A section of the effective fields about a wire an instant after a uniform current is established (same condition as in Fig 4(b). F'' and H are moving electric and magnetic fields of thickness d in the radiating electromagnetic pulse. $\mathcal{H}$ is the stationary magnetic field about the current that has just been established.

Figure 5 shows a section of the cylindrical electromagnetic pulse $F''H$ in three dimensions at the same instant at which Fig. 4 shows a section of it in two dimensions, except that only the effective fields are shown. The thickness d of the pulse depends on the length of time the electrons were accelerating and, as already stated, usually is many kilometers. The moving pulse leaves in its path the "stationary" magnetic field $\mathcal{H}$, which surrounds the conductor.

When the flow in the conductor is diminishing, the negative electron acceleration produces a similar cylindrical electromagnetic pulse with the directions of the lines of both its com-

ponents reversed. This field, moving into space, leaves in its path a magnetic field which neutralizes the appropriate part or the whole of the stationary magnetic field produced by the original electron velocity. *This partial or complete neutralization by superposition of the two magnetic fields is in reality merely a diminution of the velocity of the electron field and of the electrons*

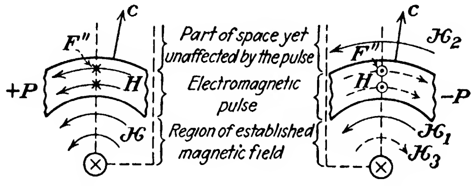


FIG 6.—Cross section of space about a conductor an instant after its free electrons ceased accelerating (*a*) in the case of positive acceleration and (*b*) in the case of negative acceleration.

themselves. The pulse carries "negative" energy and diminishes the electron-field motion. It is therefore called a *negative pulse.*

Figure 6 shows in cross section a part of the above-described electromagnetic pulses and the conductors from which they emanate. In Fig. 6(*a*) the positive pulse $+P$ has moved a short distance from the conductor. It has left in its path the

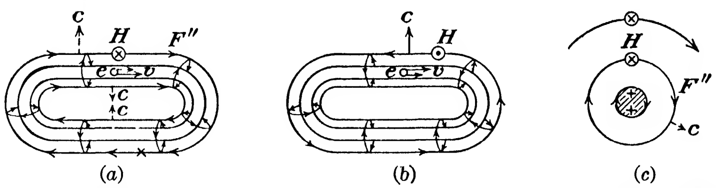


FIG 7—Electromagnetic pulse about an electric circuit and about a magnet: (*a*) circuit with positive electron acceleration, (*b*) circuit with negative electron acceleration, (*c*) the + pole face of a magnet in which the magnetization is increasing. The arrow at the circumference of the pole represents the direction of the accelerating magnetization whirl.

established magnetic field $\mathcal{H}$ and is carrying the magnetic flux which ultimately will occupy the space beyond. In Fig. 6(*b*) is shown the negative pulse $-P$, which has left in its path the reverse magnetic field $\mathcal{H}_3$ and thereby has partly neutralized the initial magnetic field $\mathcal{H}_1$ and which still is carrying the negative magnetic flux that will neutralize in part the initial magnetic field, $\mathcal{H}_2$, beyond.

3. Electric Field in the Electromagnetic Pulse Emanating from an Electric Circuit and from a Magnet.—Figure 4(*b*) shows the electromagnetic pulse about a small portion of a circuit only. The fields of the pulse have the same direction in each succeeding element of the whole circuit; hence the electric lines of force F'' make complete loops, as shown for positive acceleration in Fig. 7(*a*) and for negative acceleration in Fig. 7(*b*). The lines of force of the conservative electric field about charges at rest span the space between charges of opposite sign, while those of the nonconservative field in the electromagnetic pulse about a circuit form closed loops and can have no direct connection with electric charges. The fact that these latter lines are closed loops itself shows that the field which they represent is nonconservative and thereby known to be due to a magnetic-field motion.

When iron is being magnetized its increasing magnetization is equivalent to an accelerating cylindrical whirl (Art. VI–7); hence it follows that an electromagnetic pulse emanates from any magnet whose intensity of magnetization is changing. The electric lines of force of this pulse are circles about the axis of the magnet, as illustrated in Fig. 7(*c*). When the magnetization is decreasing the pulse components have reverse directions.

Since the electron accelerations in the opposite sides of the circuit or of the magnet have reverse directions, the electromagnetic pulses radiating in any one direction from the two sides tend to neutralize; therefore the intensity of the resultant pulse is appreciable only in the immediate neighborhood of circuits or the magnets.

4. No Radiation from Radial Accelerations in Uniform Circular Motion.—Electromagnetic pulses do not appear to emanate either from orbital electrons in their normal rotation about the nucleus or from a loop in which there is a uniform electron flow. Why electromagnetic pulses do not emanate from charges having such radial accelerations is not understood.

5. Action of the Electromagnetic Pulse on Electric Charges (Law C).—When an electromagnetic pulse passes through an electric charge, its electric component exerts a force on the charge

(Art. V–9). After the charge is in motion, the magnetic field also exerts a force (Law A).

It follows, then, that, since the direction of the electric field in an electromagnetic pulse is that of the acceleration of the electrons, neighboring electrons through which the pulse passes are urged by it in a direction opposite that of the acceleration. After the electrons are in motion, the magnetic component of the pulse pushes them in the direction of the pulse motion.

All the facts regarding the pulses may be summarized as follows:

Any acceleration of the electron flow within a conductor or a magnetization whirl evokes an electromagnetic pulse which emanates from the conductor or magnet with the velocity of light. The direction of the nonconservative electric component of this pulse is that of the acceleration, and the direction of the magnetic component necessarily conforms to Law B; from which it follows that, when an electron flow is accelerating in any direction all neighboring electrons are urged in the reverse direction (by the electric component) **and after being set in motion are also urged** (by the magnetic component) **in the direction of the pulse propagation.** **(Law C) (1)**

Either one of the following simplified restatements of Law C usually suffices:

1. Every electron acceleration evokes forces tending to accelerate neighboring electrons in the reverse direction.

2. The direction of the induced flow is the reverse of that of the electron acceleration in the inducing flow. **(2)**

It must be understood that neighboring protons are urged in a direction reverse to that of the neighboring electrons and that neutral atoms feel only a distorting force.

When a loop which is part of an electric circuit is placed about either of the circuits of Figs. 7(*a*), (*b*) so that their planes coincide, the electric field of the electromagnetic pulse urges the free electrons around the loop within the whole of the loop circuit. The nonconservative electric field of the electromagnetic pulse therefore is the e.m.f. imposed on the loop circuit (Art. XVII–2). In a similar manner an e.m.f. is imposed on a loop placed about the bar magnet, Fig. 7(*c*), when its intensity of magnetization

is changing. In either case (Eqs. XIII–4, 6) the imposed or induced e.m.f. is

$$E = 300F''l = -\frac{d\phi}{10^8 dt} \text{ volts,}$$

where F'' is the average intensity of the electric field forcing electrons around the circuit of length l.

Questions

1. What is the state of an electric line of force when the electron of which it is a part is moving with uniform velocity?

2. Explain what happens to an electric line of force when its electron is accelerating either positively or negatively.

3. Show that when the free electrons in a wire are moving with uniform velocity a magnetic field exists about the wire and no apparent electric field.

4. Show that when flow electrons are accelerating an electromagnetic pulse emanates into space Explain the presence of the nonconservative electric component of the pulse.

5. What is the direction of the electric field in an electromagnetic pulse with respect to the direction of the electron acceleration?

6. Explain the production of the electromagnetic pulse that emanates from a magnet in which the intensity of magnetization is changing.

7. Define an electromagnetic pulse.

8. Explain how an electromagnetic pulse establishes a magnetic field about a conductor.

9. What is the effect of the two sides of a circuit or of a magnet on the resultant intensity of the pulse moving in any one direction?

10. Show that the electric component of the radiating pulse about a circuit is a closed loop which neither begins nor ends at electric charges.

11. Give the relation of the kinetic energy of the flow in a wire to that of the pulse and to the stationary magnetic field about the wire

12. Show how the energy of the "electric and magnetic components" of the electromagnetic pulse changes into the energy of the magnetic field about the wire (see question 8).

13. Explain the action of an electromagnetic pulse on + and − charges through which it passes.

14. State Law C in its complete and in its abbreviated form.

15. What imposes an e.m.f. on a circuit of which a part is placed about a loop in which the electron flow is accelerating? In a loop placed about a magnet in which the intensity of magnetization is changing?

Experiments

1. A weighted string hangs vertically as it follows a uniformly moving point of support but deviates from the vertical when the point of support is accelerating.

2. A distortion travels along a long stretched rope with a finite velocity.

3. When flow electrons in a wire or coil are accelerating, those in a neighboring wire feel a force tending to accelerate them in the reverse direction.

4. A current is induced in a coil about a bar magnet when the bar is being demagnetized by pounding.

5. One end of a long bar of soft iron is placed within a magnetizing solenoid while the other end is inserted into a coil which is part of a galvanometer circuit Any change in the magnetization of the bar induces an e m f. in the coil which has the direction of the e m f. induced by an equivalent change in the current of the magnetizing solenoid when that is inserted into the coil in place of the iron bar

CHAPTER XVII

INDUCTANCE

1. E.M.F. Induced in a Neighboring Conductor.—When the wire P, Fig. 1(a), has its flow electrons accelerating in the direction of the flow, as represented, and the wire P_1, Fig. 1(b), has its flow diminishing, the electromagnetic pulses HF'' emanate outward as shown. In the neighboring wires, S and S_1, the

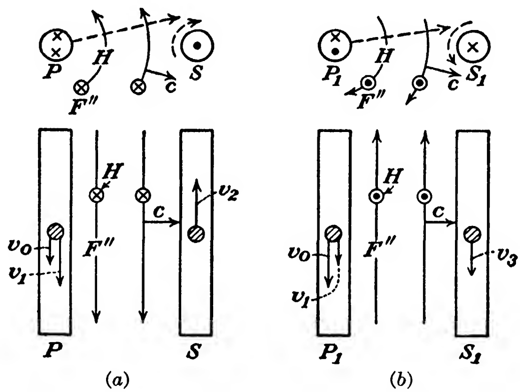


FIG. 1.—Direction of induced flow in S and S_1 when the electron accelerations in P and P_1 are those represented by the cross and dot in one view and by the changing of v_0 to v_1 in the other.

electric fields of the pulses impose e.m.fs. whose directions are represented by the dot and the cross in one view and by v_2 and v_3 in the other.

It should be noted also that the direction of the induced e.m.f. in each case is such as to tend to produce a flow whose magnetic field strengthens the magnetic component of the pulse on the cutting side, conforming with statement 1 of Lenz's law (Art. XIII–8). The direction of the induced e.m.f. may be determined from this relationship as well as from the electric component of the pulse.

Another purely descriptive but very convenient scheme can be, and often is, employed for determining the direction of the induced e.m.f. without any direct reference to the electromagnetic pulse. When a magnetic field is being established about an increasing current, the magnetic lines of force in the established field are imagined to increase in number by beginning with zero diameter at individual electrons and in expanding to cut all neighboring electrons or conductors in their path. When the intensity of the magnetic field is diminishing about a decreasing current, the lines of force while diminishing in number are imagined to be collapsing and thereby cutting through the neighboring electrons or conductors in a direction reverse to that in which they were cutting them when the field was being established.

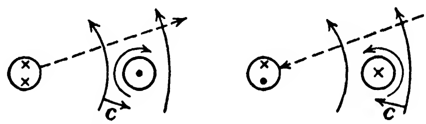


Fig. 2.—Direction of induced flow determined by the concept of (a) expanding flux and (b) collapsing flux.

The direction of the induced e.m.f. then is correctly obtained in each case by Lenz's law as is shown by inspection of Fig. 2. The direction of the induced e.m.f. is such as to tend to produce a flow whose magnetic field strengthens the cutting magnetic field on the cutting side.

The most practical method, however, for determining the direction of the induced e.m.f. in most cases is to use one of the abbreviated statements of Law C; *i.e.*, *The direction of the induced flow is the reverse of that of the electron acceleration in the inducing flow.*

If either of the wires *P* or *S* is made to approach or to recede from the other, the magnetic field due to a steady current in the wire *P* is cut, and an e.m.f. is induced in the wire *S*. This is only a special case of a wire cutting or being cut by a magnetic field.

From the foregoing considerations it is seen that in all cases in which e.m.fs. are induced, magnetic lines of force are being cut. It is immaterial with what these magnetic lines are associated. The cutting may be due to a relative motion of the wire and the

magnetic field or to an electromagnetic pulse. The only essential condition is, as already stated, that magnetic flux be cut. The direction of the induced e.m.f. may be determined by Law A, B, or C or by Lenz's law; and the magnitude of the induced e.m.f., which depends only on the rate of cutting, is calculated by Eq. XIII–6; *i.e.*,

$$E = \frac{N\Phi}{10^8 t} \text{ volts.}$$

2. Electromotive Force Induced in a Neighboring Circuit or Coil.—The two loops, Fig. 3, are shown in section in two positions with respect to each other. When the strength of the flow in

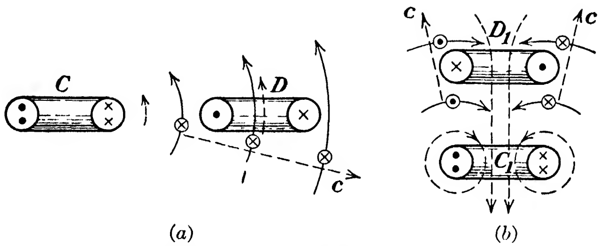


FIG 3.—Electromagnetic pulses emanating from parts of the primary loops CC_1 are shown inducing an e m f in each of two secondary loops DD_1. The resultant e m f due to all the pulses is, in each case, that due to the illustrative adjacent portion. The straight broken-arrowed lines *c* represent the directions of the moving pulses and the other broken-arrowed lines the resultant magnetic fields being established about the primary loop.

loop *C*, Fig. 3(*a*), is increasing, the electromagnetic pulse cuts the loop *D* on two sides and induces an e.m.f. in the same direction in each. After the current is established in *C*, some of the magnetic lines of force are within the loop *D* and must have cut only the nearer side. Those that have passed beyond the loop *D* have cut both sides, inducing therein equal e.m.fs. which have the same direction in space but opposite directions around the loop and therefore neutralize each other. The lines that enter loop *D* and remain within it are the ones that are effective in producing the e.m.f. in the loop. The induced e.m.f. (Eq. XIII–11) then is equal to the *rate of change of flux within the loop*/10^8. This is also true when the current in loop *C* is diminishing. The direction of the induced e.m.f. is that of the evoked forces acting on the electrons in the nearer side of the loop. This direction conforms with that deduced from Lenz's law (Art. XIII–8).

When the current in the coil C_1, Fig. 3(b), is increasing, the electromagnetic pulse cuts the coil D_1 as shown. The direction of the induced flow is reverse to that of the increasing flow in the primary loop C_1. The direction of the induced flow may be obtained most readily from Law C or from Lenz's law.

When a coil of several loops is substituted for the single loop, the total induced e.m.f. is the sum of those in the individual loops.

The magnitude of the electromotive force an accelerating flow induces in a neighboring circuit is determined by Eq. XIII–11. This equation applies to electromagnetic pulses as it does to relative motions between conductors and magnetic fields. (1)

When two coils are placed with their planes at right angles to each other and with the center of one coil in the plane of the other, inspection shows that a change in the current of one coil produces no e.m.f. in the other.

3. Quantity of Electricity Induced in the Secondary Coil.—The coil, Fig. 4, in which the inducing current is changing is called the *primary coil* and that in which the current is induced is called the *secondary coil.*

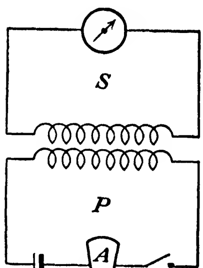


FIG. 4—Primary circuit P and secondary circuit S.

The average induced e.m.f. in the secondary coil during the interval in which the magnetic flux linking it is changing (Art. 2) is

$$E = \frac{N\Phi}{10^8 t} = iR,$$

from which

$$N\phi = 10^8 itR = 10^8 QR. \qquad (2)$$

Then

$$Q = \frac{N\Phi}{10^8 R} \text{ coulombs.}$$

This equation is applicable to all cases of electromagnetic induction. Self-induction (Art. 6) is disregarded in this development because its average effects on the increasing and the decreasing parts of the induced flow are equal and opposite.

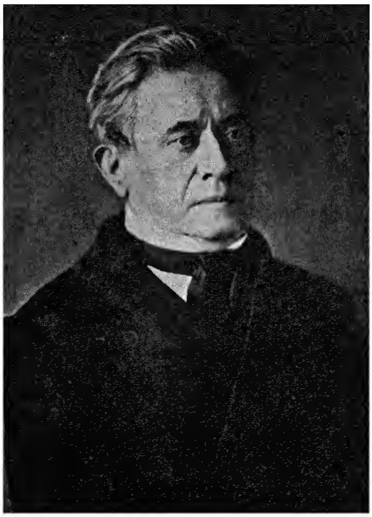
4. Mutual Inductance (Coefficient of Mutual Induction).—When the current in the primary coil is doubled, the magnetic flux linking the secondary is also doubled. It then follows that,

when the current in the primary coil is changing at twice the previous rate, the flux linking the secondary also is changing at twice that rate; therefore the e.m.f. induced in the coil is doubled. Then

$$e \propto -\frac{di}{dt} \propto \frac{I}{t},$$

or

$$e = -M\frac{di}{dt}, \qquad \text{or} \qquad |e| = \left|M\frac{I_p}{t}\right|. \tag{3}$$



Joseph Henry (1797–1878), professor of physics, Princeton College, Princeton, N. J., and in later years secretary of the Smithsonian Institution, Washington, D. C.; discovered (independently of Michael Faraday) electromagnetic induction (1831); discoverer of self-induction (1832) and of the oscillatory nature of the electric spark (1842). The unit of inductance, the *henry*, is named in his honor.

The − sign indicates that the direction of the induced e.m.f. in the secondary is the reverse of that of the flow acceleration in the primary.

Equation (3) is the defining equation for mutual inductance. The constant M is the *mutual inductance*, or, as it is sometimes called, the *coefficient of mutual induction*. Inspection shows that M represents, numerically, the e.m.f. induced in the secondary

coil when the current in the primary is changing at the rate of 1 amp/sec.

The unit of mutual inductance is called a *henry*. It follows from the defining equation that

The mutual inductance is 1 henry when a change of 1 ampere per second in the current of the primary circuit induces an electromotive force of 1 volt in the secondary circuit.

A convenient fractional unit of inductance is the *millihenry*. It is equal to 0.001 henrys.

Usually the mutual inductance between any two circuits is practically that of two adjacent coils (Fig. 4), and it can be shown that the mutual inductance between any two fixed coils is independent of which coil is assumed to be the primary.

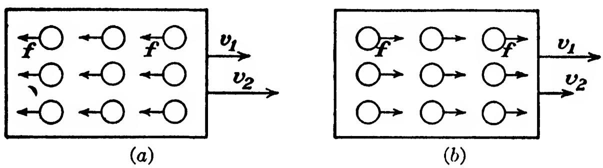


FIG. 5.—The forces f, acting on individual electrons, evoked by the neighboring electrons of an accelerating group, in the case of (a) positive acceleration and (b) negative acceleration.

Combining Eqs. (3), XIII–6, IX–7 makes the magnitude of the induced e.m.f.

$$e = M\frac{I_p}{t} = \frac{N\Phi}{10^8 t} = R_s\bar{\imath} \text{ volts,} \tag{4}$$

from which

$$M = \frac{Q_s R_s}{I_p} = \frac{N\Phi}{10^8 I_p} \text{ henrys,}$$

in which Q_s, R_s, and $N\Phi$ are, respectively, the induced quantity of electricity, the resistance, and the change in line turns in the secondary coil for the change I_p in the current of the primary coil. It also should be noted that, although the induced e.m.f. is dependent on the rate at which the current in the primary coil is changing, the quantity of electricity induced in the secondary circuit is independent of it.

The last equation shows that M equals the line turns per ampere divided by 10^8. This is equivalent, as it should be, to

the induced e.m.f., in volts, when the current in the primary is changing at the rate of 1 amp/sec.

5. Electric Inertia.—When a group of electrons is accelerating, it follows from Law C that each of its constituent electrons has exerted upon it a force whose direction is the reverse of that of the group acceleration. In positive acceleration, Fig. 5(a), each electron feels a force f opposing its acceleration and therefore that of the group. The electron group then has an inertia greater than the sum of the inertias of the same number of isolated electrons. In negative acceleration, Fig. 5(b), each electron is urged with a force f (Law C) in the direction in which the group is moving; therefore a force is exerted opposing that which is diminishing the velocity of the group; *i.e.*, the inertia of the group, in this case also, is increased.

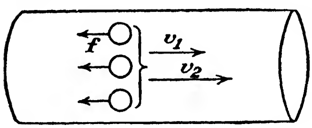


FIG. 6.—Self-induction—Electron e accelerating in a wire in the direction indicated feel forces f urging them in the opposite direction.

In a similar manner it will be shown (Art. XXVII–10) by considering the interaction between elements of an electron charge that even the inertia of individual isolated electrons or protons, and therefore of matter, may be explained in terms of these induced forces.

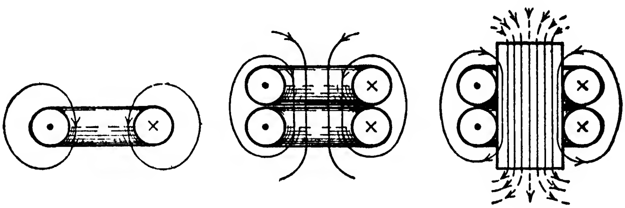
6. Self-inductance (Coefficient of Self-induction).—When the intensity of an electric current is increasing, Fig. 6, each accelerating electron urges the neighboring electrons in the reverse direction (Art. 5). This action does not differ from that in mutual induction (Art. 4) except that in self-induction the "neighboring electrons" are in the same circuit as the inducing electrons; in fact they are the accelerating electrons themselves. The force the neighboring electrons feel has a direction opposite that of the electron acceleration, be it positive or negative, and therefore opposes any change in the e.m.f. of the circuit. For this reason it is called the *counter electromotive force of self-induction.*

In order to account for the observed facts associated with induced e.m.fs., as already stated (Art. XVI–2), *the electromagnetic pulse must at its origin be assumed to contain all the magnetic lines of force which in its motion outward it leaves behind to form the magnetic field about the conductor.*

The magnetic field about a steady current was all contained, therefore, in the electromagnetic pulses that emanated outward

from each element of every accelerating electron during the period in which the intensity of the current was changing. Every element of the charge in every electron in the wire has then been cut by every magnetic line of force outside the wire and by nearly all the few lines that remain within. Since the magnetic field about a current is proportional to the current, the counter e.m.f. induced by the cutting flux is proportional to the rate at which the current is changing.

If the current in a solenoid is increasing, the magnetic field of the electromagnetic pulse which emanates from each loop cuts, in its motion outward, not only the electrons in the loop itself but also the electrons in all the neighboring loops. The counter



(a) (b) (c)

FIG. 7.—Dependence of self-inductance on the number of loops.

e.m.f. in each loop then is much greater than if the loop were isolated from the others.

If the solenoid has an iron core, Fig. 7(*c*), the increasing magnetic field within the solenoid increases the magnetization of the iron, which magnetization is externally equivalent to a cylindrical electron whirl (Art. VI–7) accelerating in the same direction as the electron acceleration within the solenoid. The accelerating magnetization whirl urges the accelerating electrons in the magnetizing solenoid in a direction reverse to that of the whirls and therefore also of their own acceleration, *i.e.*, the increasing magnetization of the iron greatly increases the counter e.m.f. of self-induction in the inducing solenoid.

This large counter e.m.f. increases greatly the energy required to give the flow electrons in any circuit a given flow velocity. Since this expended energy becomes the energy of the flow motion, the amount of energy in the same flow depends on the self-inductance of the circuit. Also, since the counter e.m.f. exists only when a magnetic field is being established or is col-

lapsing, the energy expended in giving the flow electrons a velocity may be said to have been given to the established magnetic field (moving electric field) about the circuit. The energy expended in giving the flow electrons a velocity, therefore, may be said to exist either in the established magnetic field or in the moving electrons or in both. This can be understood when it is recalled that the elemental charge and its electric field must be considered (Art. I–3) aspects of one and the same physical entity.

The equations for the e.m.f. of self-induction are the same as for any induced e.m.f. and also are similar to those of mutual induction. The induced counter e.m.f. varies as the rate of change of flux within the loop and therefore as the rate of change of current in the solenoid (without iron). Then

$$e \propto -\frac{di}{dt},$$

or

$$e = -L\frac{di}{dt}. \tag{5}$$

The constant L is called *self-inductance*, or *coefficient of self-induction*, and is defined by Eq. (5). It corresponds to the mutual inductance in every respect except that the induced e.m.f. is in the inducing circuit itself instead of in a neighboring circuit. The unit of self-inductance, therefore, is also called the *henry*. The $-$ sign has the significance of that in Eq. (3).

It follows that in any part of a circuit, containing self-inductance, in which the current is changing, the effective e.m.f.

$$e_0 + e_L = e_0 + \frac{-Ldi}{dt} = e_0 - L\frac{di}{dt} = Ri,$$

from which

$$e_0 = Ri + L\frac{di}{dt}. \tag{6}$$

The sign of di/dt is $+$ when the current is increasing and $-$ when decreasing. The part Ri of the applied e.m.f. is effective in maintaining the current and the part $+Ldi/dt$ is required to make the current increase or, if the current is decreasing, contributes toward maintaining the current.

As in mutual induction, Eq. (3),

$$e = L\frac{I}{t} = \frac{N\Phi}{10^8 t},$$

and

$$L = \frac{N\Phi}{10^8 I}\text{ henrys,} \tag{7}$$

in which $N\Phi$ is the number of line turns linking the coil. Therefore L is equal to the number of line turns per ampere divided by 10^8.

The following experiments illustrate the inertia effects of self-induction: (1) When the circuit of large self-inductance, Fig. 8, is opened, the large energy of electron motion keeps the electrons moving until a large p.d. is established across the air gap S and a comparatively large quantity of electricity passes through the ionized air (spark).

From another point of view, the electrons are displaced by the collapsing flux, which urges the electrons in the direction of their original motion. The large energy content of the magnetic field appears mainly as the energy of the flow across the gap.

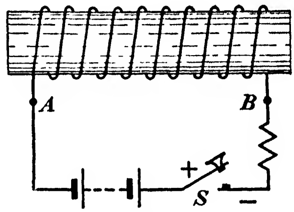


FIG. 8—Large p.d. across gap at opening of circuit of large self-inductance.

(2) If an electric lamp is connected across the points A and B, it lights more brilliantly for an instant, owing to self-inductance, both at the make and at the break of the circuit.

The voltage induced by the collapsing field of strong electromagnets, such as the field magnets of large direct-current machines, is sometimes several times that of the applied voltage; hence it is necessary to take every precaution against injury to instruments and attendants.

7. Dependence of Self-inductance on the Number of Loops.—The magnetic flux linking the coil of two loops, Fig. 7(b), is twice that linking the single loop, Fig. 7(a). Each loop produces its own magnetic field and that about the two loops is the resultant of two such superposed fields of equal strengths. If the flux that does not completely link both loops is neglected, the field of each loop links both turns; hence the flux linkage, and there-

fore the counter e.m.f., per turn in the two loops is twice that in one isolated loop. The e.m.f. of self-inductance in the two loops then is four times that in one isolated loop. It then follows that

$$L \propto N^2. \tag{8}$$

The self-inductance of a coil of wire, therefore, increases greatly with the number of turns.

The self-inductance for zero or low frequencies of a straight wire is

$$L = 0.002l\left[2\ 303 \log_{10}\frac{4l}{d} + \frac{d}{2l} - 1 + \frac{\mu}{4}\right]10^{-6} \text{ henrys,}$$

and for a circular coil of circular cross section of fine nonmagnetic wire

$$L = 0.01257an^2\left[2.303 \log_{10}\frac{16a}{d} - 1.75\right]10^{-6} \text{ henrys,}$$

where l = length, d = diameter, n = number of turns, and μ = permeability, all distances being expressed in centimeters.

8. Power Expended in Forcing an Electron Flow against an Opposing E.M.F.—If an e.m.f. E is forcing electrons against the counter e.m.f. e, the resultant e.m.f. is $E + (-e) = E - e$. Then (Art. IX–7)

$$E - e = RI.$$

The power which is expended (Art. IX–9) in forcing the electrons through the circuit and which is all transformed into heat (Art. IX–3) is

$$P_H = (E - e)I = RI^2 \text{ watts.}$$

From which the total power being supplied to the circuit at any instant is

$$P = EI = RI^2 + eI \text{ watts.} \tag{9}$$

The term EI is the total power being supplied to the circuit, and the term RI^2 the power expended in heat. The power being expended in forcing the flow against the counter e.m.f. then is eI watts and, in a simple circuit, becomes the energy of motion of the flow (energy of the magnetic field).

This conclusion is also reached by considering the work required to move electrons (Art. IX–4) against the counter e.m.f. to be

$$w_J = eQ = eIt \text{ joules.}$$

$$P = \frac{w_J}{t} = \frac{eIt}{t} = eI \text{ watts.}$$

When the change in the flow is not uniform, as is generally the case, the power expended in any instant during which the rate of change may be considered uniform is eI, in which e and I are the counter e.m.f. and the flow at the instant under consideration.

In other than simple circuits part or all of the energy represented by eI is transformed into chemical potential energy as in the charging of a storage cell (Art. XIV–8), into mechanical energy as in the electric motor (Art. XXIII–4), and into electric energy of secondary circuits as in the transformer (Art. XXII–3).

9. Energy of a Magnetic Field.—Assume the current to be increasing uniformly to its final magnitude so that the counter e.m.f. of self-induction is constant. During this accelerating period the average magnitude of the increasing current is one-half that of the final, *i.e.*, $\frac{1}{2}I$. The work done in forcing the electrons of the flow against the opposing e.m.f., $e = LI/t$, then, is

$$w_J = Qe = \frac{1}{2}It \times L\frac{I}{t} = \frac{1}{2}LI^2 \text{ joules.} \tag{10}$$

This equation enables the energy of the magnetic field to be calculated when the self-inductance is constant as it is in circuits which are not magnetizing some magnetic material.

It can be shown, also, that the energy density of the magnetic field is

$$w = \frac{\mu H^2}{8\pi} \text{ ergs/cm.}^3$$

This expression is similar to that for the energy of an electric field (Art. XII–13).

Questions

1. When the intensity of the current in a wire is changing, explain the direction of the induced flow in a neighboring wire by means of the electric component of the electromagnetic pulse.

2. What is the direction of the induced flow relative to the direction of the electron acceleration in the inducing circuit?

3. Explain how the direction of the induced flow may also be determined by means of the magnetic component of the electromagnetic pulse and by means of the concept of "collapsing flux"

4. What is the magnitude of the induced e m.f. in terms of the rate of change in the flux linking a coil of N turns?

5. Derive the expression for the quantity of electricity induced in a circuit. Show from the Eq. 2 that the induced quantity is independent of the rate of cutting, and that it depends on the total change in line turns; also that the quantity varies inversely as the resistance in the secondary circuit.

6. Write the defining equation for mutual inductance. Define mutual inductance. Define a henry.

7. Show that the mutual inductance,

$$M = \frac{Q_s R_s}{I_p}, = \frac{N\phi}{10^8 I_p}.$$

8. Why must the electromagnetic pulse at its origin contain all the magnetic lines of force that will appear in the magnetic field established thereby about the wire?

9. Explain how *the induced e.m f. of self-induction opposes any change in the intensity of the electron flow.* Explain why this opposition varies quantitatively with the arrangement of loops in the coil.

10. Does the electron flow in a circuit possess inertia? Why does the magnitude of this inertia depend on the number and position of the loops?

11. Explain why the self-inductance of a coil varies nearly as the square of the number of loops.

12. When a counter e m f exists in a circuit, show how much of the total expended power is used to force the electrons against this e.m.f and how much is spent in heating the wire. What becomes of the power used in forcing the electrons against the counter e m.f ?

13. How much energy is expended against the counter e.m.f. of self-induction while the current increases from zero to its final value? Derive the expression for the energy in terms of L and I, and show that this energy is also the energy of the moving electric field or of the motion energy given the flow electrons.

Problems

1. (*a*) What is the induced e.m.f. in a coil of 20 turns of wire when the flux within it is changing at the rate of 1,000,000 maxwells/sec? (*b*) What is the induced e.m.f. if the number of turns is 100,000?

2. A coil of 100 turns in a circuit whose resistance is 5 ohms is linked by 100,000 maxwells produced by a neighboring electromagnet. When the circuit of the electromagnet is opened, the residual flux of 2,000 maxwells in its core remains linking the coil. What is the quantity of electricity induced in the coil because of the collapsing flux?

3. If a change of 2 amp in a circuit induces 0.001 coulombs of electricity in a neighboring circuit whose resistance is 20 ohms, what is the mutual inductance of the two coils in millihenrys?

4. What is the mutual inductance of two coils, in millihenrys, if when the current in one changes 2 amp the average flux linking the 80 turns of the other changes 70,000 maxwells?

5. When the current in a coil of 120 turns changes 3 amp, the flux within the coil changes 300,000 maxwells. What is the self-inductance of the coil?

6. When a current of 12 amp energizes a solenoid whose self-inductance is 0 017 henrys, what amount of energy exists in the solenoid's magnetic field and therefore in the moving electrons of the flow?

Experiments

1. Varying the current in a coil induces currents in a neighboring coil. The directions of these induced currents with respect to the primary current shown.

2. Magnitude of the induced quantity of electricity varies directly as the current in the primary and inversely as the resistance in the secondary circuit.

3. Effects of self-induction: (1) spark at the break of a circuit containing large self-inductance; (2) a lamp shunted with the coils of an electromagnet brightens for an instant when the circuit is either closed or opened.

4. Inductance standards shown

CHAPTER XVIII

ALTERNATING CURRENTS

1. Alternating E.M.F. Induced in a Coil Rotating in a Uniform Magnetic Field.—The rectangular loop, Fig. 1, rotating on its axis in a uniform magnetic field H is represented in two positions, (*a*) and (*b*), approximately at right angles to each other. The circles C and D, Fig. 1(*b*), represent the cross sections of the two cutting sides of the loop whose plane, in the position shown,

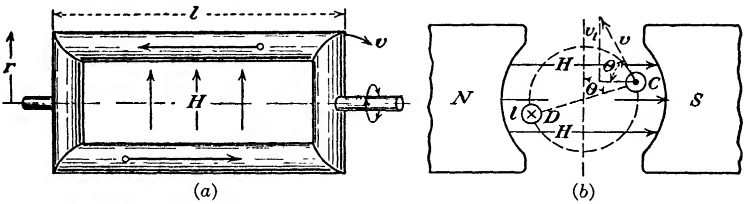


FIG. 1.—Two views of a loop rotating in a uniform magnetic field.

makes the angle θ with a plane drawn perpendicular to the flux lines. The velocity v of the side C has a component, $v_1 = v \sin \theta$, perpendicular to the flux lines. The induced e.m.f. (Art. XIII–4) at any instant, then, is proportional to this component of the velocity at that instant, and in the one side of the loop is

$$e_1 = \frac{Blv_1}{10^8} = \frac{Blv}{10^8} \sin \theta.$$

The equal induced e.m.f. in the side D is in the reverse direction. The electrons in the two sides, though moving in opposite directions, are moving in the same direction around the loop; hence the two e.m.fs. are in series, and the total e.m.f., at any instant, for N such loops is

$$e = 2e_1N = N\frac{2Blv}{10^8} \sin \theta = E_m \sin \theta,$$

where E_m is a constant and is the maximum magnitude (Art. 2) of e.

Such an e.m.f. is called a *sinusoidal e.m.f.* because its magnitude varies as the sine of the angle θ. It is represented by a sine curve, Fig. 2, in which the ordinates show the induced e.m.f. at any time t. This is a harmonic curve, which is the locus of a point

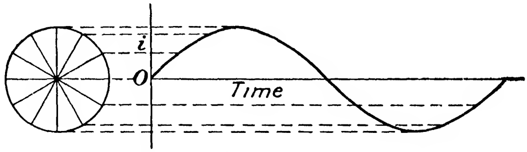


FIG. 2.—Sine curve representing instantaneous magnitudes of an alternating electromotive force. The circle at the left is the circle of reference for the simple harmonic component of the curve.

oscillating with simple harmonic motion and moving at the same time with a uniform velocity at right angles to the path of the oscillation.

The curve also shows that the induced e.m.f. reverses at every half revolution of the coil. This e.m.f. is said to be an *alternating electromotive force,* and the current produced by it an *alternating current.*

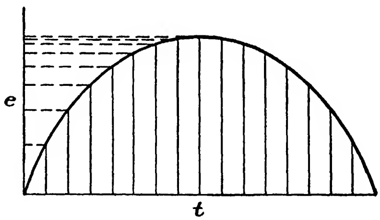


FIG. 3.—The magnitude of alternating e.m.f. in a half-cycle at successive equal intervals of time.

The magnitude of the induced e.m.f. at any instant can be expressed in terms of the angular velocity ω of the coil, the angle θ, and the maximum flux ϕ which links the coil when its plane is at right angles to the magnetic lines of force. Since $v = \omega r$, $A = 2lr$, and $\phi = BA$, the foregoing equation becomes

$$e = N\frac{2Bl\omega r}{10^8}\sin\theta = N\frac{BA\omega}{10^8}\sin\theta = \frac{\omega N\phi}{10^8}\sin\theta. \qquad (1)$$

2. Effective E.M.F. and Effective Current.—Since the e.m.f. varies continually as shown in Fig. 3, some convention must be employed for designating its various magnitudes. The *instantaneous value, e,* is its magnitude at any given instant. The *maximum value,* E_m or E_{max}, is its largest instantaneous magnitude and from Eq. (1) is $\omega N\phi/10^8$. The *average value,* E_{av}, is

the average of all its possible magnitudes in a half cycle. It can be shown, Appendix V (5), that

$$E_{av} = \left(\frac{2}{\pi}\right)E_m = 0.637E_m. \tag{2}$$

The effective value or *root-mean-square value*, E_r, is the value of a steady e.m.f. that would expend the same power as the alternating e.m.f. in forcing an electron flow through a given circuit containing resistance only. Since the power being expended (Art. IX–9) at any instant in such a circuit is $p = e^2/R$, the power expended in the large number of parts, representing instantaneous magnitudes of the e.m.f., into which a half cycle can be divided (Fig. 3) is the average power expended in these individual parts. Then the power (*i.e.*, the average power) in the half cycle and therefore for a period of any length is

$$p = \frac{p_1 + p_2 + \cdots + p_n}{n} = \frac{\frac{e_1^2}{R} + \frac{e_2^2}{R} + \cdots + \frac{e_n^2}{R}}{n} = \frac{\frac{e_1^2 + e_2^2 + \cdots + e_n^2}{n}}{R} = \frac{E_r^2}{R},$$

where

$$E_r = \sqrt{\frac{e_1^2 + e_2^2 + \cdots + e_n^2}{n}};$$

i.e., E_r has the magnitude of a steady e.m.f. that would expend the same power as the alternating e.m.f. and is equal to the *square root of the average of the squares* (abbreviated to root-mean-square) of all the instantaneous values in a half cycle. The E_r by definition then represents the effective or root-mean-square value of the alternating e.m.f. or of p.d. It can be shown, Appendix V(6), that

$$E_r = \frac{E_m}{\sqrt{2}} = 0.707E_m. \tag{3a}$$

The effective value or root-mean-square value, I_r, of an alternating current is the value of a direct current which would expend

the same power in a given resistance as the alternating current. By dividing a half cycle, now representing current, into a large number of parts, and since the power expended in each part is $p = Ri^2$, then

$$p = \frac{p_1 + p_2 + \cdots + p_n}{n} = \frac{Ri_1^2 + Ri_2^2 + \cdots + Ri_n^2}{n} = R\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n} = RI_r^2,$$

in which

$$I_r = \sqrt{\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}}.$$

Again, as in the case of alternating e.m.f., it can be shown that

$$I_r = \frac{I_m}{\sqrt{2}} = 0.707I_m. \tag{3b}$$

The effective values E_r and I_r are usually represented by E and I without the subscripts, and, unless otherwise stated, the terms alternating e.m.f. and alternating current refer to effective values.

The units of alternating current and of e.m.f. may now be defined as follows:

An effective alternating current of 1 ampere is that current which develops the same amount of heat in a given resistance as is developed in the same time by a direct current of 1 ampere.

An effective alternating e.m.f. or potential difference of 1 volt is that e.m.f. or p.d. which produces an effective alternating current of 1 ampere in a nonreactive resistance of 1 ohm.

When a 60-cycle alternating current of 1 amp is energizing a copper wire of 1-mm² cross-sectional area, the free electrons as a group are systematically displaced from their normal positions about 0.000032 cm in one direction and then in the other. These small double displacements, however, are each 1,400 times the distance between the centers of two adjacent atoms of the metal, but the total amplitude of oscillation spans a distance of only about one-half the wave length of sodium light. An alternating current, therefore, consists of minute, systematic electron tremors without a continuous electron flow.

3. Production of Alternating and Pulsating E.M.Fs. and Currents.—If a coil which is rotating in a magnetic field has its terminals connected to "slip rings," Fig. 4(*a*), the generated alternating e.m.f. is impressed on the circuit W.

If in place of the slip rings two semicircular segments, a *commutator*, are connected to the rotating coil, Fig. 4(*b*), the alter-

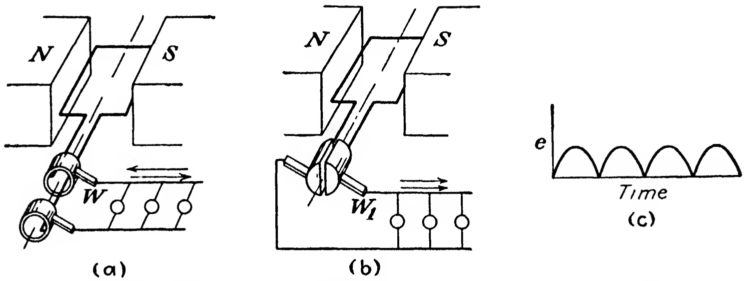


FIG. 4.—(*a*) Production of alternating e m f.; (*b*) production of pulsating e m f.; (*c*) pulsating electromotive force.

nating e.m.f. in the loop becomes a pulsating e.m.f. in the circuit W_1. The segments change positions on the brushes at the instant the e.m.f. in the coil reverses, thereby causing the impressed e.m.f. always to have the same direction in the external part of the circuit. The induced pulsating e.m.f. in the circuit is represented in Fig. 4(*c*).

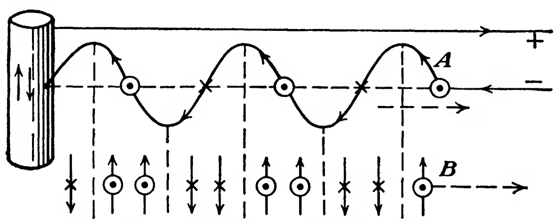


FIG. 5.—Alternating electromagnetic pulses or waves.

4. Alternating Electromagnetic Pulses (Electromagnetic Waves).—When the current in a conductor is alternating, the electrons are being accelerated first in one direction and then in the other. These accelerations produce a succession of alternating electromagnetic pulses or fields (Art. XVI–2) which emanate outward into space with the velocity of light. These pulses are represented diagrammatically in two ways in Fig. 5. The form A shows a succession of distortions in a line of force of

the electron field and the form B represents the alternating directions of the nonconservative electric field of the emanating pulse. The crosses and dots, in both cases, show the directions of the alternating magnetic field. Such alternating electro-magnetic pulses travel into space from every alternating or oscillating current.

These alternating fields are called *electromagnetic waves* and are said to consist of two components, the electric and the magnetic, at right angles to each other. When such electromagnetic waves pass through a conductor, the free electrons in the con-

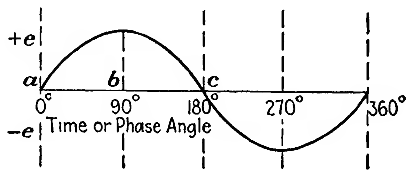


FIG. 6—A complete sinusoidal e m.f. cycle, showing the phase angles.

ductor are urged first in one direction and then in the other; *i.e.*, an alternating e.m.f. is impressed on the conductor by the waves.

5. Phase Angle—Cycle.—The instantaneous magnitude of a sinusoidal e.m.f. (Art. 1) is

$$e = E_m \sin \theta,$$

where θ is the angle (Fig. 1) between the plane of the rotating loop and a plane perpendicular to the magnetic field. The variation of induced e.m.f. in one revolution of the loop is represented by the harmonic curve, Fig. 6. When θ is zero, the e.m.f. also is zero, as represented at the point a in the figure. When the loop is 90° from that position, the impressed e.m.f. has its maximum magnitude, as represented at b. The points a and b are said to be 90° apart in phase or to differ by a phase angle of 90°. The curve showing the e.m.f. for a complete revolution of the loop, or its equivalent, is said to represent a *complete cycle*. The abscissas indicate both the phase angle in the cycle and the time. The number of complete cycles or alternations per second is called the *frequency* of the e.m.f. and is represented by f. The angle θ is usually represented by ωt or by $2\pi ft$, where t is measured from the instant at which $\theta = 0$.

6. Magnitude of the E.M.F. of Self-induction.—Equation (1), although developed for an e.m.f. induced in a special case, is applicable for determining the e.m.f. produced wherever the amount of the linking flux is changing in a sinusoidal manner. It is therefore applicable for determining the counter e.m.f. of self-induction in an alternating-current circuit, because the sinusoidal change in current produces a like change in line linkage and such a change in line linkage induces a sinusoidal counter e.m.f. The induced e.m.f. e of Eq. (1) may now be considered the counter e.m.f. e_L of self-induction. The fact that the counter e.m.f. and the current differ by 90° in phase, however, changes the sin θ to cos θ when phase angles are measured from the same point a, Fig. 8. Then Eq. (1) becomes

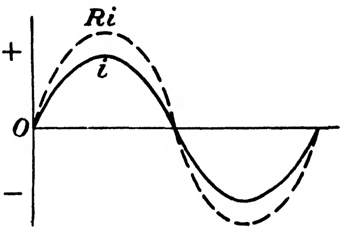


FIG. 7.—The Ri drop is always in phase with the current. Drawn for $R = 1.3$.

$$e_L = \frac{\omega N\phi}{10^8} \cos \theta.$$

But from Art. XVII–6

$$e_L = L\frac{I}{t} = \frac{N\phi}{10^8 t},$$

from which

$$N\phi = 10^8 LI.$$

Substituting this value of $N\phi$ gives for the e.m.f. of self-induction at any instant

$$e_L = \omega LI_m \cos \theta. \tag{4}$$

The maximum magnitude of the counter e.m.f. of self-induction, $E_{L \cdot m}$, in an alternating-current circuit, then, is ωLI_m volts.

7. Phase Angle between the Current and Ri Drop and E.M.F. of Self-induction.—In circuits containing resistance the magnitude of the Ri drop (potential difference across R) varies as i varies, because R is a constant. The Ri drop, therefore, is always in phase with the current, as shown in Fig. 7. The Ri drop also at any instant is equal to the algebraic sum of the e.m.fs. then in the circuit. These e.m.fs. in alternating-current circuits with

negligible capacitance are the impressed e.m.f. e_0 and (Art. XVII–6) the e.m.f. of self-induction e_L. Then

$$e_0 + e_L = e_R = Ri, \tag{5}$$

where e_0 and e_L are the instantaneous values of the e.m.fs. which may act in the same or in opposite directions and e_R is their resultant.

The curve i, Fig. 8, represents the varying magnitude of the current during one complete cycle. At the instants b and d the

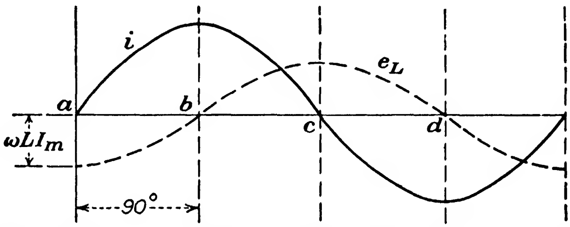


FIG 8.—Phase difference of 90 degrees always exists between the current and the electromotive force of self-induction. Drawn for $\omega L = 0.5$.

current has its maximum value, and the flow electrons have no acceleration; while at the instants a and c the current has zero magnitude, and the electron flow has maximum acceleration. The e.m.f. of self-induction, therefore, is zero when the current is maximum, and maximum when the current is zero. The sinusoidal curve e_L represents this e.m.f. of self-induction and shows

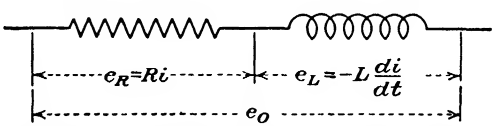


FIG. 9—Representation of impressed e.m.f., e_0, across any part of a circuit containing both L and R.

that a phase difference of 90° exists between it and the current, and also that, when the current is increasing, the e.m.f. has a negative value and, when decreasing, a positive value; *i.e.*, this e.m.f. of self-induction is always opposing any change in the current intensity. The curve representing e_L, Fig. 8, is drawn for a case in which $E_{L \cdot m} = \omega LI_m = 0.5I_m$.

8. Phase Angle between Impressed E.M.F. and Current in Circuits Containing L and R.—The self-inductance and resistance, although usually existing together in the same conductor

or coil, may be represented as though they existed in two series coils, one containing the whole of the self-inductance and the other all the resistance. This is shown in Fig. 9 where di/dt is taken to have + magnitude when the current is increasing and a − magnitude when decreasing. Then, for example, when the current is increasing, e_L has a − magnitude, *i.e.*, a direction which is the reverse of that of the electron flow in the increasing current. Then the resultant e.m.f. is

$$e_0 + \left(-L\frac{di}{dt}\right) = Ri.$$

from which

$$e_0 = Ri + L\frac{di}{dt}, \tag{6}$$

whence it is seen that when the current is increasing e_0 is larger than Ri by Ldi/dt and when the current is decreasing it is smaller then Ri by the same term.

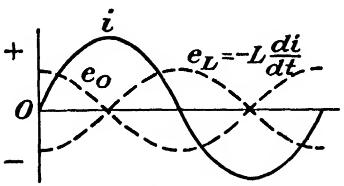


FIG. 10—Phase relationship of e_0, e_L and i in circuits containing L and no resistance. Drawn for $L\omega = 0.5$.

In a circuit containing self-inductance and zero (or negligible) resistance any unbalanced e.m.f. (*i.e.*, one without an equal opposing e.m.f.) would produce a current of infinite magnitude. The impressed e.m.f. e_0, therefore, can produce such an electron acceleration as invokes, in all parts of the cycle, a counter e.m.f. of self-induction which just balances the impressed e.m.f. This is represented in Fig. 10, where e_L must have the 90°-phase difference with i (Art. 7). The foregoing relationship may also be derived from Eq. (6), for when $R = 0$, $e_0 = Ldi/dt$. The two e.m.fs. must, of course, be oppositely directed.

In a circuit containing both L and R there is always a potential drop across R when a current is flowing, so that e_0 does not, in general, equal e_L. In Fig. 11 the e_L and Ri curves must have (Art. 7) the above-given phase relationships with i and their ordinates, for the illustrative case, the relative magnitudes. It then follows, Eq. (6), that at any instant

$$e_0 = Ri + L\frac{di}{dt},$$

where Ri has a + value when it is represented above the axis of abscissas, and Ldi/dt, when its value is below that axis because of the transposition of that term in forming the equation. The impressed e.m.f. e_0 is equal to the e.m.f. opposing the flow together with what potential drop remains to maintain the flow. In the equation, therefore, all values of e_0 and of Ri are to be taken as positive when they are above the axis, and negative when below, while for Ldi/dt the reverse is the case. Five

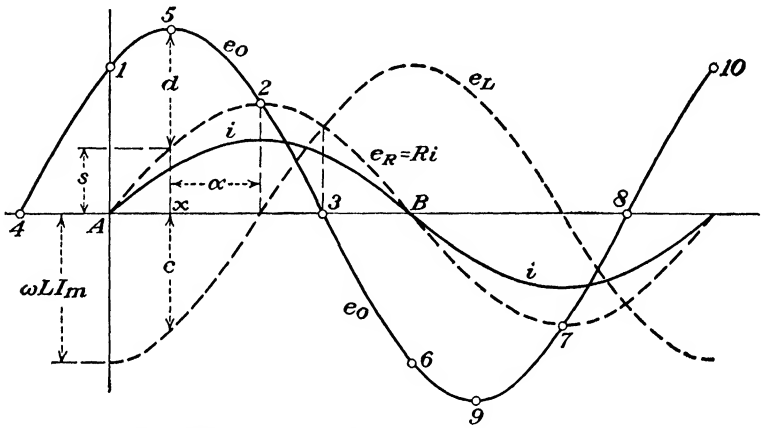


FIG. 11.—Phase relationship of i, Ri, e_L and e_0 in circuits containing L and R, shown by means of rectilinear coordinates and drawn for a case where $R = 1.5$ and $\omega L = 2$.

points on each lobe of the e_0 curve are most conveniently determined from the equation as follows:

1. When $Ri = 0$, $e_0 = Ldi/dt$, where e_0 has a + value and therefore is above the axis.

2. When $Ldi/dt = 0$, $e_0 = Ri$.

3. Where Ri and Ldi/dt curves cross, Ldi/dt has − magnitude while Ri has an equal + magnitude; then $e_0 = 0$. The counter e.m.f. due to the collapsing flux alone is causing the electron flow.

4. The distance $A4$ = the distance $B3$, because the curve e_0 has the same frequency as i.

5. The e_m point is located at x where $x3 = x4$. Here

$$e_m = e_0 = Ri + L\frac{di}{dt} = s + c = s + d.$$

Points 6 to 10 repeat the points 1 to 5 in the reverse direction.

Any desired number of points on the impressed e.m.f. curve e_0 may be obtained by the addition of the instantaneous magnitudes of Ri and Ldi/dt, as was done in the case for e_m.

Inspection of the curves shows that the current lag α is less than 90°. Resistance therefore diminishes the lag as well as the magnitude of the current. The self-inductance aids in limiting the current by diminishing the average resultant e.m.f. This is seen from the fact that the e.m.f. of self-induction opposes the impressed e.m.f. more than one-half of the time and that time includes the periods in which the e.m.fs. have their maximum values.

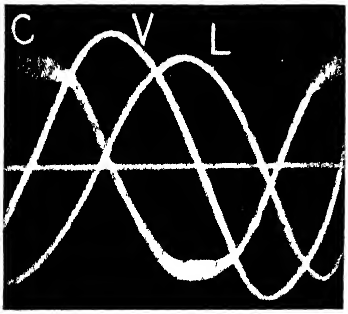


FIG. 12.—Oscillogram showing the current phase lag and phase lead in inductive and capacitive circuits. V is the impressed e.m.f. curve; L, the current curve when the current lags in phase due to self-inductance; C, the current curve when the phase is advanced (Art. 10) by capacitance.

Since the introduction of self-inductance into an alternating-current circuit diminishes the current, it provides an economical means for the control of such currents; no energy is then wasted in heating an added regulating resistance.

In a coil of large self-inductance, such as the primary of a transformer, a direct e.m.f. may produce a current large enough to melt the wire, while an alternating e.m.f. of the same effective magnitude produces a current so small that the heating is inappreciable.

The phase of the impressed e.m.f. when taken from the mains is that at the point where the e.m.f. is applied to the circuit under consideration and not that at the generator. The parts which contribute to the magnitude and phase of the impressed e.m.f. need not be considered in connection with what the impressed e.m.f. produces in a circuit to which it is applied.

An oscillogram of a current phase lag is shown in Fig. 12.

9. Magnitude of Current in Circuits Containing L and R—Impedance and Reactance.—The resultant of two simple harmonic motions in the same line and of the same period is a simple harmonic motion in that line and of that period. Its radius of

reference at any instant has the magnitude and position of the vector sum of the radii of reference of the two component motions. The impressed e.m.f., e_0, and the inductive e.m.f., e_L, are represented by such simple harmonic motions whose radii of reference are equal to their maximum values, $E_{0 \cdot m}$ and $E_{L \cdot m}$. It follows that their resultant, $e_R = Ri$, can also be represented by a simple harmonic motion and that the radius of reference, RI_m, of this motion is the vector sum of $E_{0 \cdot m}$ and $E_{L \cdot m}$. All the three simple harmonic motions are about the point S in the line ASB of Fig. 13(a).

The phase angle between the resultant, RI_m, and the component, $E_{L \cdot m} = \omega L I_m$, is necessarily 90° (Art. 7). These, therefore, are represented in the circles of reference as radii of reference at right angles to each other. The radius of reference RI_m shown in a position whose projection on the line ACB has its maximum value while that of $\omega L I_m$ has a minimum or zero value. Since RI_m is the vector sum of $E_{0 \cdot m}$ and $\omega L I_m$, the problem is to find $E_{0 \cdot m}$ as one of the components. When the parallelogram is completed, the side $E_{0 \cdot m}$ represents this component and therefore the magnitude and the direction of the radius of reference for the impressed e.m.f. This, for the instant shown, makes the instantaneous value of the impressed e.m.f. $e_0 = RI_m$. The angles between the three radii of reference remain fixed while the radii revolve about the center S counterclockwise once during the period of a complete alternating-current cycle. The projections of these radii on the line ASB give the instantaneous magnitudes of e_0, e_L, and e_R. The figure, as already stated, shows the positions of the radii of reference when e_R has its maximum value. At this instant $e_L = 0$, and $e_0 = SF = RI_m$.

The angle α_L is the phase angle between the impressed e.m.f., $E_{0 \cdot m}$, and the RI_m drop and therefore between the impressed e.m.f. and the current.

Since E_0, E_L, and RI (dropping the subscripts m) form a right triangle, then, dropping all subscripts,

$$E^2 = R^2I^2 + \omega^2L^2I^2 = I^2(R^2 + \omega^2L^2).$$

Then

$$I = \frac{E}{\sqrt{R^2 + (\omega L)^2}} \text{ amp.} \tag{7}$$

The equation is derived for the maximum values of E and I in circuits containing L and R; however, because of proportional relationship, it holds for the effective values (Art. 2) as well.

The term $\sqrt{R^2 + \omega^2L^2}$ is called *impedance* and corresponds to resistance in direct-current circuits. It is the apparent resistance and is usually represented by Z. The unit of impedance, the ohm, is that of true or ohmic resistance. The term ωL is called *reactance* and may be represented by X or X_L; its unit, also, is the ohm. In direct-current circuits ω is zero, and the equation becomes that of Ohm's law. In circuits of large induc-

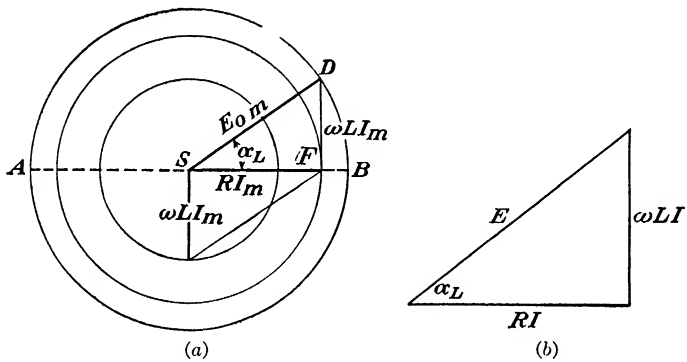


FIG. 13.—(a) "Crank-phase" diagram showing phase angle between E_m and I_m as determined from the known phase and magnitude relationship of RI_m and the counter e.m.f. of self-inductance ωLI_m. (b) Conventional diagram representing phase and magnitude relationships of E, RI and $E_L = \omega LI$.

tance, especially when the frequency ($f = \omega/2\pi$) is large, R often is negligible; then

$$I = \frac{E}{\omega L},$$

where ωL is both impedance and reactance. Reactance, as already stated, adds no energy loss to the circuit; it only limits the supply of energy.

The relationships shown in Fig. 13(a) may be represented, without details, by only the vector triangle SFD; but since the effective values have a proportional relationship to the maximum values, the triangle can be and usually is drawn to represent effective values as shown in Fig. 13(b).

10. Circuits Containing Capacitance and Resistance.—When an alternating e.m.f. is impressed upon a condenser through a negligible resistance and negligible self-inductance, Fig. 14(*a*), a current of such intensity must flow through the conductor as to keep the charge of the condenser continually at such a magnitude that its opposing potential difference, $e_c = q/C$, is at all times equal to, and therefore balances, the impressed e.m.f. e_0. There can be no Ri drop in the connecting wires when $R = 0$. In order that e_c may change in magnitude, electrons must be displaced through the circuit from one plate of the condenser toward the other. The speed of this transfer, and therefore the flow in the conductor, has its greatest magnitude when the

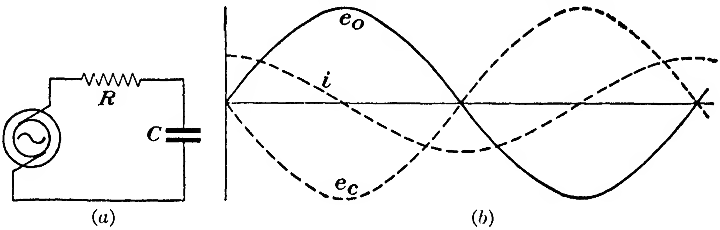


FIG. 14.—(*a*) A circuit containing capacitance, (*b*) phase relationship of e_0, i and e_c in circuits containing capacitance and zero (negligibly small) resistance. Magnitude of i drawn for a case where $\omega C = 0.5$.

impressed e.m.f. changes most rapidly; *i.e.*, as it passes through its zero value. The current then has its maximum value when $e_0(= e_c) = 0$. When $e_0(= e_c)$ has its maximum value, its rate of change is zero and therefore no electrons are being transferred. Then $i = 0$. Since the current has its maximum value when the impressed e.m.f. is zero, it is said to *lead the impressed e.m.f. in phase by* 90°. This phase difference is illustrated in Fig. 14(*b*), which also shows that the current in the circuit is alternating and that $e_c = e_0$ oppose each other and therefore differ in phase by 180°.

During one complete cycle the condenser is charged and discharged twice; therefore the full charge, $q = CE_{c \cdot m}$, of the condenser is transferred past every section of the circuit twice in one direction and twice in the other. Then

$$Q = 4q = I_{av}t = -4CE_{c \cdot m}$$

$$I_{av} = -\frac{4CE_{c \cdot m}}{t} = -4fCE_{c \cdot m},$$

where f is the frequency of the alternating current. The $-$ sign is inserted to show that when the q in the condenser is increasing the e_c has a direction reverse to that of i.
But

$$I_{av} = \frac{2}{\pi}I_m.$$

Then

$$I_m = \frac{\pi}{2}I_{av} = -2\pi fCE_{c\,m} = -\omega CE_{c\cdot m}$$

And, by using effective values,

$$E_c = -\frac{I}{\omega C}. \tag{8}$$

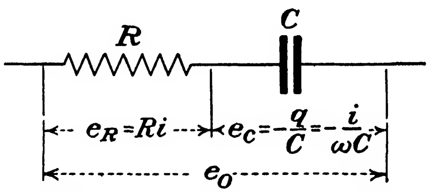


FIG. 15.—Representation of impressed e.m.f., e_0 across any part of a circuit containing C and R.

When the resistance of the circuit is not negligible, there is an Ri drop in the conductor. This Ri drop and the potential difference, e_c, of the condenser, together, Fig. 15, at any instant balance the impressed e.m.f., *i.e.*, $e_0 = Ri + e_c$. The signs of e_0, Ri, and e_c are $+$ or $-$ depending on direction. The current necessarily has its maximum value when the charge, and therefore the e_c of the condenser, varies most. This is the case when $e_c = 0$. It then follows that the phase of i necessarily differs from that of e_c by 90°. The p.d. of the condenser opposing the impressed e.m.f. increases as the condenser charge increases and the Ri drop must be in phase with i.

The phase relations of e_0 to i can then be shown, Fig. 16, by plotting the known relative values of i, Ri, and e_c on rectilinear coordinates in the same manner as was done in Fig. 11 for circuits containing L and R. Using the equation $e_0 = Ri + e_c$, the points 1, 2, 3, 4, 5, etc., for the e_0 curve are located in the order given. In this equation, too, the e_c is treated as having $+$ magnitude when drawn below the axis of abscissas.

1. When $e_c = 0$, $e_0 = Ri$.
2. When $i = 0$, $e_0 = e_c$.
3. When $e_c = -Ri$, $e_0 = 0$.

The point x and the points 4 and 5 are then located as were the corresponding points in Fig. 11. The distance α represents the

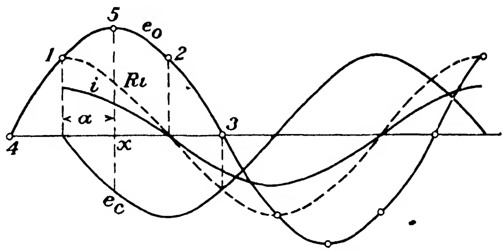


FIG. 16.—The phase relationship of i, Ri, e_c and e_0, and the angle α by which the current leads the impressed e m f in circuits containing C and R. Drawn for a circuit in which $1/\omega C = 2$ and $R = 1$ 5.

phase angle by which the current leads the impressed e.m.f. (see oscillogram in Fig. 12).

The vector diagram of Fig. 17(a) representing the phase and magnitude relationships shows RI leading E_c in phase by 90°. The vector representing E_c is drawn downward so that the phase of E_0 would be shown to lag behind that of RI because in the crank-phase diagram, Fig. 13(a), of which the vector diagram is an equivalent, the radii are assumed to rotate counterclockwise.

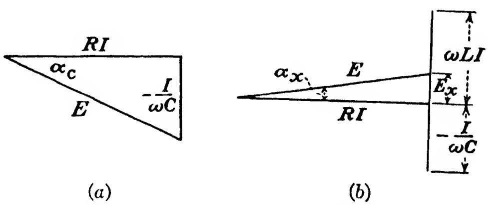


FIG. 17.—(a) Vector diagram for circuit containing C and R; (b) vector diagram for circuit containing L, C and R, drawn for a case where $\omega L > 1/\omega C$.

This representation applies for the same reason as it does in circuits containing L and R (Art. 9). The RI is the resultant of two simple harmonic motions and the quantities involved are vector quantities.

With the subscripts dropped, the vector diagram shows that,

$$E^2 = R^2I^2 + \left(\frac{-I}{\omega C}\right)^2,$$

from which

$$I = \frac{E}{\sqrt{R^2 + (1/\omega C)^2}} \tag{9}$$

Then, when $R = 0$, $I = \dfrac{E}{1/\omega C}$. This equation shows that, in a series circuit, capacitance like self-inductance limits the magnitude of a current. In such a circuit, however, an increase in capacitance and in frequency increases the current while in an inductive circuit an increase in self-inductance and in frequency decreases it. In this equation the impedance

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \text{ ohms}$$

and the reactance $X_c = 1/\omega C$ ohms.

When the circuit contains both capacitance and self-inductance, the phase lead of the capacitance tends to offset the phase lag of the self-inductance. This condition is represented by the vector diagram of Fig. 17(b). The $E_L = \omega LI$, due to self-inductance, tends to make the current lag behind the impressed e.m.f., while the $E_c = -I/\omega C$ tends to make it lead. Both differ 90° in phase from RI and are in opposition to each other. The resultant reactive p.d. then is

$$E_x = \omega LI - \frac{I}{\omega C},$$

and therefore from the vector diagram

$$E^2 = R^2I^2 + \left(\omega LI - \frac{I}{\omega C}\right)^2,$$

from which

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}. \tag{10}$$

When $\omega L = 1/\omega C$, the effects of inductance and capacitance neutralize, and the current is what it would be if there were no reactance in the circuit.

The term corresponding to resistance is called *impedance* as in the case of the simple inductive or capacitive circuits. The

term $(\omega L - 1/\omega C)$ is the reactance and may be represented by X or by $X_L + X_c$, where the numerical value of X_c is treated as having a $-$ sign.

The "equation of the (alternating-current series) circuit" containing all the elements R, L, and C, from what has been given, is seen to be

$$e_0 = Ri + L\frac{di}{dt} + \frac{q}{C}. \tag{11}$$

Because an alternating current "flows" through the wires leading to a condenser, the current is said to *flow through the condenser.*

11. Phase Difference between Impressed E.M.F. and the Current.—Referring to Fig. 13, the phase lag α_L in inductive circuits is seen to be such that

$$\tan \alpha_L = \frac{\omega LI}{RI} = \frac{\omega L}{R}. \tag{12a}$$

When R is very small compared with ωL, $\alpha_L = 90°$, nearly; *i.e.*, the current lags nearly 90° behind the impressed e m.f.

In a similar manner the vector diagram of Fig. 17(a) shows that in circuits with capacitance the phase lead is such that

$$\tan \alpha_c = \frac{I/\omega C}{RI} = \frac{1}{\omega CR}. \tag{12b}$$

In circuits containing both inductance and capacitance the vector diagram, Fig. 17(b), gives

$$\tan \alpha_x = \frac{\omega LI - \dfrac{I}{\omega C}}{RI} = \frac{\omega L}{R} - \frac{1}{\omega CR}. \tag{12c}$$

When the terms due to inductance and capacitance are equal, the impressed e.m.f. and current are in phase. The current then has the largest magnitude for any given E_0 and R and the circuit is said to be a *resonant circuit.* It should be noted that the current can be brought into phase by adjusting either L or C. The potential differences across individual reactances are not eliminated; they neutralize each other in their combined effect only, because they have oppositely directed equal magnitudes.

In low-resistance circuits, these potential differences may be very large, because they are respectively equal to ωLI and $-I/\omega C$ and the current in such circuits usually is large.

Figure 18 represents three parts, A, B, C, of a series circuit on which is impressed the e.m.f. E from the mains. The phase angle, as calculated from Eq. (12*c*), refers to the phase angle between the impressed e.m.f. E and the current and not to the phase differences that may exist between the current and the potential drops E_1, E_2, and E_3. In these parts the phase angles of the potential differences are not that of the p.d. at the mains. The phase difference between the potential differences E_1, E_2, and E_3 and the current are calculated from Eq. (12) where each potential difference is treated as an e.m.f. whose magnitude and

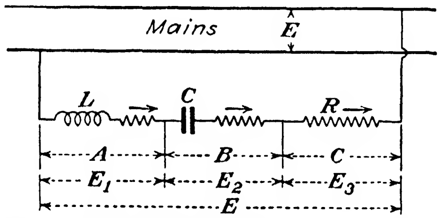


FIG. 18—Phase relationships in a series circuit

phase angle are affected by the reactance and resistance of the concerned part of the circuit. The current in all parts of the circuit, however, has the same phase with respect to the total impressed e.m.f. E. The flow vibrates as though it were an incompressible fluid and therefore in a perfect simple circuit must be the same at all points at any given instant.

12. Power in Alternating-current Circuits—Power Factor.—The power expended in, or delivered to, an alternating-current circuit during any instant is the product of the instantaneous values of the current and e.m.f. at that instant; *i.e.*,

$$p = ei = E \sin \theta \times I \sin (\theta \pm \alpha).$$

The power being delivered or received, therefore, is the average of such instantaneous values for a half cycle. This value can be determined by the method used for determining the average values of e.m.f. and of current and is shown in Appendix V(7) to be

$$P = EI \cos \alpha \qquad (13)$$

where E and I are effective values and α the phase difference between E and I. The term $\cos\alpha$ is called the *power factor.* Power is ordinarily delivered with E and I in phase as nearly as is practicable and therefore with the power factor $\cos\alpha \cong 1$.

13. Alternating Current in Parallel (or Divided) Circuits.—When an alternating e.m.f. is impressed on a divided circuit, Fig. 19, the current in each branch is calculated by Eq. (10) and the phase lag by Eq. (12) as though each were an independent series circuit. The total current in the main circuit then is the vector sum of the branch currents. An important special case of divided circuits is given in Art. XXVIII–7.

14. Choke Coils.—Self-induction is said to "choke the current." This choking effect increases, Eq. (7), with the self-inductance and with the frequency. In coils with large self-inductance or with high frequencies the counter e.m.f. is practically equal to the impressed e.m.f. and differs nearly 90° in phase. In other words, the choking effect is so great that the current becomes very small. Such coils are called *choke coils* and are used in place of resistances to diminish the intensity of alternating currents and with electric filters to prevent high-frequency currents entering any part of a complex circuit.

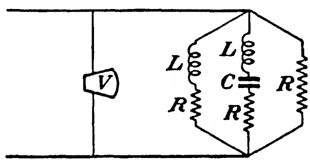


FIG. 19.—Parallel or divided circuit.

15. Phase Difference between Inducing and Induced Currents.—It was shown (Art. XVII–2) that the direction of the e.m.f. induced in a neighboring coil is the reverse of that of the inducing current when that is increasing and in the same direction when decreasing. In the case of an alternating current the inducing current increases and decreases in succession, inducing in the neighboring coil an alternating e.m.f. which is 90° out of phase with the inducing current.

If it were possible to have a loop with zero self-inductance, the induced current in it would be in phase with the induced e.m.f. The current in the neighboring coil would then be 90° out of phase with the inducing current, as shown in Fig. 20.

Parallel currents either attract or repel each other (B.P.2), and the magnitude of the force may be shown to be proportional to the product of the current strengths. The ordinates of the

curve C are made proportional to the product of the ordinates of the curves i_p and i_s. When both ordinates are on the same side of the axis of abscissas, the product is plotted above the axis of abscissas; and when they are on opposite sides, below the axis. These products represent the forces of attraction and repulsion between the loops. The parts of the curve above the

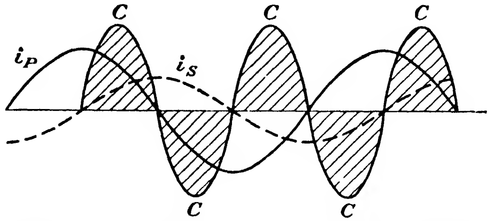


FIG. 20.—Attractions and repulsions between induced and inducing currents when $L = 0$.

axis represent attractions; and the parts below, repulsions. On account of the symmetry of the curves i_p and i_s, the attractions exactly balance the repulsions. Such loops then, with an alternating current of sufficient frequency, would appear neither to attract nor to repel each other.

In any actual case of even a simple loop self-inductance is always present, and the induced current lags behind the induced

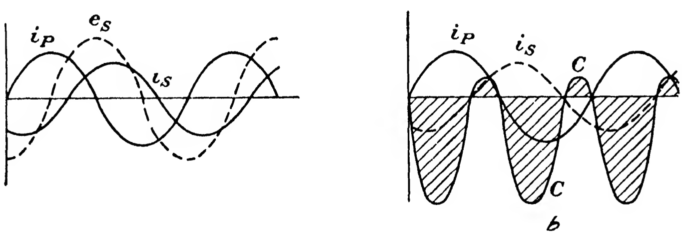


FIG. 21.—(a) Phase relationship of current in the primary, i_p, and e_s and i_s in the secondary; (b) repulsions greater than attractions because of self-inductance in the secondary circuit.

e.m.f., as shown in Fig. 21(*a*). The phase difference between the inducing and induced currents, as represented by the curves i_p and i_s, is greater than 90° but never exceeds 180°. In Fig. 21(*b*) the curve C, representing the product of the ordinates and therefore the magnitudes of the forces of attraction and repulsion, is seen to have the portions representing repulsion larger than the

portions representing attractions. There is therefore normally a force of repulsion between an alternating current and the induced current in an adjacent circuit. This effect may be shown by slipping a nonmagnetic metal ring over a projecting iron core of an electromagnet energized by an alternating current. The inducing primary current then is mainly the alternating magnetization whirl (Art. VI–7) in the iron. The ring is repelled and raised to a point where the force of repulsion equals the force due to gravity. If the circuit is closed suddenly, the ring may be thrown a considerable distance.

The induced currents in the ring are produced by electromagnetic pulses or waves (Art. 4) and illustrate what normally

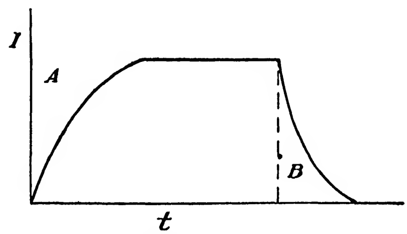


FIG. 22.—Effect of L on the establishment and the cessation of a current.

takes place in any conductor through which electromagnetic waves are passing.

16. Effect of Self-induction on the Establishment and the Cessation of a Current.—The presence of the e.m.f. of self-induction in a circuit is due to the magnetic field being established or collapsed. The e.m.f. of self-induction varies as the rate at which this changing field is cutting through the circuit. When the current is increasing, the counter e.m.f. retards the growth of the current. When an e.m.f. is applied to a circuit, the final strength of the current, therefore, is attained only after some lapse of time. The value of the current at any instant is

$$i = \frac{e - L\dfrac{di}{dt}}{R}.$$

The term Ldi/dt is the counter e.m.f. of self-induction (Art. XVII–6). As the current increases, the rate at which it is changing diminishes, until finally the term representing the

counter e.m.f. becomes negligible. The time required for the current to build up is usually a few ten-thousandths of a second in a circuit containing no iron but may be many seconds in large electromagnets.

When the impressed e.m.f. ceases, the magnetic field, in collapsing, induces an e.m.f. which forces the electrons in the direction of the original flow; *i.e.*, the electrons continue to move because of their flow inertia.

The establishment and the cessation of a current are both illustrated in Fig. 22.

Questions

1. Derive in terms of $N\phi$ and ω the expression for the e m f. induced in a rotating coil whose axis is at right angles to a uniform magnetic field.

2. Show that the variation of the magnitude of this induced e m.f. follows the law of simple harmonic motion, and that, when plotted with time on the axis of abscissas, it is represented by a harmonic curve.

3. Define instantaneous, average, maximum, and effective (root-mean-square) e.m.f. and current.

4. What is the relation of E_r to E_m?

5. What are frequency and angular velocity? Give the equation expressing their relationship.

6. Show by a simple diagram the means used for obtaining pulsating and alternating currents from coils rotating in magnetic fields.

7. Show that electromagnetic waves emanate from alternating currents.

8. Show that, when these electromagnetic waves pass through a conductor, the alternating electric field causes the free electrons in the conductor to oscillate.

9. Derive in terms of angular velocity the expression for the e.m.f. due to self-induction. Give the maximum value of the e m.f.

10. What is meant by phase angle? A cycle?

11. Explain why the current and the e.m.f. of self-induction differ exactly 90° in phase.

12. Explain why in circuits containing self-inductance and negligible resistance the current at any instant is such that the e m.f. of self-induction just balances the impressed e.m.f. Show how it follows that the phase of the current lags 90° behind that of the impressed e.m.f.

13. Explain why the Ri drop is necessarily in phase with the current.

14. Using rectilinear coordinates, draw curves showing the phase relationship between i, e_L, and Ri in alternating-current circuits containing L and R, and from these curves locate five points for a one-half cycle of the impressed e m.f. curve.

15. Show from the foregoing curves that in circuits containing L and R the current lags in phase less than 90° behind the impressed e.m.f. and that its magnitude is diminished by the self-inductance.

16. How is the resultant of two simple harmonic motions in the same line obtained?

17. Using the crank-phase diagram, derive the expression for the current in a circuit containing L and R. Draw the conventional vector diagram.

18. Draw curves showing the phase relationship of i, e_c, and e_0 in circuits containing capacitance and negligible resistance. Explain why the current leads the impressed e m.f. by 90°.

19. Draw curves showing the phase relationship of i, Ri, and e_c in circuits containing capacitance and resistance and from these curves locate five points for a one-half cycle of the impressed e m f. curve.

20. Draw vector diagram for circuits containing C and R and for circuits containing L, C, and R. Derive the expression for the current in each case.

21. Give and explain the "equation of the circuit."

22. Explain how to adjust a circuit so that the current may be in phase with the impressed e m f.

23. What are impedance and reactance? What are the units of impedance and reactance called?

24. What is the reactance in each of the three kinds of circuits: containing only L; only C; and both L and C?

25. Derive the expression for the lag in phase in circuits containing inductance; capacitance; inductance and capacitance.

26. Give the expression for the power expended in alternating-current circuits. Explain.

27. What are choke coils? Give the principle of their action.

28. Show that, if there were no self-inductance, the induced current in a neighboring circuit would lag 90° behind the current in the primary circuit.

29. Show that on account of self-inductance the induced current in the secondary lags more than 90°, but less than 180°, behind that in the primary.

30. Show that, if there were no self-inductance in the secondary circuit, the induced current produced in it by an alternating current in the primary would, on the whole, cause no attraction or repulsion between the circuits.

31. Show that, on account of self-inductance, two such coils repel each other.

32. Explain the part self-inductance plays in the establishment of any current. In the cessation of any current.

33. What is the estimated amplitude of oscillation of the electron flow in a copper wire of 1 mm cross section in which a 60-cycle current has an effective value of 1 amp?

Problems

1. A coil of 100 cm² area and 50 turns is making 20 revolutions/sec on an axis at right angles to the lines of force, in a magnetic field of 10,000 gauss. What is the effective magnitude of the induced e.m.f.?

2. What is the maximum magnitude of the counter e.m.f. of self-induction in a coil whose $L = 0.02$ henrys, when $I_m = 5$ amp and the frequency is 20 cycles/sec?

3. An effective e.m.f. of 110 volts and 60 cycles/sec is impressed on a series circuit whose $L = 0.5$ henrys. (*a*) What is the inductive reactance? (*b*) Impedance when $R = 2$? (*c*) Magnitude of I_r when $R \cong 0$? (*d*) When $R = 2$ ohms? (*e*) What is the phase lag in each case? (*f*) What power is being expended in the circuit in each case?

4. An effective e m f. of 110 volts and 60 cycles/sec is impressed on a series circuit whose capacitance and resistance are 100 microfarads and 20 ohms. (*a*) What is the reactance? (*b*) Impedance? (*c*) Magnitude of the effective current? (*d*) Phase difference?

5. An effective e.m.f. of 110 volts and 60 cycles/sec is impressed on a series circuit whose $R = 21$ ohms, $L = 0.5$ henrys, and $C = 40$ microfarads. (*a*) What is the inductive reactance? (*b*) Capacitive reactance? (*c*) Impedance? (*d*) Effective current? (*e*) Phase difference? (*f*) Power being expended?

Experiments

1. Model of a rotating coil in a uniform magnetic field shown.

2. An alternating current produced by slowly rotating a loop in a magnetic field. A pulsating current produced by using a segmented commutator.

3. An alternating current produced in a circuit by induction from an electromagnet energized by an alternating current.

4. The carbon filament of an incandescent lamp made either to deflect or to vibrate in a magnetic field depending on whether the energizing current in the filament is a direct or an alternating current.

5. Alternating-current wave form projected by means of an oscillograph.

6. Incandescent lamp, in a circuit in which L may be altered by inserting a bar of iron into a coil, is energized first by alternating current and then by direct current of the same voltage. In one case a change in L changes the current, and in the other it does not.

7. Light is cut off entirely by introducing a laminated iron bar into a solenoid which had been made a part of the lighting circuit. The bar itself is within another solenoid which, when energized by a direct current, magnetizes the iron bar in one direction and thereby diminishes L and increases the current in the lighting circuit. This illustrates a method used in the control of theater lighting.

8. An alternating current produced by an alternating e.m.f. applied to a condenser by means of a magneto.

9. A current varied by altering either L or C in a series circuit.

10. Aluminum ring placed over an iron core projecting from a solenoid is repelled when the solenoid is energized by an alternating current.

11. The ring of Exp. 10 is heated by the induced current.

CHAPTER XIX

EDDY CURRENTS

1. Eddy Currents.—*Eddy currents* are currents which are induced in any mass of metal when it is moving relative to a nonuniform magnetic field or rotating relative to any magnetic field. The current is usually in the form of a whirl or an eddy, from which it derives its name. Such currents are also known as *Foucault currents,* after their discoverer. Their presence must be considered in the construction of electric machines. In some instruments they are utilized to perform an important

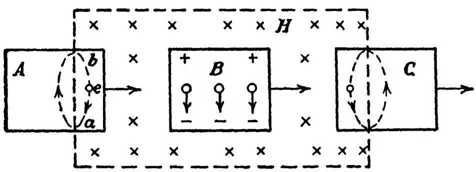


FIG. 1.—Eddy currents produced when a mass is entering into or emerging from a magnetic field.

function, but in most cases means are devised to limit their development, for not only is the energy required to produce them wasted but the heating caused by them may be extremely undesirable.

2. Masses Moving in a Magnetic Field.—The rectangular conductors A, B, and C, Fig. 1, are moving into, through, and out of a magnetic field whose direction is represented by the crosses. The conductor A has only its front end in the magnetic field. Any free electron e moving in that part with the mass has its own magnetic field and through electromagnetic reaction (Law A) is pushed downward. All free electrons in the front part of the metal are similarly urged from b toward a. The displacement of the electrons toward a causes the lowering of the potential at a and the raising of it at b. The electrons then find their way back to b in the conductor outside the field, moving

from a lower to a higher potential. The electron flow forms an eddy in the metal as represented by the broken-line loop.

The electrons in the right-hand side of the eddy are moving downward; hence the magnetic field evoked by their downward motion strengthens the magnetic field through which the mass is moving on the cutting side. A force, therefore, is impressed on the electrons which opposes the motion of the mass (Lenz's law, Art. XIII-8). The work that must be done to move the mass into the field against this force is expended only because the eddy currents are being generated and becomes the energy of the eddy current and finally heat.

A metallic mass swinging as a pendulum between the poles of an unenergized electromagnet stops suddenly when the energizing circuit is closed. If the pendulum is moved by hand, it acts as though it were being moved through thick syrup. If the metallic mass of the pendulum is replaced by one with slits so that the resistance of the eddy-current paths is greatly increased, the pendulum moves through the magnetic field with ease.

When the whole rectangular conductor is moving in a uniform magnetic field, the electrons at all points within it are pushed downward, as shown in *B*, Fig. 1. Because there is no return path for an eddy current, the free electrons are only displaced until the potential difference due to their displacement urges them upward with the same force with which the magnetic forces due to their forward motion with the mass urges them downward. In this case the force opposing the motion of the mass exists only during the instant the electrons are being displaced and is negligibly small.

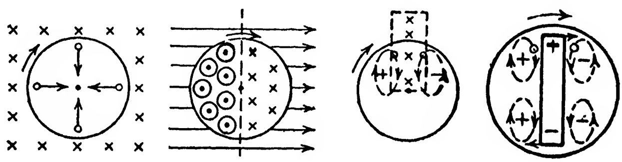
When the rectangular mass is emerging from the field or is entering a weaker portion of a field, an eddy current is again produced as shown in *C*, Fig. 1, and an opposing force also is exerted on the moving mass.

It should be noted that the eddies in *A* and *C* both have the electrons moving downward in the magnetic field and that one is a clockwise whirl while the other is counterclockwise; hence one whirl has its + magnetic pole, and the other its − pole facing the observer.

3. Masses Rotating in a Magnetic Field.—Imagine an electron moving with a rotating mass and evoking a magnetic field by its

motion (Law A). This field determines the direction in which the electron and its electron neighbors are urged, through electromagnetic induction, when the mass of which they are a part is rotating in a magnetic field.

When a mass is rotating on an axis which is parallel to the magnetic lines of force as shown in Fig. 2(*a*), the electrons are urged either toward or away from the axis of rotation depending on the direction of rotation and the direction of the magnetic field. In the case shown the electrons are urged toward the axis and thereby cause the circumference to become charged positively and the axis negatively. The electrons are displaced, but no eddy currents are produced.



(*a*) (*b*) (*c*) (*d*)

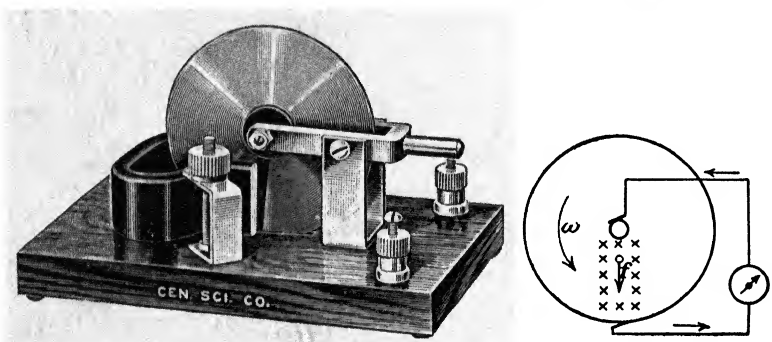
FIG. 2.—Production of eddy currents (*a*), (*b*) in masses rotating in a uniform magnetic field; (*c*), (*d*) in masses rotating by magnetic poles.

When the mass is rotating on an axis at right angles to the magnetic lines of force, as shown in the case of a rotating cylinder, Fig. 1(*b*), the free electrons on one side of the vertical plane passing through the axis are urged toward the observer and those on the other side, away from him. These displacements establish potential differences which cause electrons to be urged across the faces of the cylinder and thereby produce an eddy which evokes a torque opposing the rotation of the cylinder. The eddy current, in cases like this, is almost entirely eliminated by making the cylinder of thin circular disks coated with an insulating varnish. The armatures of generators and motors (Art. XXIII–1) are wound on such cylinders of laminated iron or soft steel.

When a metal disk is rotating between the poles of a horseshoe magnet, so that the magnetic field covers only a part of the disk, two eddy currents are formed as shown in Fig. 2(*c*), where the crosses show the direction of the magnetic lines through the disk. The work done to produce these eddy currents may be explained from two points of view: (*a*) The streams of electrons in the parts of the eddies within the magnetic field are urged

(Law A, Lenz's law) in a direction opposing that of their motion with the disk (*b*) The two eddy currents have reverse directions and therefore their unlike magnetic poles face the observer. Each of these is acted on by the adjacent pole of the magnet in a direction opposing the rotation of the disk. This form of apparatus is employed to control the mechanism of the watt-hour meter (Art. XXI–13), as an electromagnetic brake, etc.

When a metal disk is rotated below or above a magnetic needle, the needle feels a torque in the direction of rotation of the disk. The eddy currents induced in the disk, which in Fig. 2(*d*) is shown rotating below the needle, evoke forces opposing the motion of the disk. Since action is equal to reaction (Newton's third law of motion), the magnetic needle feels a torque urging it in the direction of the disk's rotation. From another point of view, the magnetic poles of the eddy currents facing the needle



(*a*) (*b*)

FIG. 3.—(*a*) Faraday's disk dynamo; (*b*) diagram of same.

are such that those moving from the needle attract it, and those moving toward it repel; hence both sets, together, produce a torque which urges the needle in the direction of the disk's rotation.

If in the cases illustrated in Fig. 2(*a*), (*c*) the axis and circumference of the rotating disk are connected so as to include the disk as a part of an electric circuit, the potential difference (e m.f.) established by the rotation causes a current to flow in the circuit. This arrangement, Fig. 3(*a*), (*b*), is an electric generator and is known as *Faraday's disk dynamo*. The diagram shows that the action is simply that of any conductor cutting a magnetic field.

4. Magnetic Flux Moving through Masses.—If a horseshoe magnet is rotated beneath a metal disk, the moving magnetic fields cuts the metal, and the associated nonconservative electric field (Art. XIII–5) produces eddy currents whose directions are such that the electromagnetic reaction gives the disk a torque in the direction of the rotating magnet.

The field of the rotating magnet, Fig. 4*A*, cuts through the glass plate *G* and the pivoted metallic disk *C*, producing the eddy

currents shown in Fig. 4*B*. This figure, *B*, represents the eddies as seen when the observer is looking upward from the rotating magnet. This action of a moving magnetic field is employed in the induction motor (Art. XXIII–5) and in the electromagnetic speedometer in which a metal cap, held by spiral springs, follows a rotating magnet until the torque of the distorted springs balances the deflecting torque.

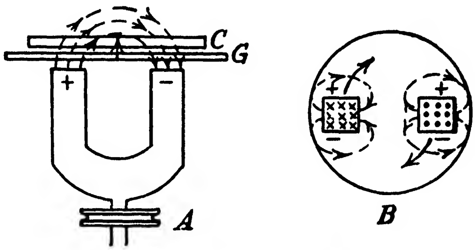


FIG. 4.—Production of eddy currents by a moving or a rotating magnetic field.

(The student should check the direction of the eddies in terms of (*a*) the associated electric field and (*b*) the strengthening of the cutting magnetic field (Lenz's law). He should check the direction of rotation of the disk (*c*) in terms of forces acting on moving electrons in a magnetic field and (*d*) with reference to action between like and unlike magnetic poles.)

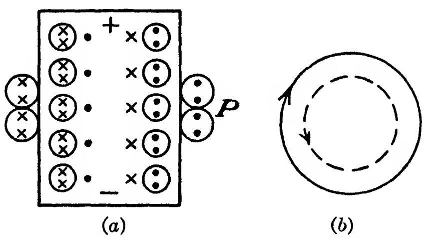


FIG. 5.—Eddy whirl in the core of an electromagnet when magnetization is increasing. (*a*) Section of core; (*b*) top view of core.

When the magnetization of the iron core of an electromagnet is changing in intensity, there is induced within the core a cylindrical eddy current whose axis is that of the magnetization whirl (Art. VI–7) of the magnet. Figure 5(*a*) represents a section of such a core, when its intensity of magnetization is increasing. The accelerating electrons of the whirl then urge the free electrons of the iron (Law C, Art. XVI–1) in the opposite direction to that of their own acceleration. The crosses and dots

which are not encircled represent the direction of this eddy current, and the encircled double crosses and double dots the accelerating magnetization whirl.

(The existence of these eddies may also be explained by referring directly to the action of the magnetic field cutting through the free electrons when it is building up or collapsing.)

The changing current in the magnetizing coil P, whose direction is that of the magnetization whirl, contributes its almost negligible share to the production of the eddy current.

Figure 5(b) shows the + pole end of the magnet. The full-line circle represents the magnetization whirl, and the broken-line circle the induced eddy when the magnetization is increasing.

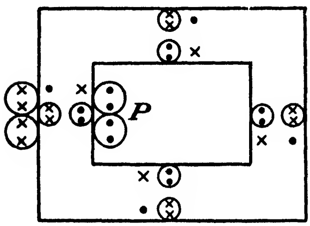


FIG. 6.—Eddy whirl in a rectangular core when the intensity of magnetization is changing.

When the magnetizing current is alternating, the eddy currents induced in the core are also alternating, absorbing much energy and heating the core. The larger the resistance along the circular path, the smaller is the induced eddy current; hence the cores of induction coils are made of thin iron wires whose surface layers of oxide give good insulation and thereby limit the eddy flow.

If the iron core of Fig. 5 be extended to form a loop as shown in Fig. 6, each molecular magnet, as it is turned by the magnetizing field, acts on those at either pole and turns them to face in its own direction. In this manner the core becomes magnetized equally around the whole loop, as represented by the encircled dots and crosses, and approximates a toroidal magnet (Art. VI-4). When the magnetization increases or decreases, a cylindrical eddy of free electrons is produced as in the case of the straight core. The dots and crosses that are not encircled represent this cylindrical eddy when the magnetization is increasing.

This form of a core, when used in transformers, is constructed of thin shellacked sheets, called *laminations*, which limit the formation of eddy currents. Another method of limiting eddy

currents is to make the core from minute granules of the magnetic material. The granules are covered with an insulating compound and formed into the desired shape under great pressure.

5. Screening Effect of Eddy Currents.—Let *A*, Fig. 7, be a section of the iron core of an electromagnet, with the magnetization whirl in the direction of the arrow; also let *B* and *C* be sections of two conducting cylinders about the core. When the magnetization of the electromagnet is increasing, for example, it induces eddy currents in both *B* and *C* in a direction opposite that of its magnetization whirl, as represented by the full-line arrows.

The eddy current in cylinder *B* is large on account of the low resistance of the cylinder and its nearness to the core. This

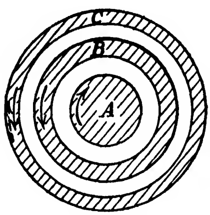


FIG. 7.—Screening effect of a metal cylinder.

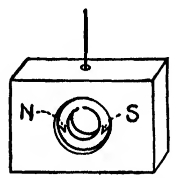


FIG. 8.—A circular magnetic needle damped by eddy currents induced in a block of copper.

eddy current, because of its reverse acceleration, neutralizes to a great extent the effect of the magnetizing core on neighboring conductors, as is shown by the broken-arrowed line in the cylinder *C*. The cylinder *C* is then said to be screened. This screening principle is used for changing the intensity of the e.m.f. in the secondary of small induction coils by inserting a cylinder of brass or copper between the primary and the secondary coils.

6. Damping of Galvanometer Needles and Coils.—The ring-magnet needle of a moving-magnet galvanometer has its poles on opposite sides of the ring as shown in Fig. 8. This needle, suspended by a fiber, hangs in a cavity within a block of copper. When the needle vibrates, its magnetic field cuts the copper block and induces eddy currents which take energy from the swinging needle and thereby damp its oscillations.

The coil of a moving-coil galvanometer (Art. XXI–3) turns in a magnetic field. If this coil is wound on a metal rectangle, a large damping current is induced in the rectangle as it turns with the coil. This induced current, however, is not an eddy current in a strict sense; *i.e.*, it is a case of a loop of low resistance turning in a magnetic field (Art. XVIII–1). The coil of such a galvanometer, when used on closed circuit of not too high a resistance, is damped without the rectangle. Enough current is induced in the coil itself to produce the damping.

Questions

1. What is an eddy current? A Foucault current?

2. Explain the direction of the eddy current induced when a rectangular piece of metal is moving into or out of a magnetic field.

3. Show that, when an eddy current is produced, a force is evoked opposing the motion of that which causes the current.

4. Show that when a mass of metal is moving in a uniform magnetic field at right angles to the lines of force no current is induced, but that a potential difference is established between two sides of the metal.

5. If the mass were moving parallel to the magnetic lines, would any effect be produced by the field?

6. How can the formation of eddy currents be limited?

7. Explain the effect of rotating a disk or cylinder in a uniform magnetic field with its axis parallel to the lines of force.

8. Explain how an eddy current is induced in the foregoing disk when the axis of rotation is at right angles to the lines of force.

9. Explain how two eddy whirls are produced when a disk is revolving between the poles of a horseshoe magnet

10. Explain what happens when a metal disk is revolving below a magnetic needle.

11. Explain the action of the Faraday disk dynamo.

12. Explain what happens when a horseshoe magnet is revolving beneath a metal disk.

13. Explain the direction of the eddy currents in the core of an electromagnet. How are these currents usually limited?

14. Explain how eddy currents are formed in the core of a transformer. Which produces the greater effect, the change in the magnetization of the iron or the change in the intensity of the magnetizing current? How is the formation of eddy currents limited in the transformer?

15. Explain how a metal tube slipped over an electromagnet "screens" the region outside.

16. Explain how the needle of a moving-magnet galvanometer is damped by eddy currents.

17. How is the coil of a moving-coil galvanometer damped?

Experiments

1. The presence of an eddy current shown by swinging a metal conductor into the strong magnetic field of an electromagnet and then by swinging a similar conductor in which the eddy is reduced by the presence of slits.

2. A cylindrical conductor hung by a twisted string so that it rotates in a magnetic field with its axis vertical is damped; a similar cylinder made of insulated circular disks is not.

3. A rotating disk inserted partly in the field of an electromagnet is damped.

4. A magnetic needle pivoted on a pane of glass placed over a rotating metal disk follows the rotations of the disk.

5. A metal disk pivoted on a pane of glass above a rotating horseshoe magnet follows the rotation of the magnet. An electromagnetic speedometer shown.

6. Iron-wire core of an alternating-current electromagnet and a laminated core of a transformer shown.

7. Screening effect of a metallic cylinder shown as applied to a medical induction coil.

8. Damping of a galvanometer coil with a damping rectangle and of a galvanometer needle by a neighboring mass of copper.

CHAPTER XX

MAGNETIC CIRCUIT—MAGNETIC PROPERTIES OF IRON

1. Number of Lines of Force Emanating from a Unit Point-pole.—For the purpose of analysis it is customary to place the conventional unit magnetic point-pole (Arts. VI–8, VII–1) wherever desired and often to disregard its connection with any real magnet. Such a point-pole has no material existence but is found to be a useful tool for calculating the field strength outside a magnet. It should be noted that the magnetic field surrounding such a pole is radial and that it extends to an infinite distance. This field, therefore, does not correspond to any actual magnetic field which always can be represented by closed loops.

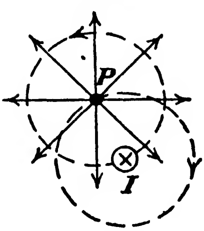


FIG. 1.—Work done in moving a unit magnetic pole around a current

Since a unit point-pole acts on another point-pole 1 cm distant with a force of 1 dyne (Art. VII–1), a unit point-pole 1 cm distant from another unit pole is in a magnetic field of 1 oersted. (Art. VII–3). Since a magnetic field of such intensity has assigned to it (Art. VII–4), by convention, one line of force per square centimeter and there are 4π cm^2 in a spherical surface of 1 cm radius, such a surface about the unit point-pole is pierced by 4π lines, all of which emanate from the unit point-pole.

2. Work Expended in Moving a Unit Point-pole around a Closed Path Linking a Coil of Wire.—An infinitely long energized wire, I, Fig. 1, when moved perpendicular to its length completely around a unit point-pole P, cuts all the 4π magnetic lines of force emanating from the pole. The fraction of the total number of lines that pass through the open ends of the cylinder cut out by such a motion of even a wire of comparatively short length is negligible. The mechanical energy expended in cutting these lines (Art. XIII–2) is

$$w_1 = \Phi_1 I' = 4\pi I' \text{ ergs.}$$

When the unit point-pole P moves completely around such a wire, all its lines of force cut the wire once; and since action equals reaction, or because it is immaterial whether the lines are cutting or are being cut (Art. XIII–5), the same amount of work is done in moving the pole around the wire as in moving the wire around the pole. While the pole is moving around the wire, the lines of force must be imagined to be moving always parallel to themselves; otherwise all of them would not cut the wire. (The lines of force of a magnet rotating on an axis passing through its point-poles would not cut the wire.)

When the unit pole moves in any path about N such wires having equal currents flowing in the same direction, the work done is

$$w = 4\pi NI' \text{ ergs.}$$

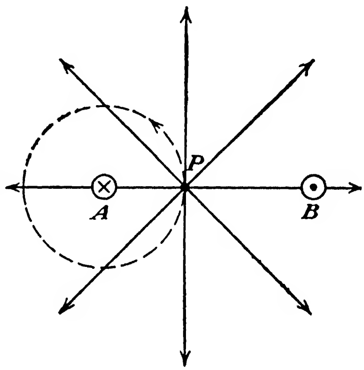


FIG. 2.—Work done in moving a unit point-pole completely around one of two parallel wires or completely around any path linking a loop carrying a current.

The two parallel wires A and B, Fig. 2, have equal currents flowing in opposite directions. Inspection shows that in moving the unit point-pole completely around one of the wires through any path all the magnetic lines of force cut the encircled wire once. Somewhat less than one-half of the lines also cut the other wire; but they cut it first in one direction and then in the other, so that the positive work due to this cutting equals the negative. The total work done, then, is that due to the presence of the encircled wire alone.

Let Fig. 2 now represent a section of an energized loop of wire and a unit point-pole being moved completely around a path linking the loop. All the lines of force cut the encircled part of the loop once and part of them also cut the unencircled section. Those that cut the unencircled section cut it, as in the case of two parallel wires, first in one direction and then in the other, contributing nothing, on the whole, to the work required to move

the pole completely around the path linking the loop. The current then may be considered as having been cut once by all the 4π lines of the pole. The total work done then is that required to move the pole around a wire of infinite length which is carrying the same current, *i.e.*, $4\pi I'$ ergs.

The work done in moving a unit point-pole around any closed path linking a coil of N turns is

$$w = 4\pi NI' = 0.4\pi NI \text{ ergs,} \tag{1}$$

which must be the average strength of the magnetic field times its length (*i.e.*, the length of the path).

If the unit point-pole is moved completely around a path inclosing both the wires A and B, Fig. 2, just as much positive

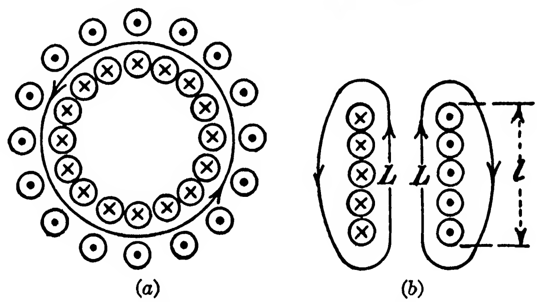


FIG. 3.—Magnetic lines of force (*a*) linking the loops of a toroid; (*b*) linking the loops of a long solenoid.

work is done because of the current in one wire as negative work because of the oppositely directed (equal) current in the other wire. This also applies to moving a unit point-pole completely around a loop. No work therefore is required to move a unit point-pole the whole distance around two such parallel wires or completely around the loop (without linking it).

3. Intensity of the Magnetizing Field within a Toroid, within a Long Solenoid, and within the Iron of an Electromagnet.—Since the magnetic lines of force within a toroid link all N loops, Fig. 3(*a*), the work required (Art. 2) to move a unit point-pole along the entire length of a line is $0.4\pi NI$ ergs. But since the intensity of a magnetic field is measured by the force acting on a unit point-pole placed in it, the work done in moving the unit pole the whole length l of a line of force within the toroid is

$$w = Hl.$$

From which

$$H = \frac{w}{l} = \frac{0.4\pi NI}{l} = \frac{1.257NI}{l} \text{ oersteds,} \tag{2}$$

where N, I, and l, are respectively the number of turns, the current in amperes, and the length of the field in centimeters. Equation (2) shows that $H \propto NI/l$ and therefore that it can be expressed in terms of ampere turns / cm or ampere turns / inch.

The magnetic field within a long solenoid is strong compared with that without; hence in moving a unit point-pole the whole length of a line L, Fig. 3(b), the greater part of the work is expended in the distance l within the solenoid. Equation (2) in which l is made to represent the length of the solenoid then holds, approximately, for the field strength, H, at and near the center of the solenoid. When l stands for the total circuit length of L, H represents the average field intensity along the whole length of the line.

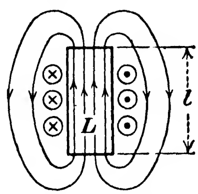


FIG. 4—Magnetic lines of an electromagnet link the magnetizing coil.

When a toroid is wound about a ring of iron, the iron is magnetized along its whole length by a magnetic field of known intensity and the resulting *toroidal magnet* has all its magnetic lines of force within the iron. A piece of iron completely within a solenoid is likewise magnetized by a field of calculable intensity. If the iron is longer than the solenoid, as shown in Fig. 4, the approximate average intensity of the magnetizing field is conveniently determined by making l represent the *length of the iron in place of the length of the solenoid.* In every case the magnetic field of the magnetized iron is superposed on the magnetizing field and has the same general direction.

4. Magnetic Potential Difference—Magnetomotive Force—Gilbert.—By analogy with electrostatic potential difference (Art. III–3), a unit of *magnetic potential difference* exists between two points in a magnetic field when 1 erg of work is required to move a unit point-pole from one of the points to the other. This unit of magnetic potential difference is called the *gilbert.* It follows then that

1. The magnetic p.d. between two points 1 cm apart in a unit magnetic field is 1 gilbert.

2. The magnetic p.d. between any two planes A and B, Fig. 5(a), in any magnetic field is $\mathfrak{F} = Hl$ gilberts, *i.e.*, the average

William Gilbert (1540–1603), English physician and physicist, proved that the compass points north and south because of the earth's magnetism and was the first to use the term, "electricity" Author of the epoch-making book "De Magnete" (1600). The unit of magnetomotive force, the *gilbert*, is named in his honor.

intensity times the length of the field, or, the work required to move a unit point-pole from one of the planes to the other.

3. The total p.d. in the complete circuit along any line of force L, Fig. 5(b), linking a magnetizing solenoid is Hl, the average

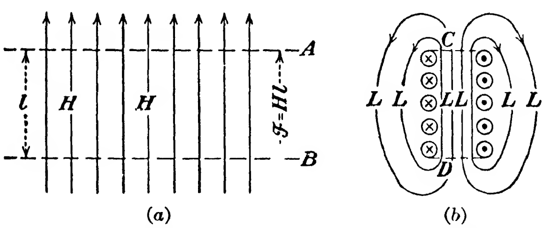


FIG. 5.—(a) Magnetic p d in a magnetic field H, (b) magnetomotive force or total magnetic p.d. impressed on a magnetic circuit L of any length by a magnetizing solenoid.

strength of the field throughout the length of the line times this length. The total p.d. is measured, therefore, by the work required to move a unit point-pole throughout the whole length

of the circuit. This is the case regardless of whether or not the path follows any line of force and regardless of the length of the path.

But it has been shown (Art. 2) that the work expended in moving a unit point-pole through the length of such a linking circuit is $0.4\pi NI$ ergs. Then the total magnetic p.d. (magnetomotive force) in the magnetic circuit is

$$\mathfrak{F} = Hl = 0.4\pi NI = 1.257NI \text{ gilberts.} \tag{3}$$

The greater part of the p.d., of course, is within the solenoid between the planes C and D.

Since the magnetizing power of any magnetic field is proportional to both the intensity and the length of the magnetic field, the magnetizing power, or *magnetomotive force,* then is identical with the applied potential difference. It is represented, therefore, by the same symbol as p.d. and is measured in terms of the same unit by the application of Eq. (3).

Since the solenoid is generally employed as the magnetizing agent, and because the intensity of magnetization produced by any given magnetizing field depends also on other factors than the impressed magnetic potential difference, it is found convenient in the case of a magnetizing solenoid to restrict the term magnetomotive force to the total magnetizing power. In a transformer, for example, the magnetizing field makes a complete circuit through the material of the core (Fig. 9A), while in an electromagnet (Fig. 9B) it passes for a part of the distance through the air gap. The total p.d. in the field linking the solenoid, however, is taken to be the impressed m.m.f. in both cases. In the electromagnet, therefore, the air gap is assumed to be a part of the circuit upon which the m.m.f. is impressed.

Magnetomotive force is that which causes a magnetic substance to become a magnet, but the term is generally restricted to apply to the total magnetizing power throughout any circuit linking the magnetizing solenoid and is measured by the total impressed potential difference ($0.4\pi NI$) in that circuit.

The term *gilberts per centimeter* is often used to designate the intensity of the magnetic field H in place of oersteds. This is possible because the p.d. per centimeter and the intensity of the field are numerically equal.

The product NI, to which the intensity of the magnetizing field and the m.m.f. are proportional, is for convenience called *ampere-turns*. The m.m.f. remains unchanged regardless of the strength of the current, providing the product of the strength of the current and the number of turns remains constant. The m.m.f. then is 1.257 gilberts/ampere-turn, so that its magnitude is often expressed in ampere-turns in place of gilberts.

5. Magnetic Circuit.—Equation (2) shows that $H \propto NI$ and Eq. (3) that $\mathfrak{F} \propto NI$.

Then

$$\mathfrak{F} \propto NI \propto H \propto \phi,$$

from which

$$\mathfrak{F} = \mathfrak{R}\phi \text{ gilberts},$$

or

$$\phi = \frac{\mathfrak{F}}{\mathfrak{R}} \text{ maxwells.} \tag{4}$$

The "constant" $\mathfrak{R}$ is called *reluctance*, and the equation is to magnetic circuits what Ohm's law (Eq. IX–7) is to the electric. The $\mathfrak{F}$, ϕ, and $\mathfrak{R}$ correspond to E, I, and R, respectively.

The unit of reluctance has no universally accepted name. In the United States it has generally been called *oersted*, but an international committee on units is now recommending that name for the unit of magnetic-field intensity (Art. VII–3). The recommendation of this committee is followed in this text.

When a magnetic material such as iron is inserted into any part of a magnetic circuit, the magnetized iron adds to the linking flux; hence the total flux is greatly increased without any change in the m.m.f. From this it follows that $\mathfrak{R}$ is decreased by the presence of the iron. The reluctance, then, is a variable quantity depending on the nature and amount of the magnetic material in the circuit and the degree of its magnetization. It has, however, a constant value in any given circuit if the circuit contains no magnetic material.

6. Magnetic Field Density (Magnetic Induction).—When an iron rod is magnetized, its magnetic flux has the direction of the magnetizing-field flux; hence the total flux per square centimeter within the iron is the sum of the two superposed fluxes. This resultant *flux density* is represented by the letter B and is

also called *magnetic induction.* When A is the cross-sectional area of the iron, $\phi = BA$ (see also Art. 7).

The practical limit of attainable field density, using ordinary iron, is about 12,000 gauss; but where efficiency is not a consideration and the pole faces have the form of truncated cones, densities of 40,000 gauss can be produced.

7. Magnetic Permeability.—When a magnetizing solenoid contains an iron core, the atomic whirls of the iron are turned to face in the same direction as the loops of the solenoid Each of these whirls then increases the intensity of the magnetic field within the solenoid. And all the whirls together produce a magnetic field which may be ascribed to the conventional magnetization whirl (Art. VI–7). The magnetic field due to the magnetization of the iron is superposed on that of the magnetizing solenoid and differs practically only in that its intensity is usually 500 to 2500 times greater. The intensity of the resultant field H_r then is equal to the sum of the intensities of the superposed fields; *i.e.*,

$$H_r = H_s + H_m,$$

where H_s is the field due to the solenoid, and H_m that due to the iron magnet. When the flux density is made (by convention) numerically equal to the field intensity in each of the fields,

$$B_r = B_s + B_m = \mu H_s.$$

The total flux density (magnetic induction) in the space or in the material within the solenoid, with subscripts dropped, then is

$$B = \mu H \text{ gauss.} \tag{5}$$

The factor μ is called *magnetic permeability* of the space or of any material through which the magnetizing field makes a complete circuit. Because the magnitude of permeability depends on a property of the medium (space or material), it is now treated by many as a dimensional quantity rather than as a simple numerical constant. When so treated, the dimensional equations, Appendix VI, give an added check against errors in mathematical calculations. In Eq. (5) the B and H, then, have different dimensions and therefore are treated as different kinds of quantities. It is for that reason that even when $\mu = 1$, the

expression for field density is written $B = \mu_0 H$. In the equation, H always represents the intensity of the magnetizing field and not the total intensity of the field within the solenoid; B represents the total flux density within the solenoid and itself is numerically equal to the total field intensity H_r.

What the degree of magnetization in any magnetic field is depends not alone on the properties of the material being magnetized, because each magnetized section of the material aids in the magnetizing of the neighboring sections and superposes external parts of its own field on the internal parts of the fields

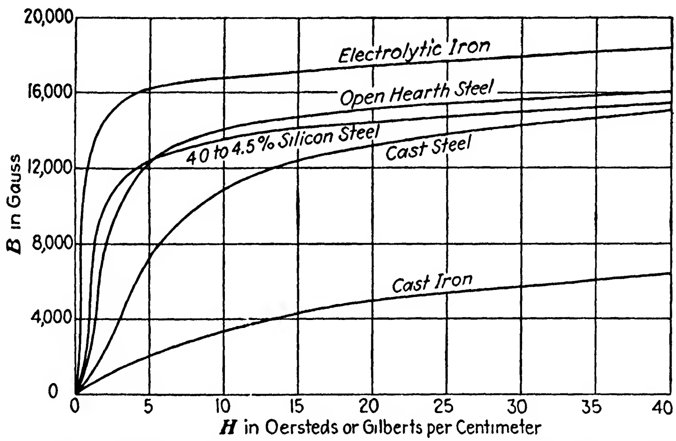


FIG. 6.—Magnetization curves for materials used for electromagnets, armatures, and transformer cores.

in those sections. The degree of magnetization in any field, therefore, depends on the length of the material being magnetized and on whether or not the material forms a partial or complete loop linking the magnetizing solenoid. The term *permeability of a material*, to have any useful meaning, must therefore be taken to be the ratio of B to H ($\mu = B/H$) as it is in a closed-circuit magnet.

The variation of B with change of H is shown for some substances in Fig. 6. It should be noted that in iron and steel the value of B is very small in weak magnetizing fields, increases most rapidly in fields of from 2 to 5 oersteds, and reaches almost its maximum value in fields of 8 oersteds. Beyond that the

increase in B is contributed mainly by the magnetizing field itself.

The variation of permeability with the intensity of the magnetizing field is shown in Fig. 7.

The permeability of each sample of iron or steel must be known before exact calculations for any particular magnetic circuit can be made.

An alloy of iron and nickel called *permalloy* (Ni 78.5 + Fe 21.5) has a permeability many times greater than iron in weak mag-

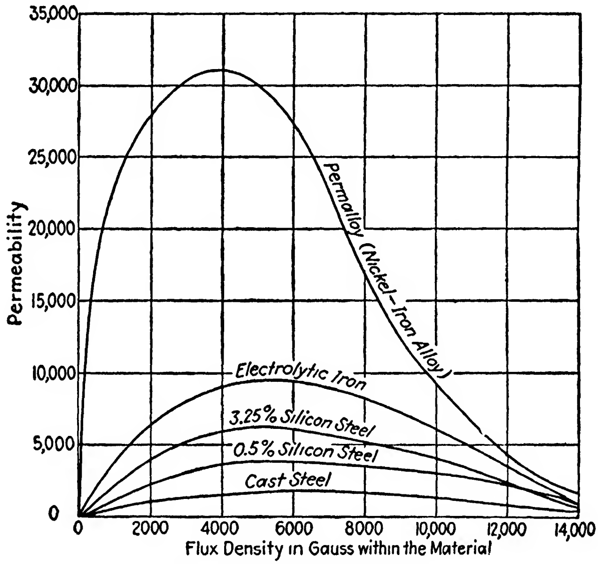


FIG. 7.—Permeability curves.

netizing fields, as is seen by comparison of the magnetization curves in Fig. 8. This alloy is of great service in long-distance telephony (Art. XXIX–4). Another useful alloy *perminvar,* composed of nickel, cobalt, and iron, has a constant permeability over a wide range of magnetizing force.

The strongest permanent magnets are made of cobalt steel and are three or four times stronger than ordinary magnets which are made of tungsten steel (Art. 11).

8. Reluctance.—The magnetizing solenoid S, Fig. 9A, is magnetizing a rectangular piece of iron which is at all points in the linking magnetizing field. When the magnetic iron atoms

within the strongest part of this magnetic field are turned, they affect the neighboring atoms; these neighboring atoms affect those ahead of them, etc., until most of the atoms even in the weakest part of the magnetizing field face in the same direction around the circuit. The broken-arrowed line L within the iron represents the direction of the resulting magnetic flux. The

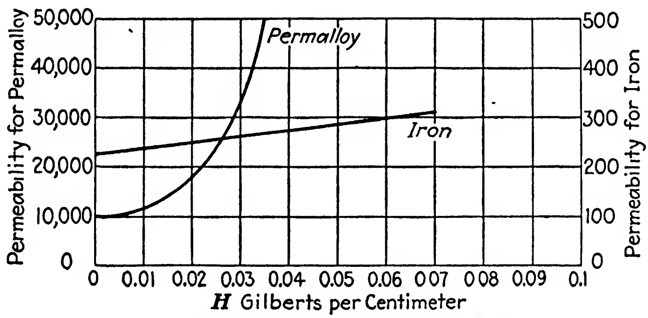


FIG. 8.—Permeability of permalloy in low magnetizing fields.

rectangle approximates a toroid and practically all its magnetic lines of force are within the iron circuit (Art. VI–4), the "leakage" of the lines being comparatively small. The reluctance of such a magnetic circuit depends on the materials in it and on the dimensions. From Eq. (4)

$$\mathcal{R} = \frac{\mathfrak{F}}{\phi} = \frac{Hl}{BA} = \frac{Hl}{\mu H \cdot A} = \frac{l}{\mu A} \text{ units.}$$

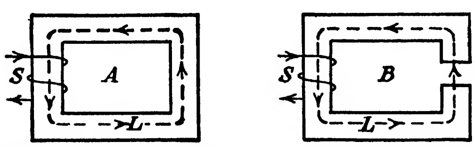


FIG. 9.—(a) Complete iron circuit linking a magnetizing solenoid; (b) an air gap in a part of a magnetic circuit.

If the iron circuit is not continuous, Fig. 9B, the magnetic flux must pass through an air gap, in which the permeability is unity. The reluctance of the air gap, then, is

$$\mathcal{R}_1 = \frac{l_1}{A} \text{ units,}$$

where l_1 is the length and A the cross section of the air gap. This equation shows that a 1-cm cube of air or of any non-magnetic material has a reluctance of 1 unit.

The total reluctance of a magnetic circuit is the sum of all the reluctances which are in series. In the case of a circuit with a short air gap the reluctance is

$$\mathcal{R} = \frac{l}{\mu A} + \frac{l_1}{A} \text{ units,} \tag{6}$$

of which the greater part is usually in the air gap. In order to produce strong magnetic fields, therefore, the air gap is made as short as is possible. The expression for the reluctance of the air gap is only approximately correct, especially in long gaps, because the lines of force in the gap curve outward and therefore penetrate a greater area than the cross section of the iron.

If the iron is a straight bar, the air-gap area is difficult to estimate because the lines of force spread out in all directions; but if the length of the bar exceeds forty times the diameter, the great spreading of the lines is equivalent to so large a cross-sectional area that the reluctance of the air space becomes negligible. The reluctance of such a straight rod is practically the same as though it were in the form of a closed loop.

The reluctance of a centimeter cube of iron or steel used in electric instruments ranges from 0.00025 to 0.016 units.

9. Intensity of Magnetization—Magnetic Susceptibility.—The term *intensity of magnetization* is used to designate the degree of magnetization of any piece of iron apart from the field that is magnetizing the iron. It is defined as the magnetic moment per unit volume and can be shown to be equal to the pole strength per square centimeter of cross-sectional area. It is represented by the letter J.

Magnetic susceptibility is the ratio of the intensity of magnetization to the intensity of the magnetizing field; *i.e.*,

$$\text{Magnetic susceptibility } \kappa = \frac{J}{H}.$$

10. Hysteresis.—If the flux density B of a piece of iron in an increasing magnetizing field is represented by the curve a of Fig. 10, that in a decreasing magnetizing field is represented by b.

The curve *b* does not follow the curve *a* on account of residual magnetism. It is seen that some magnetism remains in the iron after the magnetizing field becomes zero.

When the magnetizing field alternates, the curve for the magnetic induction incloses an area shown by the full-line curve *bcde* (called *hysteresis loop*). The area bounded by this curve can be shown to represent the amount of work per cycle done against the residual magnetism. In alternating currents this

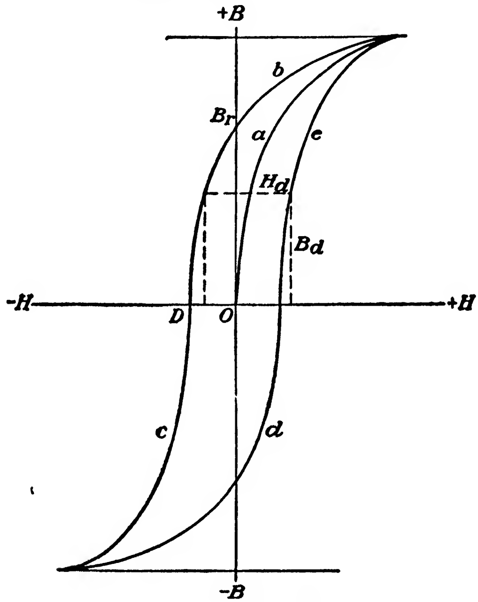


FIG. 10.—Hysteresis loop.

energy loss (hysteresis loss) is considerable and is transformed into heat in the iron.

The term *coercive force* (coercivity) is applied to the demagnetizing field *OD* required to remove the residual flux density B_r. This residual flux density is called *retentivity*. The area B_dH_d, called *energy product*, is a measure of the quality of the steel with reference to its retention of magnetism. B_d is any value less than B_r which gives the energy product its maximum value.

11. Properties of Permanent-magnetic Steels.—The following table gives the composition and properties of steels used in the construction of permanent magnets.

PERMANENT-MAGNET STEELS

	Materials other than iron, percentage of mass						B_r*	H_c*	(B_dH_d)* max
	Co	W	Cr	C	Mn	Si			
Carbon steel	. .	. .	. .	0.9	0 5	0 2	8,800	48	180,000
Chromium steel. . . .	. .	. .	2	0 95	0 5	. . .	9,500	55	230,000
Tungsten steel	. .	5	. .	0.6	0 5	0.2	10,000	65	260,000
Cobalt-chromium steel.	15	. .	10	0 9	0.2	0 2	8,300	200	650,000
Cobalt steel standard	35	4	2	0 9	0 5	0 2	10,000	250	1,100,000

* B_r = residual flux density; H_c = retentivity; (B_dH_d) = energy product (Art. 10).

12. Production of Strong Magnetic Fields.—The strongest electromagnets have cores of soft iron or soft steel and as strong a magnetizing field as is possible. Toroidal electromagnets, such as shown in Fig. 11(*a*), have less flux leakage than other

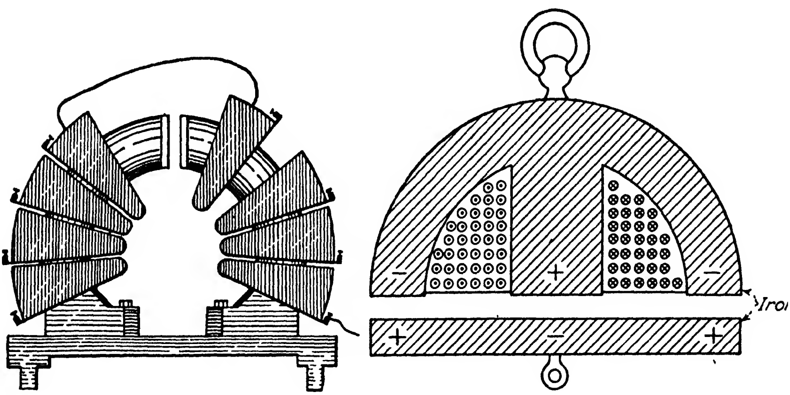


FIG. 11.—(*a*) A good form of an electromagnet; (*b*) a diagram of a lifting electromagnet.

forms. Lifting magnets usually have the form represented in Fig. 11(*b*). The magnetizing solenoids are often made of water-cooled copper tubing, as in the electromagnet of Fig. 12, to withstand greater magnetizing currents. The electromagnets of transformers, generators, and motors are shown in Chaps. XXII and XXIII.

The electromagnets used with alternating currents are made of thin sheets, called *laminations.* These laminations are made of high-silicon steel in thicknesses varying from 0.01 to 0.03 cm and are coated with enamel or varnish. Silicon steel has a high resistance and thereby contributes to the limiting of the eddy

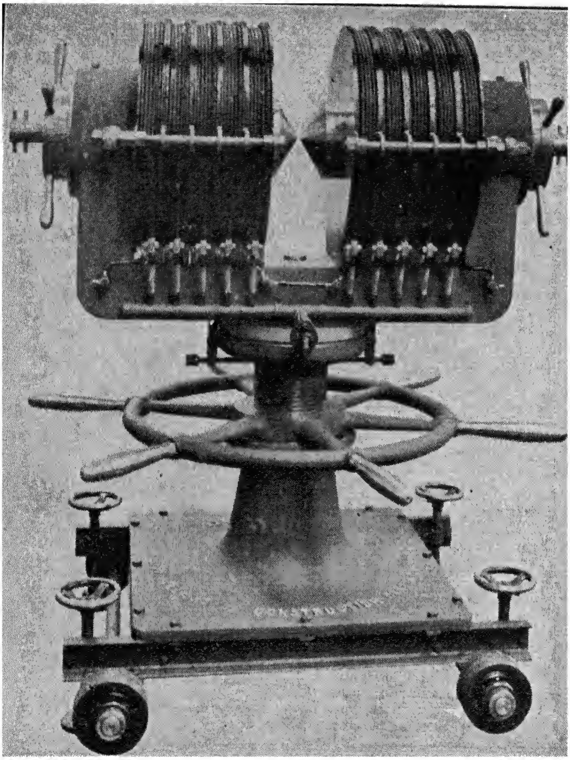


FIG 12.—Large electromagnet with magnetizing solenoids made of copper tubing. (*Williams, Magnetic Phenomena.*)

currents within individual sheets, which currents always exist in addition to the comparatively small currents which may flow through the well-insulated laminations.

Electromagnets are also made of insulated granules (Art. XIX–4) of some magnetic material pressed together to form one coherent mass. Permalloy magnets are usually made in that manner. They are expensive and therefore generally used

only in telephone and radio transformers and in loading coils (Art. XXIX–4).

When only one pole of a magnet can be utilized, the electromagnet takes the form, Fig. 13, of that used by oculists for extracting iron or steel particles imbedded in the eyeball.

The permanent magnets of galvanometers, ammeters, voltmeters, and similar instruments are U shaped with soft-iron pole pieces as shown in Fig. XXI–5.

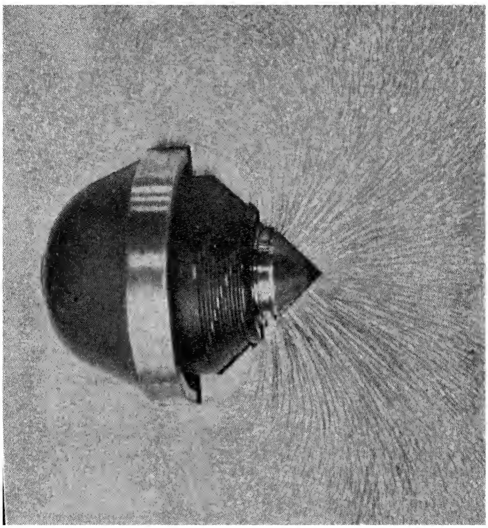


FIG. 13.—Electromagnet used by oculists. (*Williams, Magnetic Phenomena.*)

A soft-iron armature placed so as to complete the magnetic circuit of a permanent magnet aids in preventing demagnetization with time.

Questions

1. What is the conventional unit point-pole? Show that 4π lines emanate from it.

2. Show that it takes $4\pi I'$ ergs of work to move a unit point-pole around a wire of infinite length. How much work does it take to move the point-pole around a path linking a loop of wire? A coil of N loops? Completely around a loop (or coil) without linking it?

3. Develop the expression for the intensity of the magnetic field within a toroid. Within a long solenoid. Develop the expression for the average magnetizing field in a long piece of iron magnetized by a short coil.

4. Define the unit of magnetic potential. The gilbert.

5. Derive the expression for the total potential difference in any complete circuit linking the magnetizing coil.

6. Define magnetomotive force.

7. What interpretation is given to m.m.f. when the material being magnetized does not occupy the whole length of a loop linking the magnetizing solenoid?

8. What is meant by gilbert per centimeter? What relation has it to the intensity of the magnetic field?

9. If the m.m.f. and the complete loop length of the magnetized iron are given, what is the average intensity of the magnetizing field within the iron?

10. What is a magnetic circuit? Compare it with an electric circuit.

11. Define ampere-turn.

12. Give the expression for the magnetic flux in terms of the m.m.f. and reluctance.

13. Define magnetic induction and flux density.

14. The total field intensity within the iron core of a solenoid is the resultant of what two field intensities? The total flux density of what two flux densities?

15. Define magnetic permeability. The meaning of B and H in $B = \mu H$.

16. Give the interpretation of Eq. (5) when μ is assumed to be a dimensional quantity and when taken to be a simple nondimensional constant.

17. Explain from the magnetization curves of Fig. 6 why permeability changes with the magnetizing force.

18. Derive the expression for reluctance in terms of l, A, and μ.

19. How does an air gap in an iron circuit affect the reluctance?

20. Define intensity of magnetization and magnetic susceptibility.

21. What is magnetic hysteresis?

22. What core materials have the smallest hysteresis loss?

23. Define coercive force and retentivity.

24. Why is silicon steel used in transformer cores?

25. Distinguish between resistance, impedance, and reluctance.

Problems

1. What is the magnetic potential difference in a complete closed path linking a coil of 100 turns carrying 5 amp?

2. A circular bar of iron 8 cm^2 in cross-sectional area and 60 cm in length is inserted into a solenoid which is 10 cm in length and has 50 turns. This bar is then bent into the form of a rectangle making a complete magnetic circuit. The permeability of the iron is 2,500 when the magnetizing current is 0.5 amp. (*a*) What is the number of ampere-turns? (*b*) What is the average intensity of the magnetizing field impressed on the magnetic circuit? (*c*) The total p.d. in this field? (*d*) The m.m.f.? (*e*) What is the

reluctance? (*f*) What is the total flux? (*g*) The flux density or magnetic induction B within the iron?

3. The iron rectangle of Prob. 2 is opened so that an air gap 2 cm in length is formed. (*a*) What is the reluctance of the air gap? (*b*) The total reluctance of the circuit? (*c*) Assuming the current and the permeability to remain unchanged, what is the total flux? (*d*) The flux density B? (*e*) By what factor has the total flux been diminished by the introduction of the air gap? (*f*) If the air gap is made 4 cm in length and the permeability remains unchanged, what are the reluctance and the total flux?

Experiments

1. Model of unit point-pole rotating with its field about a current.

2. Magnetic circuits:

a. Loop of wire.

b. Solenoid and toroid with and without iron.

c. Electromagnets with and without air gap.

d. Permanent magnets.

3. The relative magnitudes of the flux in a magnetic circuit with and without an air gap, measured by means of a galvanometer in series with a few turns of wire linking the circuit.

4. Magnetism of permalloy rod in earth's magnetic field.

PART II

This part includes the more specialized subjects, any of which may be omitted in an abridged course.

CHAPTER XXI

MEASURING INSTRUMENTS

1. Absolute Measurement of Current.—The *tangent galvanometer* (Art. VIII–5) is the simpler of the two absolute methods for measuring an electric current. The *electrodynamometer*, which is capable of giving a somewhat greater accuracy, however, has supplanted the tangent galvanometer for measurements of precision. It consists of a coil of small diameter, Fig. 1, suspended by two conducting leads so that it hangs at the center of a large coil. The same current passes through the two coils in series and deflects the smaller one. The magnitude of the current, in absolute measure, is calculated from the observed deflection, the determined torque of the fiber, and the dimensions of the coils.

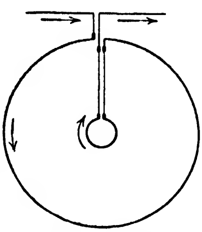


FIG. 1.—Diagram of absolute electrodynamometer.

2. Moving-magnet Galvanometer.—The expression for the current in the tangent galvanometer (Art. VIII–5) is

$$I = \frac{5Hr}{\pi N} \tan \theta.$$

The equation shows that in order to construct a more sensitive instrument on the same principle, the terms H and r must be made as small as possible; and N must be as large as is practicable. The torque of the fiber by which the needle is suspended is ordinarily negligible but becomes the main control when H is greatly reduced. It too must be diminished as much as possible.

When the radius r is made small, the current no longer is proportional to the tangent of the angle of deflection; hence such

an instrument must be calibrated or used simply as a current detector. The magnitude of H is diminished by means of a control magnet which is turned until its field almost neutralizes that of the earth. In some instruments the same effect is accomplished by employing a needle made of two magnets connected to the same vertical rod one above the other and with their like poles facing in opposite directions. The earth's field then exerts but a slight torque.

The damping of the needle is sometimes effected by attaching to the needle a strip of paper which offers a large air resistance

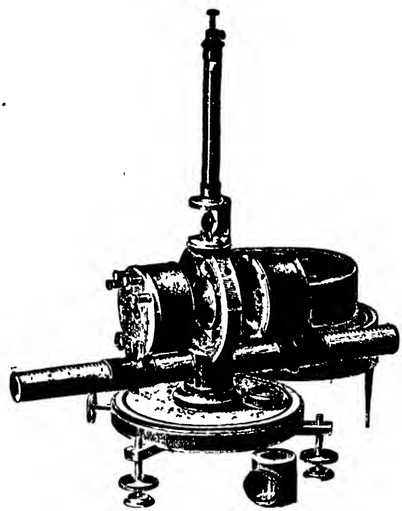


FIG. 2.—Moving-magnet sensitive galvanometer (an old type), needle, not shown, is within the copper block.

to the motion; at other times damping is accomplished by having the motion of the needle generate eddy currents in a block of copper (Art. XIX–6) which surrounds the needle, as in the instrument shown in Fig. 2.

The suspension is either an unspun silk or a quartz fiber. The sensitivity can be made such that a deflection of 1 mm is obtained by a current of 10^{-12} amp on a scale placed at a distance of 1 meter. A beam of light reflected from a mirror attached to the coil then takes the place of a material pointer.

Unless this instrument is inclosed within at least six concentric cylinders of iron to shield it from external magnetic disturbances,

the variations in H due to stray currents in the earth make its use impractical in large cities.

3. Moving-coil Galvanometer.—The *moving-coil galvanometer* consists of an open coil of wire, Fig. 3(a), suspended in a uniform radial magnetic field by means of a strip, S, of phosphor-bronze, silver, or gold, which also serves as one of the current leads. The other end of the coil is connected to a spiral, T, which serves as the other lead. The radial field is produced by means of a soft-iron cylinder placed between properly shaped poles of a horseshoe magnet, as shown in top view in Fig. 3(b). A line through the axis of the cylinder coincides with the axis of the coil; therefore the latter turns about the cylinder on an axis

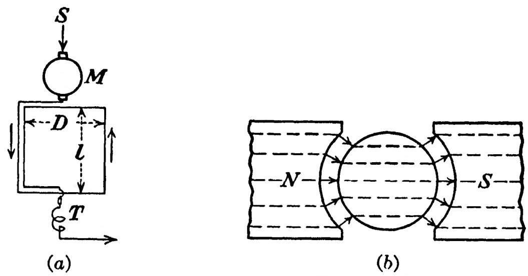


FIG. 3.—(a) Open coil of a moving-coil galvanometer, (b) radial field produced in the air gaps between the poles of the magnet and the soft iron cylinder in a moving-coil galvanometer.

which passes through the center of the radial field. Whatever may be the deflected position of the coil, its plane is in the plane of the magnetic lines of force, and the force of reaction between the current and the field is always at right angles to the plane of the coil.

Since the current has opposite directions in the two sides of the coil and the magnetic field has the same direction, the forces evoked on the two sides turn the coil in the same direction. Calling N the average of the number of wires on the two sides of the coil and D the distance between them, the torque due to the current (Art. VIII–6) is

$$L = BlNDI'.$$

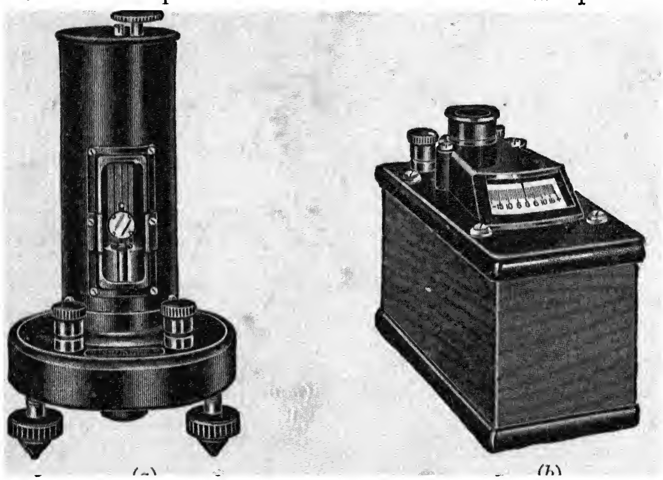
When a current flows, the coil turns until the upper twisted fiber and the lower spiral together exert an opposing torque which is equal to the torque produced by the current. When T represents the torque per radian due to the fiber and the spiral,

$$L = BlNDI' = T\theta.$$

$$I = 10I' = \frac{10T}{BlND}\theta = K\theta = K'd. \tag{1}$$

Although the deflection d on a circular scale is proportional to the angle of deflection and to the current, exact proportionality is not fully realized in practice.

The readings are usually taken either by means of the telescope-mirror-scale method, where the mirror M, shown in Fig. 3(a), reflects different portions of a scale into a telescope which is



(a) (b)

FIG. 4.—Modern types of the moving-coil galvanometer. (a) High-sensitivity galvanometer, (b) portable galvanometer. (*Central Scientific Company.*)

provided with an index line, or by means of a beam of light reflected from the mirror and focused on a scale.

The portable moving-coil galvanometer has its coil on metallic pivots in agate bearings. The current is led in and out by means of spiral springs which also furnish the opposing torque. An aluminum pointer attached to the coil indicates the deflections on the scale. A more sensitive portable galvanometer, Fig. 4(b), has a light coil, without pivots, held in place by short upper and lower phosphor-bronze or gold strips under tension. A moving-coil galvanometer of high sensitivity is shown in Fig. 4(a).

4. Direct-current Ammeter.—The portable galvanometer when provided with a scale such that the deflections indicate

the current in amperes is called a *direct-current ammeter.* Since large currents cannot be sent through the fine wire of the coil, the instrument is provided with a shunt which takes most of the current. The current flowing through the coil, however, is proportional to the total flow; therefore the scale can be graduated to indicate the total current flowing through the circuit into which the ammeter is inserted.

Let S and G, Fig. 5(a), represent the resistances of the shunt and of the galvanometer-coil branch, respectively. The current divides in passing through the ammeter. The part i flows

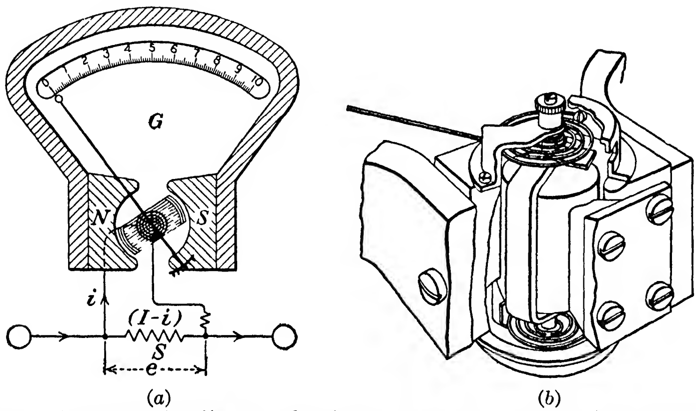


FIG. 5.—Ammeter: (a) diagram showing shunt S; (b) control spiral springs and coil.

through the coil branch, and the part $(I - i)$ through the shunt. The potential difference e is the same for both the coil branch and the shunt. Then from Ohm's law (Art. IX–7),

$$e = iG = (I - i)S,$$

from which

$$\frac{i}{I - i} = \frac{S}{G}, \quad \frac{i}{I - i + i} = \frac{S}{G + S},$$

$$i = \frac{S}{G + S}I. \tag{2}$$

The shunt is either internal or external. The internal shunt is a permanent part of the instrument and is usually within its case. The same galvanometer may have a complete set of external shunts, each of which permits a full-scale deflection for some particular current magnitude in the circuit.

For the purpose of reducing the temperature coefficient of the instrument, the galvanometer-coil branch G contains a manganin (Art. X–7) resistance four or five times larger than that of the copper coil.

The portable galvanometer is usually provided with such a resistance in its circuit that 0.050 volts (50 mv) between its binding posts produce a current which gives a full-scale deflection. Shunts are then made that produce a potential difference of 50 mv between their binding posts when a current of the rated value is flowing through the circuit into which they are inserted. Such shunts are interchangeable; *i.e.*, they can be used with any instrument constructed on this principle.

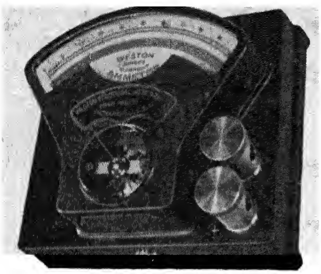


FIG. 6.—Direct-current ammeter.

Since the resistance of an ammeter, on account of the low resistance of its shunt, is small, its introduction into a circuit does not alter the current appreciably. One commercial form of the direct-current ammeter is shown in Fig. 6.

5. Alternating-current Ammeter.—An alternating current produces no deflection in an ammeter such as described above. Alternating-current ammeters are of several types:

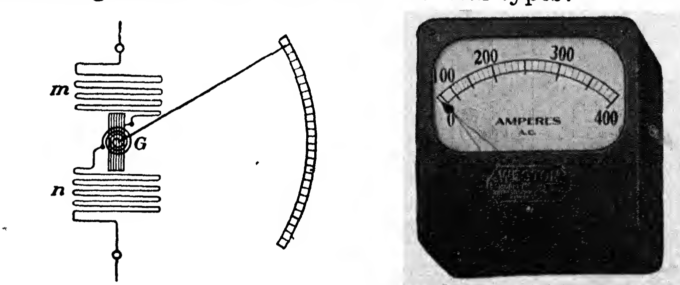


FIG. 7.—Ammeter of the dynamometer type. (*a*) diagram; (*b*) a commercial form.

1. *The electrodynamometer type*, Figs. 7(*a*, *b*), is one in which the current flows through two coils in series, one of which is stationary and divided into two parts, *mn*, and the other coil, *G*, is free to turn against the torque of its control springs. The

current reverses in both coils at the same time; the forces acting (B.P.2) therefore exert a torque always in the same direction. The coil, on account of its inertia, cannot follow the varying torque acting during a cycle; hence the deflection is constant for any particular current and depends on the average torque. The instrument is graduated to indicate effective amperes (Art. XVIII–2) and therefore may be used to measure direct as well as alternating currents.

2. *The thermocouple type* consists of a thermocouple (Art. XV–8) in a circuit with a moving-coil galvanometer. One of the junctions of the couple is attached to or is formed in part

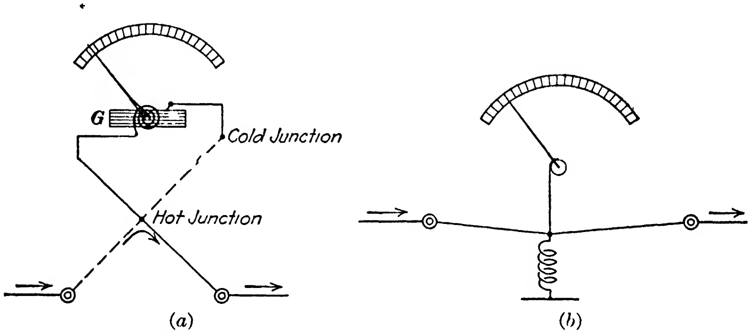


FIG. 8.—(*a*) Thermocouple-type ammeter, (*b*) hot-wire type ammeter.

by a fine wire of high resistance through which the alternating current flows as shown in Fig. 8(*a*). The wire, heated by the current, warms the junction and thereby generates a current which deflects the coil of the galvanometer. The scale is appropriately graduated to give the effective current in amperes.

3. *The hot-wire type,* Fig. 8(*b*), contains a wire which when heated by the current expands and enables a stretched spring attached to its center to contract and operate a mechanism which moves the index pointer.

4. One form of the *soft-iron type* consists of a solenoid in which is a semicircular piece of soft iron attached to a spindle carrying a pointer. This spindle, controlled by spiral springs, turns on its axis so that the iron piece moves by another similar piece which is fixed in position. The alternating current magnetizes the two pieces so that, even though their magnetism follows the cycles of the alternating current, like poles are always facing each other.

The amount of rotation produced by the repulsion then depends on the strength of the current.

6. Voltameter.—The *voltameter,* Fig. 9, method of measuring electric current (Art. XI–4) is of interest mainly because the international ampere is defined in terms of the amount of silver deposited by a coulomb of electricity from a silver nitrate solution. Precision measurements with an electrodynamometer show that 0.0011180 grams of silver are deposited by 1 coulomb of electricity.

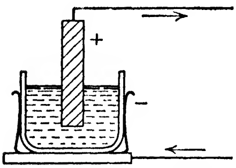


FIG. 9.—Silver voltameter.

Then

$$I = \frac{m}{0.001118\,t}.$$

7. Kelvin Balance.—The *Kelvin balance,* Fig. 10, consists of two movable coils, one on each end of a balance beam, placed directly above two stationary coils. The current flows through

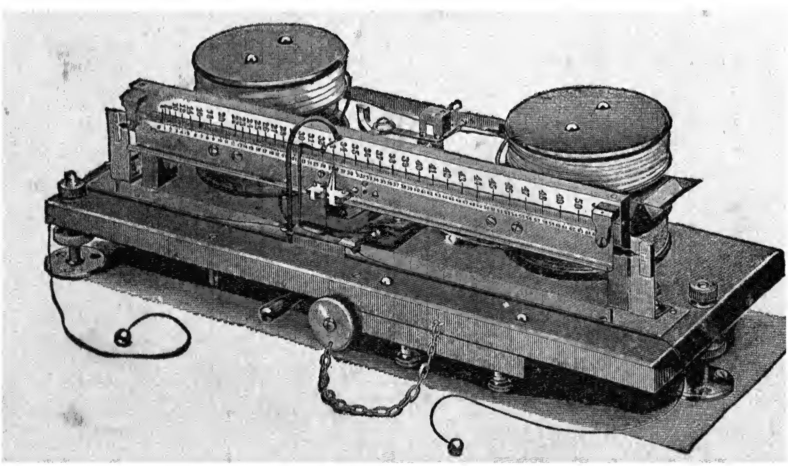


FIG. 10.—Kelvin balance.

the four coils in such directions that when one pair is attracting, the other is repelling. Balance of the beam is obtained by sliding a weight, which in the position of equilibrium gives the magnitude of the current. This type of current-measuring instrument is not in general use.

8. String Galvanometer—Oscillograph.—A fine silver or platinum wire or strip hung loosely in a magnetic field, Fig. 11(*a*), deflects when a current is passed through it. The deflections are observed by a reading telescope and are a measure of the magnitude of the current. The galvanometer employing this principle is called a *string galvanometer* and is of special service in the study of alternating or rapidly varying currents. In the study of such currents the image of a section of the string is projected on a screen or on a moving photographic film, after being reflected from a rotating mirror. The persistence of vision

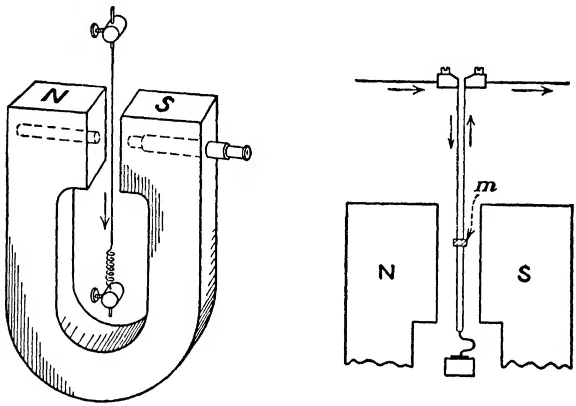


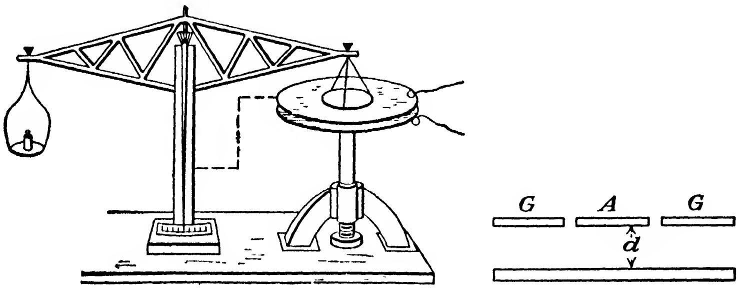
FIG. 11 —(*a*) String galvanometer; (*b*) two-string oscillograph.

enables the changes in the deflection that take place in 0.7 sec to be seen simultaneously.

The term *oscillograph* is usually applied to this galvanometer together with its accessory attachments which are used for projecting or photographing the varying deflections. The oscillograph is often made with two strings, Fig. 11(*b*), which, deflecting in opposite directions, change the angular position of an attached mirror *m*.

Another form of the oscillograph employs a thin beam of cathode rays which impinges on a fluorescent screen. This beam usually passes in succession between two sets of parallel plates, which are placed at right angles to each other and charged with the alternating p.d. under observation. One set of the energized plates oscillates the beam in one plane and the second set of

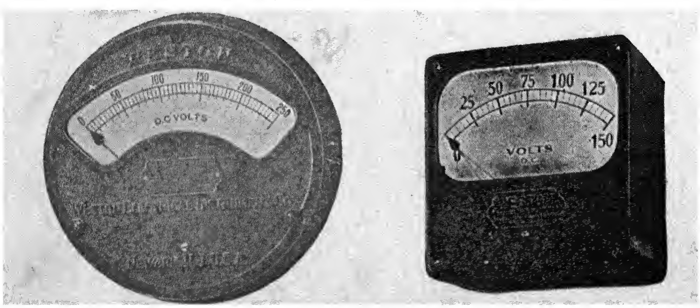
plates gives this oscillating beam a similar oscillation at right angles to the first. The resulting motion of the beam causes a compound harmonic curve to be traced on the screen, from which curve the form of the alternating p.d. may be derived.



(*a*) (*b*)

FIG. 12.—(*a*) Disk electrometer; (*b*) diagram of the attracted disk *A* and of its guard ring *G*.

9. Absolute Measurement of Potential Difference.—The calorimeter method (Art. IX–6) is an absolute method for measuring potential differences. Another method is that employing a disk electrometer, Fig. 12, in which a disk of area A, attached



(*a*) (*b*)

FIG. 13.—(*a*) Switch-board type d.c. voltmeter; (*b*) portable-type d.c. voltmeter.

to one side of a balance beam, is attracted by a stationary disk at a distance d below it. The upper disk has the smaller diameter and is surrounded by a guard ring [Art. XII–7, Fig. 12(*b*)] in order that the electric field between the plates may consist of parallel lines throughout. This condition enables a calculation of the potential difference in terms of the weight, mg, required to restore balance. It can be shown, Appendix V(4), that

$$V'' = d\sqrt{\frac{8\pi mg}{A}}.$$

10. Direct-current Voltmeter.—The *direct-current voltmeter,* Fig. 13, consists of a portable galvanometer, such as is used in an ammeter, together with a high resistance which is connected in series with the deflecting coil. The resistance is made such that the desired maximum potential difference to be measured produces the current that gives a full-scale deflection. Other deflections then represent corresponding potential differences existing between the binding posts of the instrument.

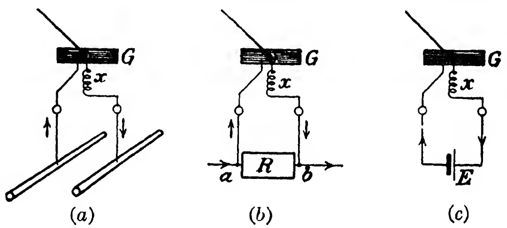


Fig. 14.—Voltmeter connected: (*a*) to measure the potential difference of mains; (*b*) to measure the potential difference between points *a* and *b*, (*c*) to measure electromotive force of a cell.

If a galvanometer having a resistance of 12 ohms gives a full-scale deflection with 0.008 amp, the resistance X that must be added to make of it a voltmeter whose full-scale reading is 1.5 volts is determined as follows:
From Ohm's law,

$$E = IR = 0.008(X + 12) = 1.5,$$
$$X = 175.5 \text{ ohms}.$$

In this particular instrument 1.5 volts between the binding posts always produce a current which deflects the pointer full scale. When the voltmeter is connected to two points on a circuit, Fig. 14(*b*), between which the resistance is comparatively low, the small current taken by the voltmeter does not appreciably change the current in the main circuit between these points. The attaching of the voltmeter to these points, then, does not change the potential difference; hence the voltmeter in this case shows the potential difference which existed between the points before the voltmeter was attached. With larger resistance between the points *a* and *b* an appreciable part of the

current passes through the voltmeter; this decreases the current in R and therefore the potential difference between the points. The voltmeter then indicates this altered and not the original potential difference. The voltmeter, however, always gives the potential difference as it is between the points when the voltmeter is connected, the potential difference in the connecting leads being negligible.

If a voltmeter is connected to a voltaic cell, Fig. 14(c), it indicates only the potential difference between the binding posts of the instrument and not the total e.m.f. of the cell. If the resistance of the battery, for example, is the same as that of the voltmeter, only one-half of the total RI drop (Art. X–4) is between the binding posts of the voltmeter. However, since the resistance of cells is usually small, the potential difference between the binding posts is practically equal to the e.m.f. of the cell.

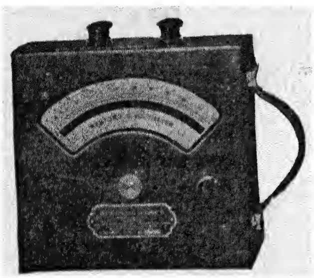


FIG. 15.—Alternating-current voltmeter.

11. Alternating-current Voltmeter.—An *alternating-current voltmeter*, Fig. 15, is an alternating-current ammeter of high resistance whose scale is graduated to indicate the potential difference between its binding posts.

Alternating-current potential differences are also measured by means of *electrostatic voltmeters*, described in Art. 12.

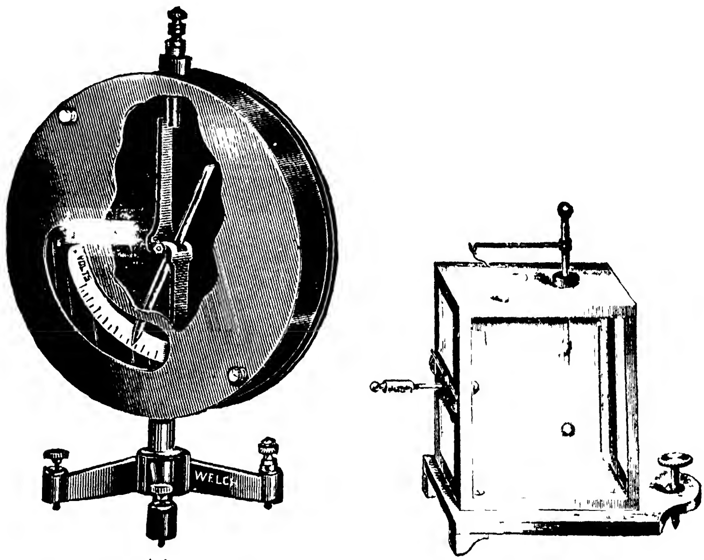
12. Electrostatic Voltmeter.—Two plates or sets of plates, one stationary and the other free to rotate and insulated from the first, when connected to a battery or to two points on a circuit (Art. XII–9) become charged oppositely. The plates in attracting each other turn the index pointer attached to the movable plate or set of plates. The moving set either twists a suspending fiber or raises its own center of gravity an amount that depends on the potential difference.

In a modified form, Fig. 16(a), the instrument consists of a vertical strip with a horizontal saddle at its center on which an uninsulated weighted needle hangs vertically. The needle and the strip acquire the same potential, and the repulsion of the two

produces a deflection whose magnitude depends on that of the potential.

An electroscope may be provided with a scale which is graduated to indicate potentials, Fig. 16(*b*), or the divergence of the leaf may be read more accurately on the scale of an observing telescope.

Very sensitive instruments of this type are called *electrometers*, Fig. 17. These consist of a very light needle suspended between four hollow quadrants by a metal-coated quartz fiber. The



(*a*) (*b*)

FIG. 16.—(*a*) Electrostatic voltmeter; (*b*) electroscope with graduated scale.

alternate quadrants are metallically connected and the needle charged to some constant high potential. The needle then deflects an amount dependent on the p.d. that may be impressed on the two pairs of quadrants.

13. Wattmeter—Watt-hour Meter (**Service Meter**).—An electrodynamometer, Fig. 7(*a*), may be graduated to indicate the power in watts being consumed between any two points in a circuit. The low-resistance field coil is connected into the circuit so that the whole current flows through it. The movable coil in series with a high resistance is connected to the two points

between which the power consumed is to be measured. The current flowing through this coil is proportional to the potential difference. The deflecting torque, then, varies with the product of the current and the potential difference; hence the scale may be marked to indicate this product, *i.e.*, the watts being consumed in the circuit between the two points. A commercial type of the instrument is shown in Fig. 18(*a*).

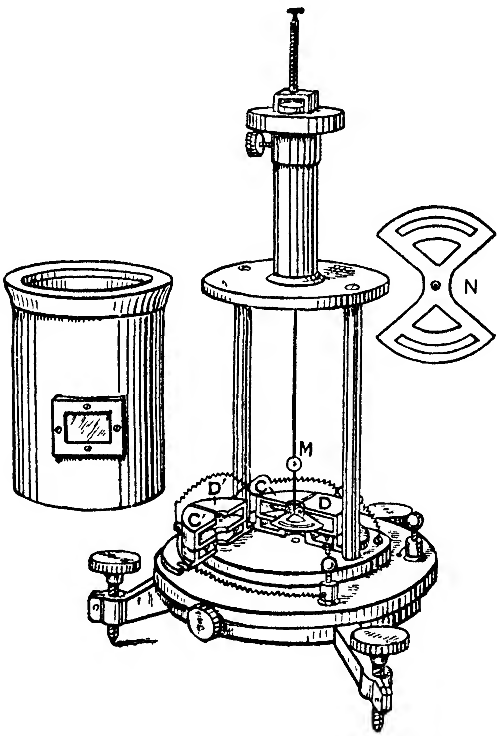
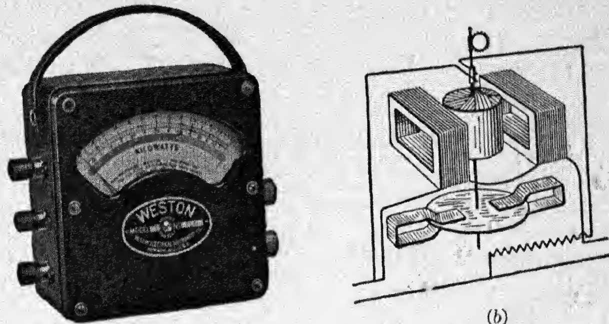


FIG. 17.—Quadrant electrometer showing the needle N, the mirror M and the two pairs of quadrants CC' and DD'. Two of the quadrants are out of position to show the needle.

If the movable coil of the wattmeter is replaced by a small armature with an appropriate commutator, it revolves like the armature of a motor, always in the same direction. If this armature is on a shaft with a copper disk which rotates between the poles of a permanent magnet, the eddy currents induced in the disk (Art. XIX–3) produce an opposing torque. The relationship

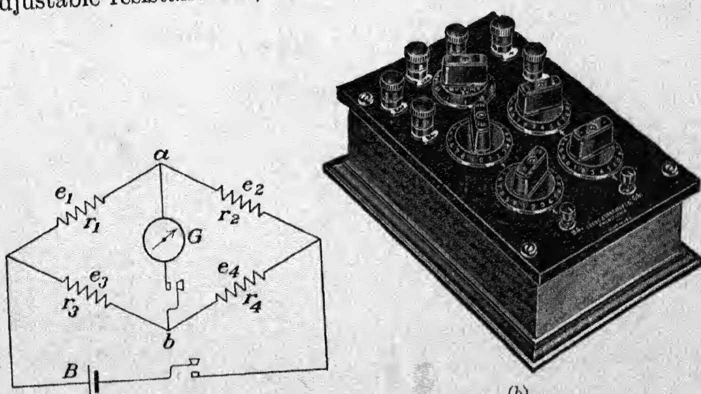
of the torques is such that the speed of rotation produced is proportional to the power being consumed. The rotating shaft operates indexes on dials that register the watt-hours or kilowatt-



(a) (b)
FIG. 18.—(a) Wattmeter; (b) service meter.

hours of energy consumed. The instrument, Fig. 18(b), is called a *watt-hour meter* or a *service meter*.

14. Wheatstone Bridge.—An unknown resistance r_1, three adjustable resistances r_2, r_3, and r_4, a sensitive galvanometer G,



(a) (b)
FIG. 19.—(a) Diagram of Wheatstone bridge; (b) Wheatstone box bridge.

and a battery B are connected as shown in Fig. 19(a) to form a *Wheatstone bridge*.

The known resistances are adjusted until no current flows through the galvanometer, *i.e.*, until there is no potential differ-

ence between the points a and b, to which the galvanometer is connected. The potential difference e_1 then equals e_3, and e_2 equals e_4. The current in the branch r_1 is the same as that in r_2; that in r_3 as that in r_4.

From Art. X–4,

$$\frac{e_1}{e_2} = \frac{r_1}{r_2}, \qquad \frac{e_3}{e_4} = \frac{r_3}{r_4}.$$

But since $e_1 = e_3$, and $e_2 = e_4$, e_1/e_2 and e_3/e_4 are identical, so that

$$\frac{r_1}{r_2} = \frac{r_3}{r_4}.$$

If the resistances in three of the arms are known, the unknown resistance

$$r_1 = \frac{r_3}{r_4} r_2.$$

The resistances r_3 and r_4 may consist of a uniform straight wire (slide wire bridge) upon which slides a contact connected to the galvanometer branch. This sliding contact may be moved along the wire, increasing the resistance on one side of the contact and decreasing it on the other. Then

$$\frac{r_3}{r_4} = \frac{l_3}{l_4},$$

because the resistance of a uniform wire is proportional to its length (Art. X–1).

It follows then that

$$\frac{r_1}{r_2} = \frac{l_3}{l_4}. \tag{3}$$

The box bridge, Fig. 19(b), consists of three sets of adjustable resistances in one box to which the resistance to be measured is connected to form the fourth arm of a Wheatstone bridge.

15. Potentiometer.—Accurate comparison of e.m.fs. or of potential differences is made by means of an arrangement called the *potentiometer*, Fig. 20. The line CD represents a uniform high-resistance wire. This wire is often replaced by resistance boxes which serve the same purpose. The main circuit includes

this wire or the equivalent resistances and the battery B. If the whole current flows through all parts of the wire, the potential difference between any two points on the wire is proportional to the distance between them (Art. X–4). The slide S, when moved along the wire, varies the potential difference between the points C and S. This potential difference tends to force a current through the secondary circuit CGS as well as through the part CS of the main circuit wire.

The secondary circuit contains a standard cell E_s whose e.m.f., which is accurately known (Art. XIV–7), opposes the potential

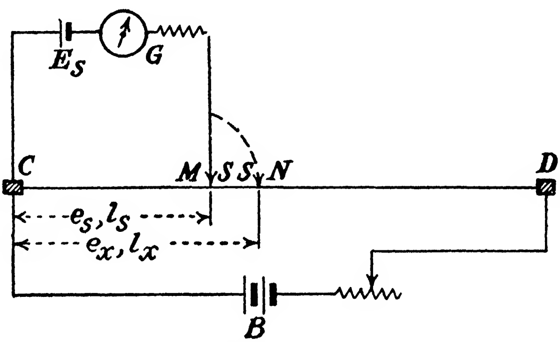


FIG. 20—Diagram of potentiometer.

difference e_s. Any current that flows through the secondary circuit is due to the difference between this e.m.f. and the opposing potential difference. The slide is moved until the galvanometer shows no deflection. The e.m.f. then is equal to the potential difference; *i.e.*, $E_s = e_s$. The standard cell is then replaced by the cell whose e.m.f. E_x is to be measured. The slide is again moved until a balance is obtained, for example, at some point N. Then $E_x = e_x$. The current in the main circuit must remain unchanged during the time of the two comparisons. Then

$$E_x = e_x = Ir_x,$$
$$E_s = e_s = Ir_s,$$

and

$$\frac{E_x}{E_s} = \frac{e_x}{e_s} = \frac{r_x}{r_s} = \frac{l_x}{l_s},$$

from which the value of E_x is determined.

Two points on any circuit may replace the voltaic cell E_x in the secondary circuit and the potential difference between them be compared with that of the standard cell.

When the potential difference between the ends of a standard resistance is measured, the current flowing through that resistance can be determined from the equation $I = E/R$.

Since the potentiometer may be used to measure both potential differences and currents, it may be employed to calibrate both voltmeters and ammeters. The cadmium standard cell, whose

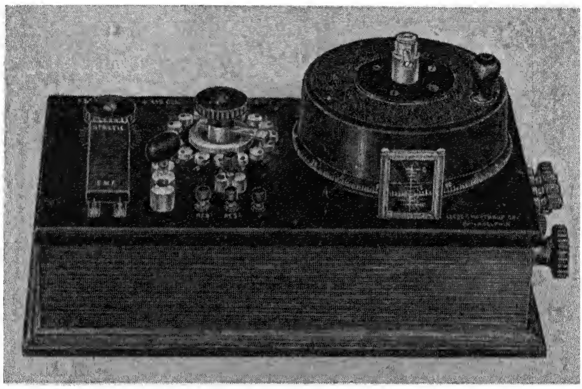


FIG. 21.—A commercial form of the potentiometer

e.m.f. is 1.01830 volts, is the most reliable type of standard cell and is almost universally used with the potentiometer.

A commercial type of the potentiometer is shown in Fig. 21.

16. Ballistic Galvanometer—Fluxmeter.—The *ballistic galvanometer* measures directly the quantities of electricity which are instantaneously discharged through it. Although this galvanometer is usually undamped, any galvanometer may be used as a ballistic galvanometer.

The quantity discharging, from a condenser for example, produces an instantaneous current which evokes a torque (Art. 3)

$$L = BlND\bar{I}' = I_0\bar{\alpha},$$

where I_0 is the moment of inertia of the coil, and $\bar{\alpha}$ is the average angular acceleration. Multiplying both sides of the equation by the time of the discharge t gives

$$BlND\bar{I}'t = I_0\bar{\alpha}t = I_0\omega.$$

From which, and because the kinetic energy of the coil becomes the potential energy of the twisted suspensions and $I't = Q'$,

$$Q = 10Q' = \frac{10I_0}{BlND}\cdot\omega = K_1\omega = K_2\theta = K_3d. \tag{4}$$

The angular velocity ω given the coil is seen to be proportional to the quantity of the discharge. Since the energy of rotation is proportional to ω^2 and the energy of twisted suspensions to θ^2, the Q which varies as ω must also vary as the angle, θ, of the observed throw. The quantities of electricity then are to each other as the angles of throw and therefore on a circular scale as the observed throws.

A standard condenser of known capacitance charged by a standard cell gives a known quantity of electricity Q_s ($= C_sE_s$) with which other quantities are compared:

$$\frac{Q_x}{Q_s} = \frac{Q_x}{C_sE_s} = \frac{d_x}{d_s}.$$

When the galvanometer is used on closed circuit, the comparison cannot be made with a throw which is obtained on open circuit because of the large difference in the amount of damping. If the condenser is used as a standard, the circuit must be closed, before the coil moves an appreciable distance, through the same resistance as that used when the unknown quantity produced its throw. For closed-circuit work either a standard mutual inductance (Art. 18) or a magnetic standard (Art. 19) is preferably used in place of a standard condenser.

A *fluxmeter* is an overdamped ballistic galvanometer with a theoretically negligible torque in its suspensions. The magnitude of the throw is limited practically only by the closed-circuit damping of the coil. It has the advantage over an ordinary ballistic galvanometer in that the discharge need not be completed before the coil turns an appreciable distance. It is used on closed circuit only and mainly for the measurement of flux density.

17. Comparison of Capacitances.—A *standard condenser* and the condenser whose capacitance is to be measured are charged with the same battery and discharged through a ballistic galvanometer. Then

$$Q_x = C_xE = Kd_x,$$

and

$$Q_s = C_sE = Kd_s.$$

Hence

$$\frac{C_x}{C_s} = \frac{d_x}{d_s},$$

or

$$C_x = \frac{d_x}{d_s}C_s.$$

It is observed that the capacitances are proportional to the throws.

18. Mutual-inductance Standard.—It was shown (Art. XVII–4) that the mutual inductance

$$M = \frac{Q_sR_s}{I_p},$$

from which

$$Q_s = \frac{MI_p}{R_s}. \tag{5}$$

When M is known, as it is in an inductance standard, Q is calculable, and other quantities may be compared with it from the proportionality of the throws.

From Eq. XVII–4

$$N\phi = 10^8MI.$$

This relationship enables the mutual-inductance standard to be employed also as a magnetic standard because its $N\phi$ is calculable.

19. Magnetic Standard.—A *magnetic standard* (magnetic inductor) has a known or a calculable $N\phi$ from which the unknown density or intensity of any magnetic field may be measured by comparison

1. In the mutual-inductance type [Art. 18, Fig. 22(*a*)] of the magnetic standard, $N_s\phi_s$ is calculated from its equivalent, 10^8MI.

2. In the permanent-magnet type, Fig. 22(*c*), the $N_s\phi_s$ is known.

3. A modification of the permanent-magnet type, Fig. 22(*b*), employs the radial field of a moving-coil galvanometer such as

that of an ammeter or voltmeter. A spring "snaps" the coil of known number of turns through any desired known number of flux lines.

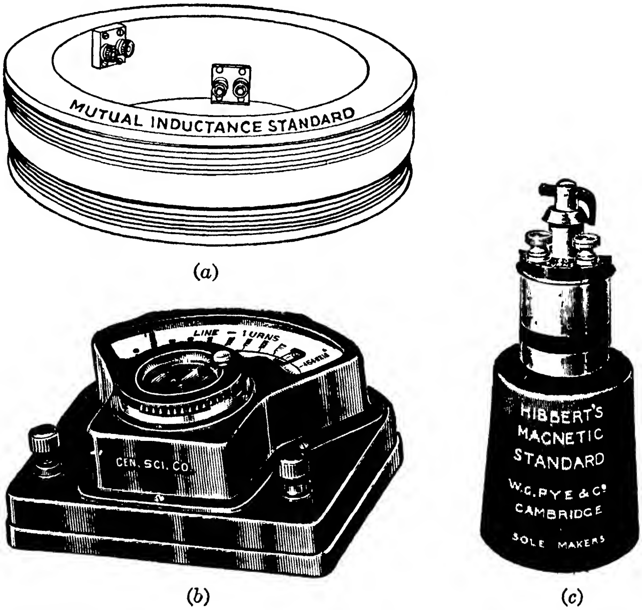


Fig. 22.—Magnetic standards· (a) mutual-inductance type; (b) permanent-magnet type with variable line turns; (c) Hibbert permanent-magnet type.

20. Measurement of the Intensity and Flux Density of a Magnetic Field.—The strength of a magnetic field H, Fig. 23, is obtained by jerking an *exploring coil* of N_x turns out of the

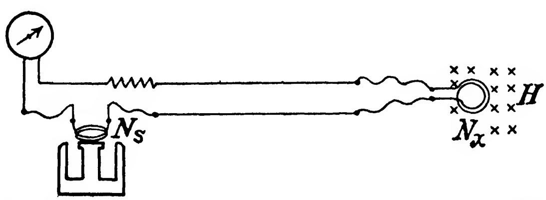


Fig. 23.—Apparatus for measuring the intensity of a magnetic field by comparison with a magnetic standard.

field. The change of magnetic flux within the coil induces a current and forces a quantity of electricity through the galvanometer. This quantity produces a throw from which, by

comparison with a magnetic standard (Art. 19), the intensity of the field is determined as follows:

From Eq. XVII–2, on jerking the exploring coil from the magnetic field,

$$N_x\phi_x = 10^8 Q_x R.$$

On dropping the coil N_s into a known magnetic field, or on opening the primary circuit if a mutual inductance standard is used,

$$N_s\phi_s = 10^8 Q_s R.$$

Then

$$\frac{N_x\phi_x}{N_s\phi_s} = \frac{Q_x}{Q_s} = \frac{d_x}{d_s}. \tag{6}$$

$$\phi_x = \frac{N_s\phi_s}{N_x}\frac{d_x}{d_s} \text{ maxwells.}$$

$$H \equiv B = \frac{\phi_x}{A} \text{ oersteds,}$$

where A is the cross-sectional area of the exploring coil. It should be noted that the resistance of the circuit cancels and that its measurement is not required provided it has the same value in the two observations.

Questions

1. What is meant by an "absolute" measurement of a quantity?
2. What instruments are employed in the absolute measurement of an electric current? Give the principle of each.
3. What is a moving-magnet galvanometer? Explain how the sensitivity of such instruments is increased.
4. Explain two methods for damping the needle of a moving-magnet galvanometer.
5. How is such a galvanometer screened from external magnetic fields? Why is this necessary?
6. Give the construction of a moving-coil galvanometer. How is the magnetic field made nearly uniform and radial? How should the coil hang with respect to the radial field?
7. Prove that the currents flowing through such a galvanometer are to each other as the deflections produced on a circular scale. Why is this not fully realized in practice?
8. How are portable moving-coil galvanometers constructed?
9. How are direct-current ammeters constructed? What are internal and external shunts? What are interchangeable shunts?

10. Develop the expression for the current in a shunted galvanometer in terms of the total current in the circuit.

11. How is an ammeter connected into a circuit in which the current is to be measured? Does its introduction into the circuit change the current appreciably?

12. Give the principle of each of the four types of alternating-current ammeters. Can they be used for measuring direct currents?

13. Show how a current is measured by means of a silver voltameter. Why is this method used?

14. Explain how a current is measured by means of a Kelvin balance.

15. State, in general terms, how absolute measurements of potential difference are made. Give two methods.

16. Explain the construction of a direct-current voltmeter.

17. Explain why the potential difference read by the voltmeter may not be the potential difference between the points before the voltmeter is connected.

18. Explain why a voltmeter connected to a battery of high resistance does not give its e.m.f. What does the reading give?

19. How is an alternating-current voltmeter constructed?

20. Give the principle of the electrostatic voltmeter and the electrometer.

21. Can an electroscope be used to measure potential or potential difference?

22. Explain the construction and the action of a wattmeter; a watt-hour meter (service meter).

23. Explain how resistances are measured by means of a Wheatstone bridge. Develop the equation.

24. Explain how e.m.fs. and potential differences are compared with the e.m.f. of a standard cell. What is a potentiometer?

25. Show how the potentiometer is used to measure currents.

26. What is a ballistic galvanometer? A fluxmeter?

27. Show that, when a ballistic galvanometer is used to measure quantities of electricity in instantaneous discharges, $Q \propto d$.

28. How is a standard condenser used with a ballistic galvanometer in measuring unknown quantities of electricity?

29. How are capacitances measured by comparison with a standard condenser?

30. How does a mutual-inductance standard give a known quantity of electricity? A known $N\phi$?

31. What is a magnetic standard? Give three types.

32. Derive the expression for the intensity of a magnetic field as measured by comparison with a magnetic standard.

Problems

1. A coil of a moving-coil galvanometer consists of 200.5 turns and is suspended in a magnetic field of 350 oersteds. The length of the coil is 4 cm and the width 2 cm. (*a*) What torque is acting on the coil when the current in it is 0.000002 amp? (*b*) How far, in degrees, does the coil

deflect when the torsional moment T of the suspension and spiral is 1.2 c g.s. units?

2. A portable galvanometer gives a full-scale deflection with a potential difference of 50 mv. What is the resistance of an interchangeable 10-amp shunt for the galvanometer?

3. (*a*) What is the potential difference, in volts, between the extremities of a wire immersed in water when a current of 5 amp raises the temperature of 800 grams of water 8°C in 5 min? (*b*) What is the resistance of the wire?

4. A portable galvanometer whose resistance is 20 ohms gives a full-scale deflection with a current of 0 0075 amp. What resistance must be added to make a voltmeter of it which reads 150 volts at full scale?

5. When a slide-wire potentiometer balances a standard cell whose e.m.f. is 1.0183 volts, the length of the slide wire between the extremities of the secondary circuit is 59.32 cm. What is the e m.f. of a battery which produces a balance on the same wire (with no change of current) when this distance is 73.47 cm?

6. When charged with a cadmium standard cell (E = 1.0183) a condenser whose capacitance is 1 microfarad produces a throw of 10 cm on a ballistic galvanometer. What is the quantity, in coulombs, that produces a throw of 16 cm on the same instrument?

7. A standard condenser and a telephone condenser are charged with the same dry cell. The standard condenser has a capacitance of 1.003 microfarads and produces a throw of 8.25 cm. The telephone condenser gives a throw of 16.50 cm. What is the capacitance of the telephone condenser?

8. The mutual inductance of two coils is 10 millihenrys; the resistance of the secondary circuit containing one of the coils is 20 ohms; and the current in the primary coil is 0.1 amp. What quantity of electricity is induced in the secondary on opening the primary circuit?

9. An exploring coil of 18 turns and 2 cm² cross-sectional area produces a throw of 8.42 cm when it is jerked out of a magnetic field. In the same circuit a magnetic standard cutting a flux of 22,000 maxwells with 10 turns gives a throw of 19.30 cm. What is the intensity of the magnetic field cut by the exploring coil?

Experiments

1. Instruments shown: (1) tangent galvanometer; (2) sensitive moving-magnet galvanometer; (3) moving-coil galvanometers including working model; (4) ammeters; (5) external shunts; (6) voltmeters, including electrostatic voltmeters, electrometer, and electroscope with graduated scale. Lecture table galvanometer made into an ammeter and a voltmeter; (7) wattmeter; (8) Wheatstone bridges; (9) potentiometer; (10) standard condenser; (11) standard inductances; (12) magnetic standards; (13) fluxmeter; (14) exploring coil.

2. A hollow iron cylinder suspended by a spiral spring above a solenoid energized by an alternating current is drawn into the cylinder more or less

depending on the strength of the current. The current may be varied by inserting an iron rod into an additional coil which forms a part of the circuit. This illustrates an old form of a current-measuring instrument.

3. Comparison of capacitances by means of a ballistic galvanometer.

4. Measurement of the intensity of a magnetic field by means of an exploring coil and a magnetic standard. By means of a fluxmeter.

CHAPTER XXII

TRANSFORMER

1. Choking Effect in Circuits of Large Self-inductance.—Figure 1 is Fig. XVIII–11 redrawn, except that the self-inductance in the circuit is assumed to be large and the resistance small, as is the case in transformers. Inspection of the figure shows that under these conditions the phase difference between the current and the impressed e.m.f. is nearly 90°. Because of this phase lag the impressed e.m.f. is opposed by the e.m.f. of

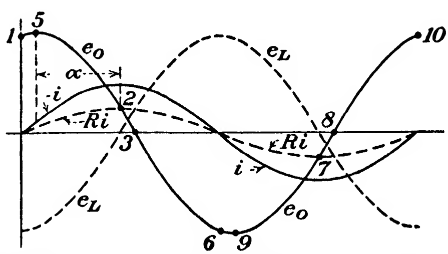


FIG. 1.—Choking effect in circuits of large self-inductance and small resistance ($\omega L = 2$, $R = 0.5$).

self-induction during nearly the whole cycle. This opposition leaves only a very small part of the impressed e.m.f., the RI drop, effective in forcing the current through the coil; hence the current is said to be choked.

These facts may be observed also by inspection of Eqs. XVIII–7,12; *i.e.*,

$$I = \frac{E}{\sqrt{R^2 + (\omega L)^2}}, \qquad \text{and} \qquad \tan\theta = \frac{\omega L}{R}.$$

With the same R and ω, the larger the magnitude of L, the larger is the angle of lag and the smaller the current. A decrease in R increases both the current and the angle of lag.

Why the high counter e.m.f. exists in a circuit linked by an iron core was shown in Art. XVII–6. Its cause, however, should again be traced by inspection of Fig. 2. When the electrons in the magnetizing coil P are accelerating, as shown by the double crosses and double dots, the rectangular iron circuit is being magnetized in the direction represented by the broken-arrowed line. Disregarding the effect of hysteresis, the elements m of the equivalent magnetization whirl are at every instant in phase with the magnetizing flow. Each element of the whirl is accelerating and thereby contributes (Law C) to an e.m.f. in the coil P. Since the acceleration in the nearer side of each element determines the direction of this e.m.f., every element of the whirl contributes to an e.m.f. which opposes the electron acceleration in the coil P. This is represented by the cross and dot placed below the coil. The changing magnetization of the iron core, therefore, evokes in this manner practically the whole of the large counter e.m.f. in the coil.

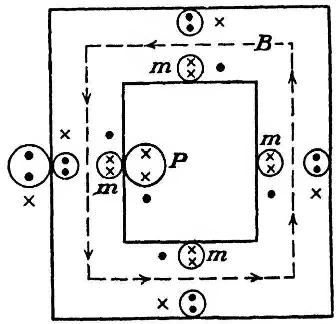


FIG. 2.—Relation of the eddy whirl to the magnetization whirl, when the flow in the magnetizing coil P has positive acceleration.

2. Effect of the Eddy Current in the Core of the Electromagnet on the Current of the Magnetizing Coil.—The alternating magnetization whirl of the iron core, represented by the loops m, Fig. 2, is practically in phase with the magnetizing alternating flow in the coil P. The accelerating whirl impresses an e.m.f. on the neighboring free electrons within the iron. This e.m.f. produces an eddy whirl, which, if the core is laminated, may consist mainly of whirls in individual sheets. These component whirls, then, are treated as though they were one equivalent larger whirl. The phase lag of this eddy is small, mainly on account of high resistance, Eq. XVIII–12, so that the eddy is practically in phase with the e.m.f. that produces it and therefore 90° out of phase with the magnetization whirl, as represented by the unencircled crosses and dots. The two oppositely accelerat-

ing whirls, then, evoke oppositely directed e.m.fs. in the magnetizing coil *P*. The eddy whirl, therefore, reduces the counter e.m.f. of self-induction due to the magnetization whirl. The eddy whirl, then, enables the impressed e.m.f. to force a larger flow through the coil *P* and, thereby, more energy to be taken from the mains.

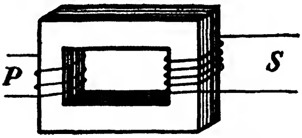


FIG. 3.—Core-type transformer.

It should be noted that the e.m.f. causing the eddy whirl in the core and the counter e.m.f. of self-induction in the magnetizing coil are both produced by the same acceleration of the magnetization whirl and, therefore, that these alternating e.m.fs. are in phase.

3. Explanation of the Action of a Transformer.—The transformer is an instrument for transferring the energy of one alternating-current circuit into another with any desired change in e.m.f. The transformer makes long-distance power transmission economically possible because it raises or lowers alternating e.m.fs. with almost negligible loss of energy and without

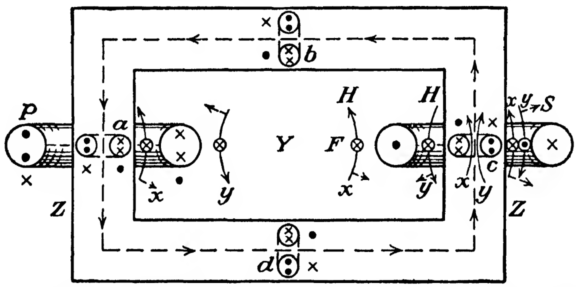


FIG. 4—Core-type transformer showing how electromagnetic pulses, x and y, emanate from two paired elements, a and c, of the magnetization whirl when the intensity of the magnetizing current in loop P is increasing.

moving parts. The core-type transformer consists of a soft-steel laminated core linking two solenoids, Fig. 3, which are parts of separate circuits. The loop *P*, Fig. 4, represents a section of one of the turns of the magnetizing solenoid in the "primary" circuit from which the energy is to be transferred. The loop *S* is a section of one turn of the solenoid in the "secondary" circuit, the circuit which is to receive the energy.

1. *Magnetic Field within and outside the Core and the Electric Field Evoked by Changing Magnetization.*—If the iron core were

toroidal (Art. VI-4), the magnetized core would contain all the magnetic flux due to the iron, while the space in the regions YZ, Fig. 4, would contain only a part of the comparatively weak field due to the magnetizing solenoid. The rectangular core of the transformer approximates this condition. By neglecting hysteresis, the magnetization whirl is in phase with the magnetizing current.

When the magnetization of the core is increasing, inspection of Fig. 4 shows that in the plane shown the magnetic fields of the electromagnetic pulses cancel in the region Y and Z and reenforce within the iron core; while the electric fields reenforce in the region Y and cancel in the regions Z. If a circle were drawn about either coil P or S in the plane of the coil, the phase and intensity relationships of the electric fields in the superposed pulses along the circumference of this circle would be such as to cause a gradual change from complete reenforcement in the region Y to complete cancellation in the region Z. It is seen that, although in the space surrounding the core the elemental magnetic fields completely neutralize, the electric fields of the electromagnetic pulses do not. They therefore impress e.m.fs. on both the coils P and S.

2. *Choking of No-load Current in the Primary.*—The alternating current in the primary circuit P usually is a 60-cycle current; *i.e.*, it flows sixty times in each direction per second. This current is choked by the e.m.f. of self-induction, as explained in Art. 1, so that, although the resistance of the solenoid P is small, the current is also small and lags in phase practically 90° behind the impressed e.m.f. This current is called the *exciting* or the *no-load current.* It consists of two components: one the *magnetization current* (90° out of phase with the impressed e.m.f.), which magnetizes the iron; the other the *power component* (in phase with the impressed e.m.f.), which supplies energy consumed by eddy currents and hysteresis. The latter component is small compared with the former, and both together are small compared with the load current which is superposed on the no-load current as soon as the secondary circuit is closed. The magnetization current, I_M, reaches only such a magnitude as is required to maintain an alternating magnetization of the core which produces

equilibrium between the impressed e.m.f. and the counter e.m.f. of self-induction and the RI_M drop, *i.e.*, so that

$$E_P = N_P e_P + RI_M.$$

The transformer is so designed that this magnetization of the core takes place along the whole length of the approximately straight part (between the knees) of the magnetization curve.

3. *Eddy Currents Suppressed.*—The eddy whirl that would form under such conditions in the steel core is almost entirely suppressed by lamination (Art. XIX–4) and the use of high-resistance silicon steel limits the eddy currents in the individual sheets.

4. *Relation of the Changing Flux within the Core to the Induced E.M.Fs.*—Imagine the coil P alone to be acting on the coil S. The electromagnetic pulse emanating from coil P then cuts through the coil S and induces in it an e.m.f. which, although caused by the nonconservative electric field, Art. XIII–5, can be measured in terms of the rate with which the magnetic flux is changing within the coil. Each element of the accelerating magnetization whirl of the iron core impresses an e.m.f. on each of the coils P and S in the same manner. Since, when using the concept of the magnetization whirl, the magnetic field within the core is established or diminished only by the superposed contributions to it by the electromagnetic pulses, the total e.m.f. induced in each turn of the coil then is equal to the rate with which the magnetic flux within it and therefore within the core is changing. It then follows that

The induced electromotive force in abvolts per turn in each of the coils of a transformer is equal to the rate at which the flux is changing within the core.

The induced e.m.f. in each coil, then, is

$$e = -N\frac{d\phi}{10^8 dt},$$

where N is the number of turns in the coil under consideration. The action of the iron core in producing the e.m.fs. in the coils P and S may be expressed also in terms of the magnetic flux of individual magnetons. Some of the flux of each magneton

occupies the space within the core and links each of the coils: The magnetic field within the core is composed entirely of such superposed, reenforcing elemental fluxes. When the magnetons are being forced from one alignment into the other, this combined linking flux of all the magnetons necessarily cuts each of the loops at the same rate with which the flux is changing within the iron core.

5. *Induced E.M.F. per Turn in Primary and Secondary and the Relation of the Impressed E.M.F. in the Primary to the Induced E.M.F. in the Secondary.*—An inspection of the magnetic loops or elements a, b, c, d of the magnetization whirl shows, Fig. 4, that when the magnetization of the core is changing, increasing in intensity, for example, every elemental magnetic locp of the accelerating whirl forces the electrons in both of the coils P and S toward the observer in the parts of the loops located in the open space Y. This force opposes the acceleration of the electrons in the coil P and is there the e.m.f. of self-induction as already explained in Art. 1; and in the coil S, the force is the e.m.f. induced in the secondary circuit. Since the change in magnetization of the core is the same in all parts and is accompanied by electromagnetic pulses which cut through the coils at the same rate at all points, the e.m.f. induced in any one turn of the primary coil P is equal to that induced in any one turn in the secondary coil S. It then follows, since $E_P \cong N_P e_P$ and $E_S = N_S e_S$ and $e_P = e_S$, that

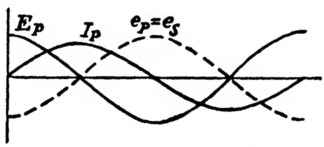


FIG. 5.—Phase relationship between impressed and induced e.m.fs. and the magnetization current in a one-to-one ratio transformer.

$$\frac{E_p}{E_s} = \frac{N_p}{N_s}. \tag{1}$$

The equation states that the e.m.fs. in the two coils of a transformer are to each other as the number of turns in the coils. Whether a transformer "steps" the voltage up or down, therefore, depends on whether the primary coil has more or less turns than the secondary.

Figure 5 shows the phase relationships between the magnetization current, I_p, and the impressed and induced e.m.fs. in a one-to-one ratio transformer.

6. *Phase Relationships in the Secondary Circuit.*—The e.m.f. e_s, Fig. 5, induced in the secondary circuit is practically 180° out of phase with the impressed e.m.f. E_p in the primary. But the phase of the current in the secondary may be anything from in phase with the induced e.m.f. which produces it to a lead or lag of nearly 90°, depending on the magnitudes of L, C, and R in the secondary circuit (Eq. XVIII–12).

7. *The Load Current in the Primary Circuit Due to the Current in the Secondary Circuit.*—The current induced in the secondary coil impresses a m.m.f. of $0.4\pi N_s I_s$ gilberts which opposes the changing of the magnetization of the core and thereby tends to diminish the counter e.m.f. in the primary. The impressed e.m.f. therefore can and must force such a current through the primary which just prevents a change in this counter e.m.f. and therefore in the necessary equilibrium described in paragraph 2 of this article. Its m.m.f. then must just balance that of the secondary coil. This current, called the *load current*, is superposed on the no-load current, and differs 180° in phase with the current in the secondary. It exists only because energy is being supplied to the secondary circuit and therefore it is through its action that the secondary circuit receives its energy from the mains. From the equality of the opposing m.m.fs.,

$$0.4\pi N_p I_p = 0.4\pi N_s I_s,$$

from which

$$\frac{I_p}{I_s} = \frac{N_s}{N_p}, \tag{2}$$

where I_p is the load current in the primary and I_s the current induced in the secondary.

8. *Power Factor in Primary and Secondary.*—Since the no-load current in the primary is small compared with the load current, the whole primary current may be considered as being 180° out of phase with the secondary current. Then since the (impressed) primary voltage also differs 180° in phase from the secondary voltage, *the power factors in primary and secondary circuits are practically equal.* From this and Eq. XVIII–13 it follows that

$$E_s I_s \cos \alpha \cong E_p I_p \cos \alpha. \tag{3}$$

This equation neglects the no-load current and makes no allowance for the energy lost in the direct heating of the primary coil.

In a practical transformer, Fig. 6, the total loss in energy from these causes is only from 1 to 4 per cent.

4. Transformer Cores.—As to types of cores, transformers may be divided into two main groups: the *core type* and the

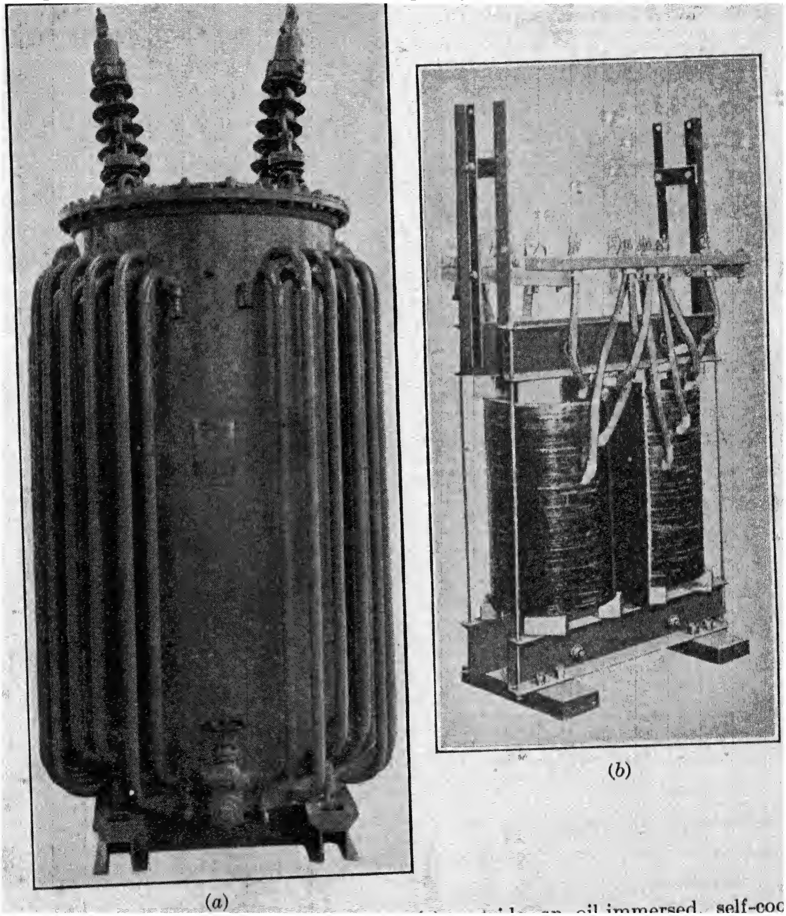


FIG. 6.—Commercial transformers: (*a*) outside an oil-immersed, self-cooling transformer, (*b*) inside a small, single-phase, high-voltage transformer (core type). (*Westinghouse Electric and Mfg. Co.*)

shell type. The core type, Figs. 6, 7(*a*), consists of a laminated soft-steel core linking the two solenoids P and S which may be wound one over the other, side by side on the same "leg," or on separate "legs."

The shell type, Fig. 7(*b*), consists of a laminated soft-steel core upon the center leg of which the two coils may be wound, either one over the other or side by side.

To fulfill better the requirements of some transformers, core modifications employing the features of both the core and shell types have been designed.

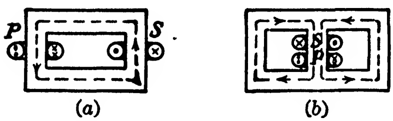


FIG. 7.—Transformer cores: (*a*) core type; (*b*) shell type.

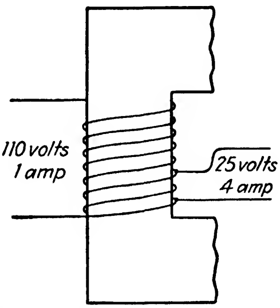


FIG. 8.—The single tapped winding of an auto-transformer.

5. Autotransformer.—A useful modified form of the transformer, called an *autotransformer*, consists of a single continuous winding, tapped so that any desired part of the winding is common to both the circuits as shown in Fig. 8. Which circuit is

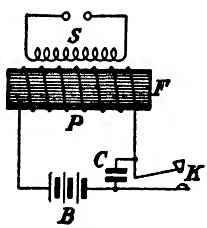


FIG. 9.—Spark coil.

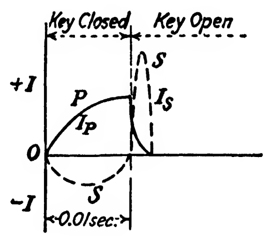


FIG. 10.—Curves *P* and *S* represent currents in the primary and the secondary of a spark coil.

used as the primary depends on whether it is desired to step the voltage up or down. Because the load current of the primary circuit and the current in the secondary must differ in phase by 180° (Art. 3), the current in the part of the winding which is common to both circuits has a magnitude which is equal to the difference in the current values of the two circuits. This trans-

former differs from the ordinary transformer in that its two circuits are connected electrically as well as inductively. It is used chiefly because of its lower cost for those purposes where it answers just as well as the standard form.

6. Spark Coil.—A simple spark coil, Fig. 9, consists of the primary coil P wound about an iron core F which consists of a bundle of iron wires. When the battery circuit is closed by the key K, the current builds up slowly on account of the large self-inductance. When the circuit is opened, the magnetic field collapses in a shorter time, as is illustrated by the rapid change in the primary current I_p, Fig. 10.

Over this primary coil is wound the secondary coil S of many turns of fine wire. Since the current in the primary builds up more slowly than it diminishes, the magnetic field builds up more slowly than it collapses. The induced e.m.f., and therefore the induced current in the secondary, is smaller when the primary circuit is closed than when it is opened. This is shown by the broken-line curve I_s. The curve also shows that the direction of the induced current at the make of the primary circuit is the reverse of that at the break. The larger induced e.m.f. at the break lasts a shorter length of time than the smaller induced e.m.f. at the make. If the secondary circuit is closed so that a current flows in it, the quantity of electricity induced is the same in both cases, for Q depends only on the change of line turns (Art. XVII–3) and not on the rate of cutting.

If the secondary circuit is open, the gap can be made of such a length that the smaller potential difference at the make in the primary is not sufficient to produce a spark; therefore no current can flow. Under such conditions only the larger potential difference at the break produces a spark. Repeated makes and breaks produce intermittent, unidirectional currents in the secondary circuit. The condenser C connected across the key K diminishes the intensity of the spark at the key. A large part of the surge due to the collapsing flux (self-inductance, inertia of the flow) flows into the condenser, which, as soon as the e.m.f. ceases, discharges through the primary coil and, in so doing, destroys the residual magnetism of the iron core. If the condenser is removed, the electrons which would have entered the condenser continue to flow toward the opened part of the circuit

and establish thereby a much higher potential difference across the gap. This potential difference produces a vigorous spark which is injurious to the material at the contact points of the

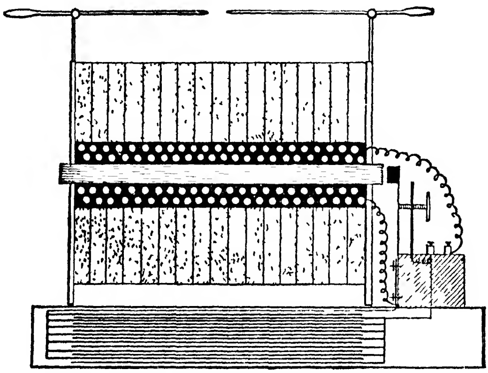


FIG. 11.—Cross section of induction coil

key and reduces the suddenness of the break and thereby the induced potential difference across the gap S of the secondary.

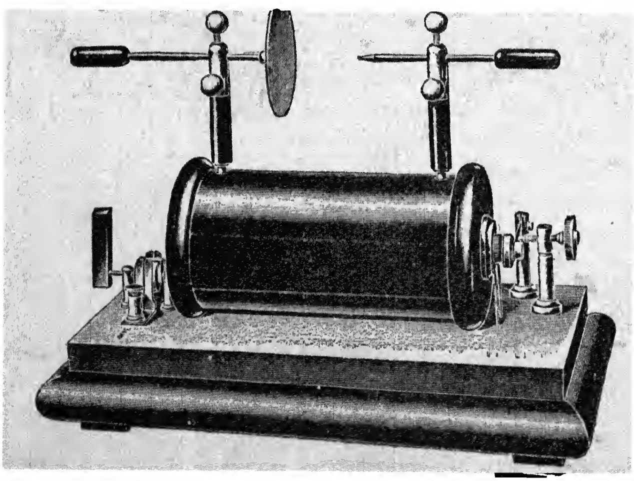


FIG. 12.—Induction coil.

Spark coils are used to ignite the gases in the engine of the automobile.

7. Induction Coil.—The *induction coil* is a spark coil supplied with an automatic make and break, as illustrated in Fig. 11.

When the iron core becomes magnetized, an iron piece attached to a spring is attracted and breaks the circuit. The iron then demagnetizes, and the iron piece springs back. The make and break are then indefinitely repeated. At each break the large e.m.f. due to self-induction produces a hot spark; this spark is greatly reduced by a condenser as explained in Art. 6. The condenser also increases the potential difference across the gap of the secondary because it lessens the time of the break and makes possible the use of larger currents in the primary.

The induction coil is used for the production of high alternating potentials (much higher when the electron displacement is

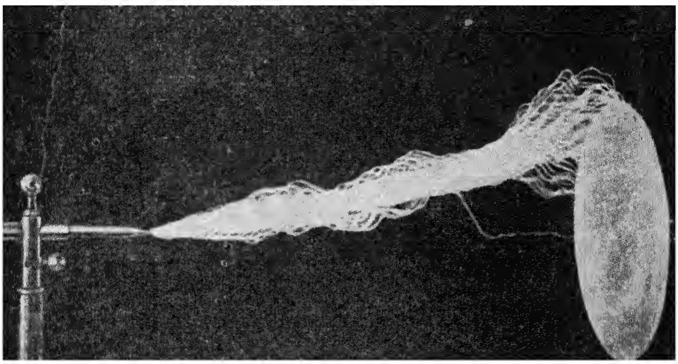


Fig. 13.—Photograph showing many sparks, in succession, to the edge of a plate when the plate is the + electrode of an induction coil.

in one direction than when it is in the other) which can be applied to an x-ray tube, for example, in which the exciting potential is higher than the lower one produced by the induction coil; hence the coil can be considered as furnishing a unidirectional intermittent potential. In the case of the x-ray tube, therefore, each electrode may be assumed to have a definite polarity which may be determined by using a circular plate with rounded edges, Fig. 11, at one of the electrodes and a pointed rod at the other. When the spark takes place as shown in Fig. 13, the plate is the positive terminal. The sparks jump to the center of the plate when the plate is negative. This phenomenon may be explained as follows: (1) The density of the charge is greater on the thin edge of the circumference than on the flat surface of the plate. (2) When the plate is positively electrified, the free electrons

which are ejected by the natural ionization of the air (Art. XI–7) are drawn in large numbers from a distance into the space in which the electric field has an ionizing potential gradient; while when the plate is negatively charged, the electrons in that space are repelled into a region where the electric field is too weak to give them ionizing velocities. (3) When the plate is charged positively, the air around the circumference of the disk, therefore, becomes highly ionized through the action of the large number of attracted electrons. The positive of these ions then are drawn in sufficiently large numbers toward the negative electrode to form a conducting path through which the spark takes place more readily than through the shortest distance between the point and the plate. (4) When the plate is charged negatively, the large ionization is produced at the pointed electrode because it is then positively charged. The positive ions then are drawn toward the center of the plate.

Questions

1. Show by diagram and by equations that in circuits of large self-inductance and small resistance an alternating current lags nearly 90° in phase behind the impressed e.m.f. and that the current is greatly reduced thereby.

2. Explain why self-inductance is increased by inserting an iron rod into a solenoid. Why is it increased more if the rod is bent into the form of a closed loop?

3. Show that, when the magnetization of the iron is increasing, the phase difference between the elemental magnetic loops of the magnetization whirl and the induced eddy current is 90° when the resistance of the eddy circuit is high.

4. Show that the eddy current increases the current supplied to the magnetizing coil.

5. Describe the two principal types of transformer cores, and state what a transformer accomplishes.

6. Explain why practically no current flows through the primary of a transformer when the secondary is open and why it does when the secondary circuit is closed.

7. How are eddy currents limited?

8. Show why the e.m.f. per turn in the secondary is the same as the counter e.m.f. per turn in the primary.

9. Explain the effect of the current in the secondary on that of the primary, and state how the energy required for the production of the current in the secondary is supplied.

10. Show that a phase difference of nearly 180° exists between the currents of the primary and the secondary regardless of whether or not the current in the secondary is in phase with the e.m.f. which causes it.

11. Explain how the magnetic components of the electromagnetic pulses due to the changing magnetization neutralize one another outside the iron core and why they reenforce one another within the core.

12. Show why the electric components of the electromagnetic pulses completely reenforce in one region and completely interfere in another.

13. Explain the production of the e.m f. in the secondary and the counter e.m f. in the primary by means of the passage of electromagnetic pulses through the material of the wire of which the solenoids are made Also by the application of Law C (Law XVI–2).

14. Explain how all the flux in the iron core of the transformer has cut all the loops of the primary and the secondary during its period of establishment and again during the period of collapse.

15. Derive the expression, in terms of the number of turns in the solenoids, for the relation between the impressed e.m.f. and that induced in the secondary.

16. Derive the expression, in terms of the number of turns, for the relation between the currents.

17. Explain the action of a simple spark coil. Why is the potential difference in the secondary greater at the break of the primary than at the make?

18. Explain the action of an induction coil.

19. What is the function of the condenser?

20. Explain how an induction coil can produce intermittent unidirectional currents.

21. When the + electrode of an induction coil is a circular plate and the − electrode a pointed rod, explain why the discharge takes place between the circumference of the plate and the pointed rod.

Problems

1. A transformer has 50 turns on the primary and 1,000 on the secondary. (*a*) When the primary is connected to 110-volt alternating-current mains, what is the induced e.m.f. in the secondary? (*b*) What is the load current when the current in the secondary circuit is 2 amp?

2. What is the magnitude of the self-inductance in the primary of a transformer when the magnetization current (part of the no-load current) in the primary is 0.1 amp and the 60-cycle impressed e.m.f. is 100 volts?

Experiments

1. Simple transformer composed of a coil slipped over the projecting core of an alternating-current electromagnet. The induced current lights a lamp.

2. A lamp in the primary circuit of a transformer shows that more current is supplied from the mains when energy is being taken from the secondary.

3. Type of transformers shown, also sheets of transformer steel.

4. An iron nail melted when made a part of the secondary circuit of a step-down transformer.

5. High-tension transformer. "Jacob's ladder" shown.

6. Two similar transformers, one having a laminated and the other a solid core, tested.

7. Induction coil provided with a disk electrode. The sparking to the circumference of the disk and then to its center, shown.

8. Medical induction coil with cylindrical screen for the primary shown in action.

9. Large induction coil in action.

CHAPTER XXIII

GENERATION AND TRANSMISSION OF ELECTRIC POWER

1. Direct-current Generator.—Direct currents for commercial purposes are produced by means of a *direct-current generator*, which consists of coils wound on a laminated iron cylinder, called the *armature*, rotating between the poles of a strong electromagnet. When a coil is rotating in a magnetic field, the flux inclosed within it is changing, and the induced e.m.f. (Art. XIII–4) is

$$e = -\frac{Nd\phi}{10^8 dt}.$$

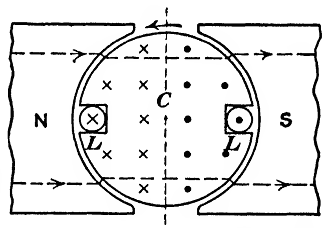


FIG. 1.—Air gaps between the armature and the pole pieces of the field magnet of a generator.

The rate at which the lines of force are being cut, and therefore the rate at which the flux within the coil changes, varies with the position of the plane of the coil with respect to the plane perpendicular to the lines of force. The induced e.m.f. in any position has been shown (Art. XVIII–1) for a uniform straight field to be

$$e = \frac{\omega N\phi}{10^8}\sin\theta = K\sin\theta.$$

If a loop of wire represented in cross section by *LL*, Fig. 1, were alone rotating in the space between the magnetic poles *NS*, the magnetic field that could be produced with any practicable m.m.f. would, on account of the great reluctance of the air space, be too weak for efficient generation of electric power.

If the loop is wound about an iron cylinder, represented in cross section by *C*, magnetic lines of force pass through the cylinder and the air gaps as shown. The air gaps, one on either side, together offer a much smaller reluctance (Art. XX–8) than

the whole space between the poles. These gaps are still further shortened by setting the loop into a slot.

As the armature rotates in the magnetic field, the electrons in it (iron and loop) are urged in the directions indicated by the crosses and the dots. This displacing of the electrons causes a current to flow around the circuit in the wire (Art. XVIII–1) and an eddy current to be produced in the iron cylinder (Art. XIX–3). The eddy current, however, is practically eliminated (Art. XIX–3) by making the iron cylinder of insulated circular disks. The high permeability of the iron cylinder causes the magnetic field in the gap to be radial, so that, in general, the

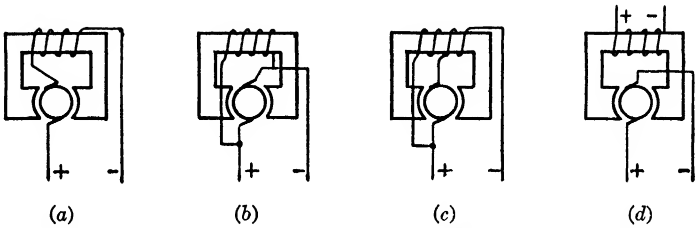


FIG. 2.—Four types of field excitation in a generator or a motor.

induced e.m.f. and current do not accurately follow the sine law of Eq. XVIII–1.

The magnetizing force for the electromagnet is usually made large enough so that the magnetic induction is well above the knee of the magnetization curve (Art. XX–6). Any reasonable change in the magnetizing current, then, produces but a slight change in the intensity of the magnetic field and therefore in the induced e.m.f.

There usually is enough residual magnetism in the field of a generator to produce a small e.m.f. as soon as the armature begins to rotate. This induces a current which in part is utilized to increase the strength of the field. This increased magnetization increases the induced e.m.f., step by step, until finally a steady state is reached. The current on leaving the armature winding is rectified by means of a commutator as described in Art. XVIII–3. This rectified pulsating current may flow through the field winding and the outside circuit in series, as shown in Fig. 2(*a*). This type is called a *series-wound generator*. The main current may flow to the outside circuit directly, while a

part is shunted through the field winding as shown in Fig. 2(*b*). This type is a *shunt-wound generator*. A *compound-wound generator* has part of the current shunted through a part of the field winding and the main current flows through the remainder, Fig. 2(*c*). The field may also be separately excited as shown in Fig. 2(*d*), or the field may be produced by a permanent magnet. In the latter case the generator is usually called a *magneto*.

The single turn on the armature may be replaced by a coil of several turns. In place of one coil, however, the armature is usually wound in many grooves as represented in cross section for the case of a two-pole direct-current generator in Fig. 3. This

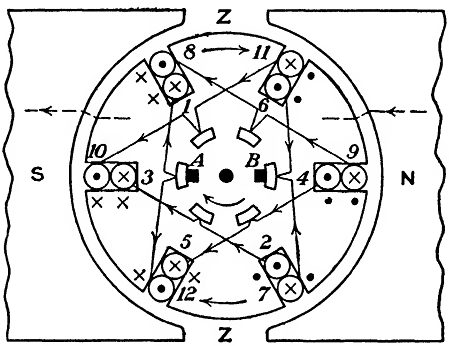


FIG. 3.—Diagram of the commutator end of a drum armature of a two-pole generator, showing the brushes, *A* and *B*, on the inside in place of the outside of the commutator. The crosses and dots in the cross sections of the wire indicate directions of winding and those outside, the directions of the induced e.m.f. in each numbered length of wire.

illustrated arrangement shows as large a number of segments in the commutator as there are grooves in the core, a condition which is not necessary but is usual. The wires are wound in the order and the directions indicated. For example, the wire is wound through the groove (1) away from the observer. On the other side of the core (not shown) it is carried across the core and through the groove (2) which is diametrically opposite the groove 1; then it is carried across the face of the core, skipping one groove, to groove 3. The process is then followed, in the order and the directions shown, until the wire reaches the observer side of the core at 12. The wire is then soldered to the wire at the starting point (1), and the centers of the parts on the observer side of the core are soldered to the individual segments

of the commutator. The winding then consists of a continuous circuit which is made up in part of sections, represented by circles, that cut magnetic flux when the armature rotates. Inspection of the figure shows that, when the armature rotates clockwise in the given flux, the free electrons are urged away from the observer in all the active sections located on the left of the plane ZZ and in the reverse direction on the right side, as represented by the crosses and dots just outside the individual sections. The arrowheads on the wires crossing the face of the core indicate the directions in which electrons are urged away from and toward the brushes A and B.

On each side of the plane ZZ some one active section is always approximately in the position where the induced e.m.f. has its maximum value, while other active sections are in positions less and less effective. The resultant e.m.f., which is the sum of these various e.m.fs., therefore is very nearly constant, and the current forced through the brushes into the external circuit is a steady direct current.

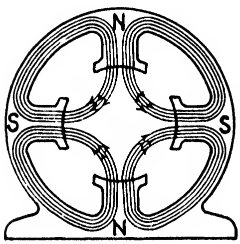


FIG. 4.—Magnetic circuits of a four-pole generator.

The armature described is called the *drum armature* which now is the armature generally employed with direct-current generators.

Generators are usually made with four or more poles in place of two. The magnetic circuits of a four-pole machine are shown in Fig. 4 and a photograph of a drum armature and a four-pole frame for the armature, in Fig. 5. As many pairs of collecting brushes are used as there are pairs of magnetic poles. The alternate brushes are connected together and then each set is attached to one of the mains. The armature, too, has to be wound appropriately so that the induced e.m.f. has the same direction from one of the mains to the other through every path in the winding. This is accomplished by the successive turns of the winding skipping the appropriate number of grooves. With this type of winding there are as many paths through the armature as there are poles in the field magnet. The number of grooves in the armature is necessarily much larger than that shown in Fig. 3.

Practical direct-current generators are constructed for voltages as high as 1,500 volts. In electrified railroads two such generators were usually connected in series, giving 3,000 volts for the desired potential difference between the mains. Rotary converters (Art. 6), however, are now supplanting the larger direct-current generators for the production of direct currents. Direct

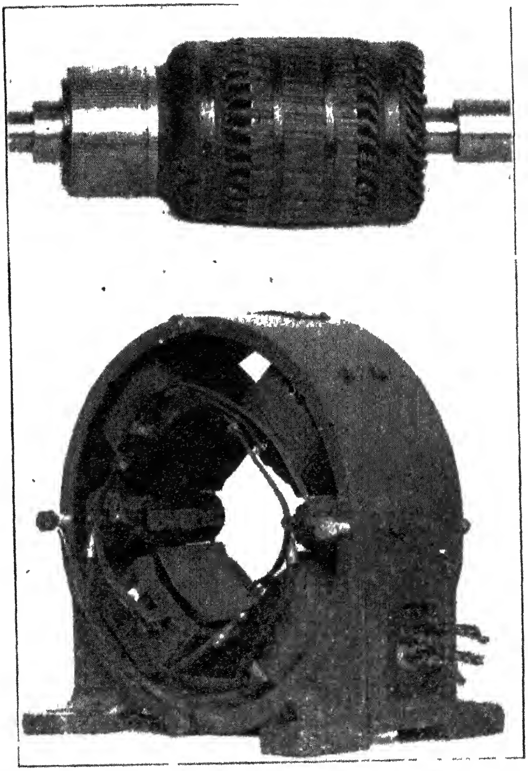


FIG. 5.—Drum armature and four-pole frame. The two interpoles are for the purpose of improving commutation. (*General Electric Co.*)

currents are used in the operation of electric trains and elevators because these currents produce a larger starting torque in motors. Either water power or steam is generally used to operate the generator. A standard form of d.c. generator is shown in Fig. 6.

The energy loss in the operation of a generator is due to (1) RI^2 losses in the armature winding, field coils, and brushes; (2) resistance of the air and friction of bearings and brushes; (3)

hysteresis; and (4) eddy currents. The losses, however, are not great, for the efficiency of a generator is ordinarily well above 90 per cent and may be as high as 98 per cent. The commercial efficiency of a generator is the ratio between the amount of electrical energy supplied by it to the mains and the amount of mechanical energy required to operate it.

2. Single-phase, Alternating-current Generator (Alternator). The production of an alternating current in a coil rotating in a

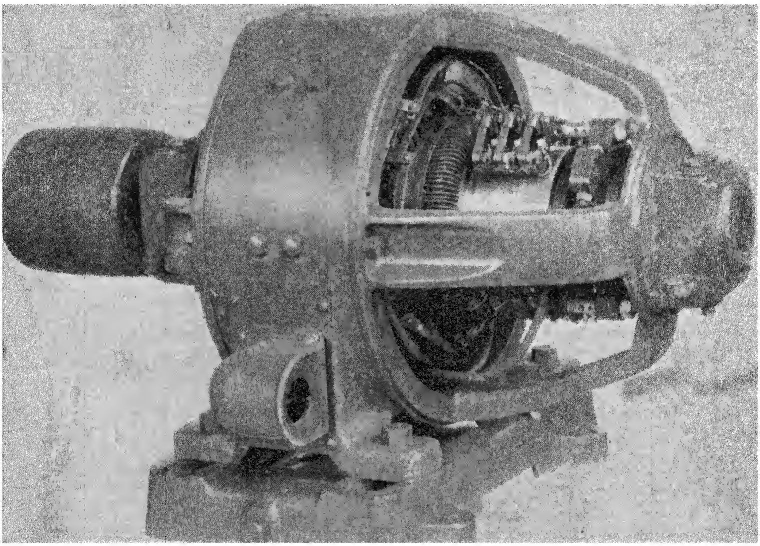


Fig. 6.—15-kw direct-current generator. (*General Electric Co*)

magnetic field was considered in Art. XVIII–3. To produce large effects, coils must be wound within slots on laminated iron as in the direct-current generator, Fig. 1. To reduce the required speed of rotation for producing an alternating current of any given number of cycles per second, the wires of the armature cut through several magnetic fields during each revolution, as shown in Fig. 4. The manner of excitation and the windings of such a generator having four poles are shown in Fig. 7. When the generator has more than four poles, the windings are appropriately modified.

Since alternating currents are employed for long-distance

of the generating unit as high as practicable before transforming it to a still higher potential by means of a transformer. Practical alternators are made giving e.m.fs. as high as 13,200 volts. Such high voltages are dangerous, and therefore it is found desirable to eliminate exposed metal parts, including collecting brushes. This is accomplished by making the rotating part (rotor) an electromagnet which has two or more poles and whose rotation induces the high voltage in a *stationary armature* (stator). The stationary armature connects directly to the mains without the use of any high potential collecting brushes or exposed parts. The magnetizing current is led to the rotor by brushes at low potentials and is furnished by a separate exciter (generator). The revolving-field magnets and the stationary armature of such generators are shown in Figs. 8, 9.

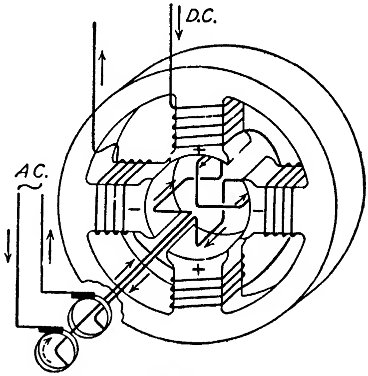


FIG. 7.—A diagram of a four-pole, single-phase alternator showing the induced e.m.f. in the different parts of the rotating armature at a given instant.

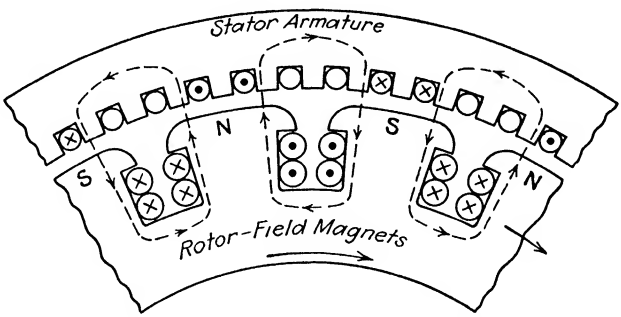


FIG. 8.—Diagram showing in section a part of the stationary armature and of the rotating field magnets. The broken-line loops represent, at a given instant, magnetic circuits, while the crosses and dots represent the directions of the steady magnetizing current and those of the induced alternating e m.fs. One cycle of the alternating e.m f. is completed in the time interval the rotor moves a distance equal to that between the centers of two consecutive + poles.

Since it has been found that a current of 60 cycles (120 alternations/sec) has the desirable frequency for most commercial

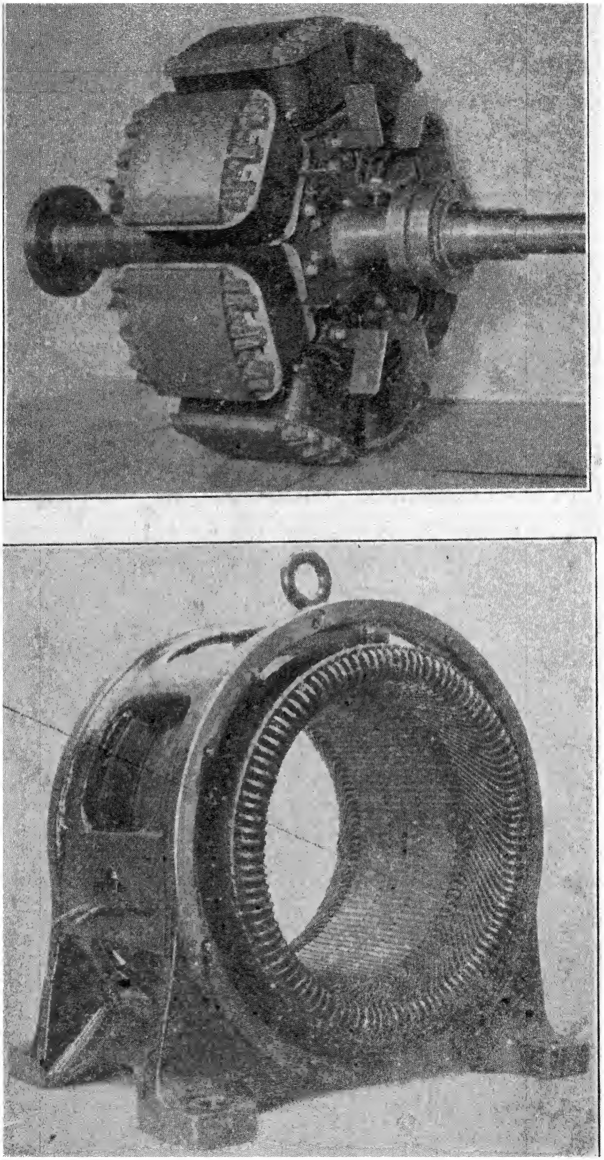


FIG. 9.—Revolving-field magnets and stationary armature of 312-kva alternator. (*Westinghouse Electric and Mfg Co*)

purposes, a generator having only two poles must rotate at too high a speed. Alternators are therefore constructed with several pairs of poles, alternately + and − around the circumference of the rotor, as shown in Figs. 8, 10. The direction of the e.m.f. induced in the conductors facing the + poles is the reverse of that induced in those facing the − poles; but the adjacent con-

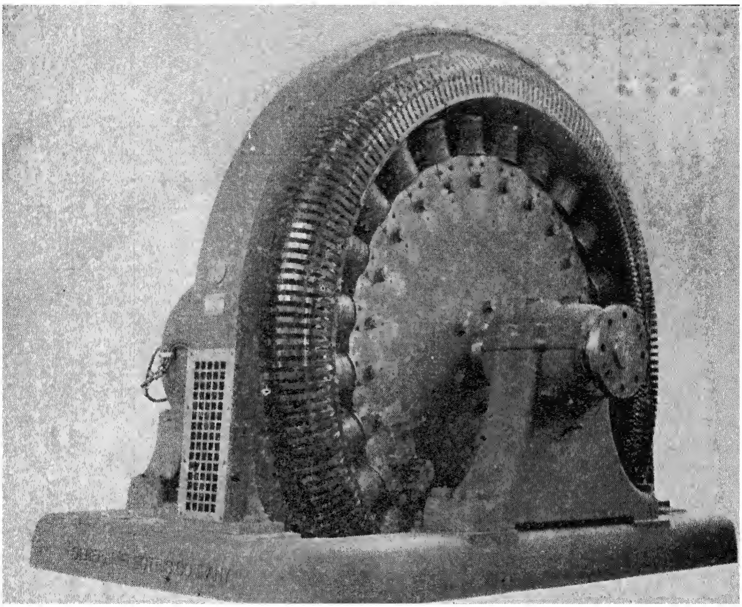


FIG. 10.—2,300-volt slow-speed alternating-current generator. (*General Electric Co.*)

ductors of the armature are parts of windings so placed that the induced e.m.f. has the same direction in all parts of the circuit from one terminal to the other. The e.m.f. in any one conductor reverses direction every time it is at the midway point between the revolving poles, and consequently the number of alternations per second is the number of revolutions per second multiplied by the number of poles. The frequency (number of cycles per second) is then half of this number of alternations, or $f = np/2$, in which f is the frequency, n the number of revolutions per second, and p the number of poles. Alternators are rated in kilovolt-amperes (kva) in place of kilowatts because their

capacity depends on the voltage and the current and not on the power delivered. The power in kilowatts is less than in kilovolt-amperes whenever the power factor is less than unity.

Generators rated for 120,000 kva have been built, and those of 60,000 kva are common.

3. Three-phase Alternating-current Generator (Alternator).—If either a rotating or a stationary-type armature, Figs. 8, 9, has three similar independent sets of windings which are so placed about the circumference of the armature that the e.m.fs. induced

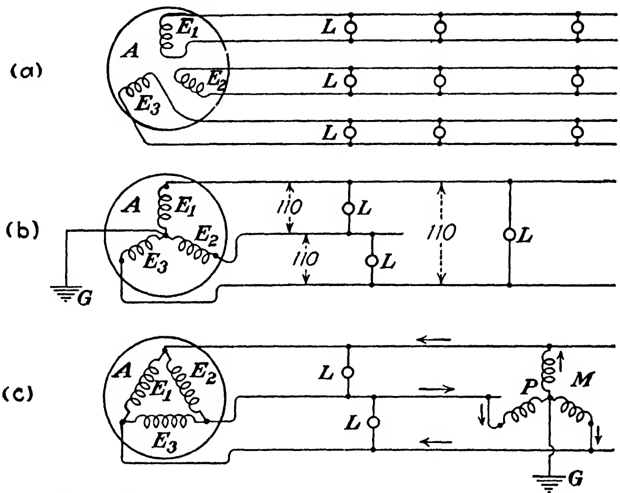


FIG. 11.—Three-phase alternating-current circuits, including the alternator, *A*, and the independent armature windings E_1, E_2, E_3. The coils *M* represent windings of an induction motor energized by three-phase currents from the mains, and the small circles, lamps energized by single-phase currents.

in them differ 120° in phase, the generator can impress these e.m.f. on each of three independent circuits. The coils E_1, E_2, E_3, Fig. 11, represent, in diagram, the three independent sets of windings. It is found economical and practical to connect these windings as shown either in Fig. 11(*b*) or in Fig. 11(*c*). Only three transmission lines or mains are required then in place of six [Fig. 11(*a*)]. Inspection shows that the mains are charged with alternating potentials which differ 120° in phase; and that when a circuit is connected to any two of the mains, the circuit has impressed upon it the same single-phase effective e.m.f. regardless of which two of the three mains are employed.

If three coils of like impedance are connected to the three mains as shown for the coils M, Fig. 11(c), the current at any instant in one of them has the reverse direction with respect to the common point P, as represented by arrows, and is equal to the algebraic sum of the currents in the other two coils. That the impressed e.m.fs. all contribute to the flow without interference is understood by inspection of Fig. 12. The algebraic sum of the three e.m.fs., at any instant, at any point is zero; *i.e.*, any one of them is equal to the sum of the other two and has the reverse direction. Such connections are important because they are used in induction motors (Art. 5) and whenever it is desired to produce a rotating magnetic field.

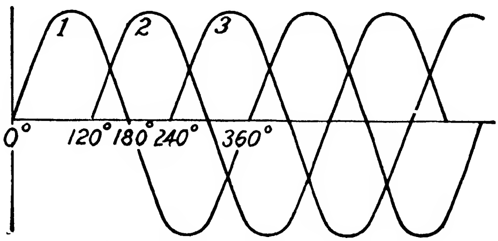


FIG. 12.—Sine-wave e m fs of a three-phase generator.

The three-phase generator is easily distinguished by its three slip rings and its three brushes in place of two. Practically all transmission lines now employ the three-phase system.

Generators transform mechanical energy into electrical. The mechanical energy is obtained from waterfalls or wind or the chemical energy of coal.

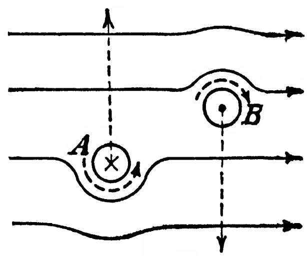


FIG. 13—Counter e.m.f. in an energized wire moving in a magnetic field.

4. Direct-current Motor.—The wire A, Fig. 13, shown at right angles to a magnetic field, has its electron flow toward the paper and is thereby urged upward from the strengthened to the weakened part of the field (Law A). When the wire moves upward, it cuts the magnetic flux and, because of this cutting, the electrons are urged toward the observer as represented by another view B; *i.e.*, a counter e.m.f. is induced in the wire

opposing the motion of the electrons in it. If there were no friction, the wire would finally acquire a velocity at which this counter e.m.f. just equals the impressed e.m.f. The current then would be zero. The counter e.m.f. in abvolts at any instant is equal to the rate at which the flux is being cut. The "flux cut" is that of the original field because the field surrounding the current and superposed on the original field is not being cut.

A *direct-current motor* is constructed like a generator, or a generator itself may be used as a motor. When the motor is connected to the mains, the current flowing through the armature causes a torque, and the resulting rotation produces a counter e.m.f., as explained above for the case of a single wire. If there is no load, the armature accelerates until the counter e.m.f. nearly equals the impressed e.m.f. The current then is small and therefore also the energy, EI, which is being supplied to the motor.

Let E be the impressed e.m.f. and e the counter e.m.f. Then for a series motor

$$E - e = RI,$$

where I is very small when e is nearly equal to E.

From this, as has been shown in Eq. XVII–9,

$$EI = RI^2 + eI \text{ watts,}$$

where EI represents the total power taken from the mains, RI^2 the power expended in heating the circuit, and eI that expended in doing mechanical work, but apparently in forcing electrons against the counter e.m.f. The counter e.m.f. is there because the armature turns; however, any appreciable current is present only when work is being done so that the term eI is that part of the supplied energy not expended in heating the circuit and therefore must represent the mechanical work done by the motor; this work includes friction and windage (air resistance) and the useful work.

Since its resistance is usually less than 0.1 ohms, the armature at rest if connected directly to the 110-volt mains would for an instant be energized with a current of more than 1,100 amp. This current would destroy the armature before sufficient speed was attained. It is necessary, therefore, except with small

inefficient motors having a comparatively high resistance, to use a *starting resistance* in series with the armature. This resistance is gradually reduced (often automatically) as the speed increases, until finally it is cut out entirely.

The counter e.m.f. generated by the motor varies directly as the speed of the armature and the strength of the exciting field. It then follows that the stronger the exciting field the smaller is the speed of rotation required to attain the necessary counter e.m.f. This means that the speed of a direct-current motor can be controlled by varying the field excitation.

When a load is placed on the motor, the speed of rotation diminishes and, therefore, also the counter e.m.f. This decrease in the counter e.m.f. allows a greater current to flow through the armature and more power to be supplied and expended.

The efficiency of a motor, being the ratio of mechanical output to electrical input, varies with the load, usually reaching a maximum somewhere between three-quarters and full load. An average working efficiency for a direct-current motor is about 85 per cent, but that of large motors sometimes exceeds 95 per cent.

5. Alternating-current Motor.—There are several types of alternating-current motors:

Small single-phase commutator motors are made in which the alternating current from the mains enters both the armature and the field coils. The field magnet as well as the armature must then be made of laminated iron to prevent eddy currents. The starting resistance is often made to turn off automatically when the rotor acquires sufficient speed. The resistance of small alternating-current motors is often made such as to require no starting resistance, but this is done at the expense of efficiency as already noted in Art. 4.

Induction motors are generally operated by three-phase currents which produce a rotating magnetic field that cuts the rotor. The current induced in the rotor by the cutting causes the rotor to turn in the direction of the rotating field, as explained in Art. XIX–4. The rotor is often made of bars of copper welded together in the form of a cylindrical cage, Fig. 14, which then is called a *squirrel-cage rotor.* It has no electric connections and operates entirely by the action of the induced currents.

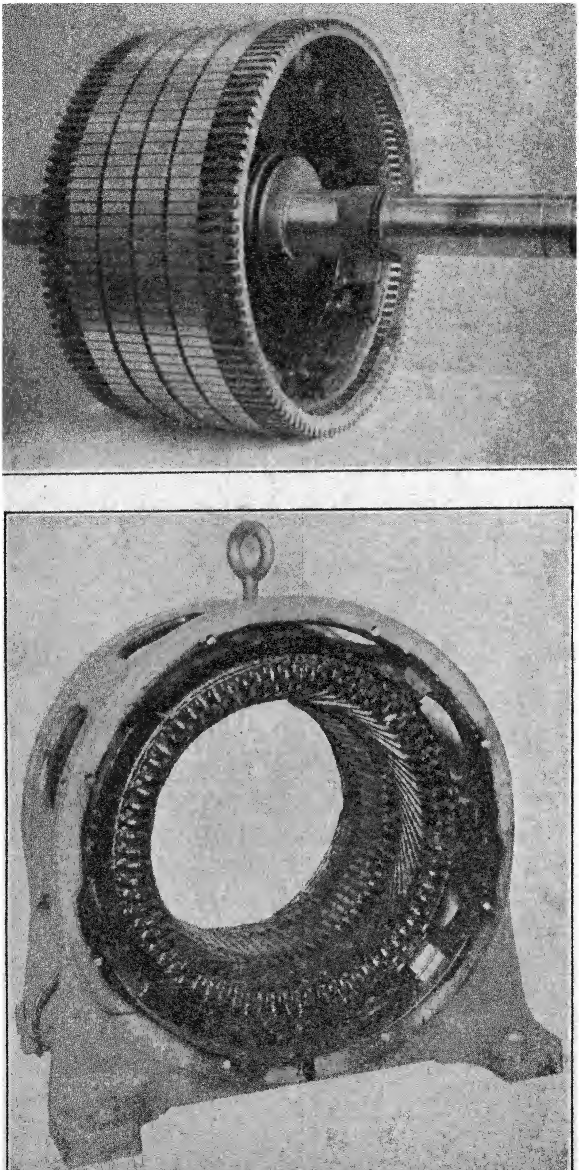
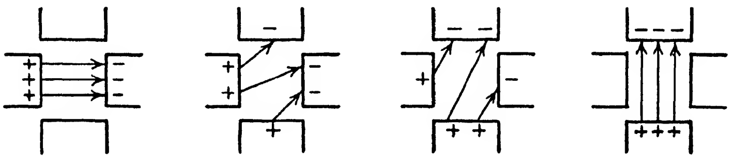


FIG. 14.—Squirrel-cage rotor and the frame containing the windings which produce a rotating magnetic field in an induction motor. (*Westinghouse Electric and Mfg Co.*)

The production of the rotating field by the more complicated three-phase windings of the stator, Fig. 14, is illustrated by the two-phase arrangement of Fig. 15. The horizontal poles are the ends of an electromagnet energized by one of the two-phase currents, and the vertical poles are part of an electromagnet energized by the other. The two two-phase currents differ 90° in phase, so that, when one current has the minimum value, the other has the maximum; and when one is increasing the other is diminishing in intensity. The strengths of the magnetic poles and of the crossed magnetic fields, then, change in a similar manner; and since observed lines of force never cross the resultant



(a) (b) (c) (d)

FIG. 15.—Rotating magnetic field produced by two-phase currents

flux takes the positions in succession as shown in (*a*), (*b*), (*c*), and (*d*). The magnetic field, in this manner, makes one complete revolution during each cycle of the current. Three-phase currents produce a rotating field in a similar manner. It is not necessary for the poles to be the ends of independent magnets, for the field rotates within the open space of the frame, Fig. 14, which contains the field windings.

Synchronous motors are the most efficient of the motors. The armature is the stator and, in the type here described, is energized by a three-phase alternating current which produces a rotating magnetic field. The field windings of the rotor, Fig. 16, are energized by a direct-current generator through ordinary slip rings and energize the successive field magnets in reverse directions, but so that each magnet continuously retains an unaltered polarity. A squirrel-cage "winding" is imbedded in the pole faces of the magnets and acts as an induction motor to bring the rotor into synchronism, *i.e.*, gives it the speed of the rotating magnetic field of the stationary armature and thereby keeps the torque due to the energizing current always acting in the same direction. When the rotor begins to gain or lose speed relative

to the revolving magnetic field of the stator, the magnetic field cuts the squirrel cage and the current induced by the cutting evokes a torque that brings the rotor back into synchronism.

Single-phase induction motors and various modifications of those described above are beyond the scope of this text.

Electric motors transform electrical energy into mechanical.

6. Motor Generator and Rotary Converter.—It is often desired to use a potential difference of 110 volts when the mains are furnishing some higher potential difference, usually 500 to 1,500 volts. The transformation is accomplished by operating a motor with the potential difference furnished by the mains, which in

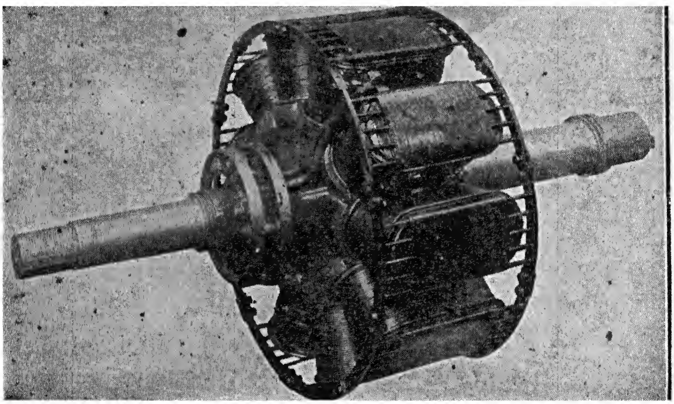


Fig. 16.—**Rotor** of a synchronous motor showing the field magnets and the imbedded squirrel cage. (*General Electric Co.*)

turn operates a generator constructed to give the desired voltage. The motor and generator are usually either on the same shaft or on coupled shafts shown in Fig. 17. Motor generators may be constructed to transform any voltage, direct or alternating, to any other desired voltage within the proper limits.

A *rotary* or *synchronous converter* is a form of motor generator which consists of a combination synchronous motor and a direct-current generator in which both motor and generator use the same armature windings. The alternating current is supplied through slip rings on one end of a rotating-type armature, and a direct current is taken by brushes from a commutator on the other end. These converters are used whenever a large amount

of power, transmitted by alternating currents, is to be converted to constant-potential direct-current mains, usually for use in electrolysis or the operation of electric trains.

7. Transmission of Electric Power.—It has been shown (Arts. 4, XVII–8) that, when a counter e.m.f. exists in a circuit, the power expended in the circuit, when the power factor is unity, is

$$P = EI = RI^2 + eI \text{ watts;}$$

where RI^2 is the power wasted in heating the circuit, and eI the power expended in forcing electrons against the counter e.m.f. The power expended against the counter e.m.f. appears in other

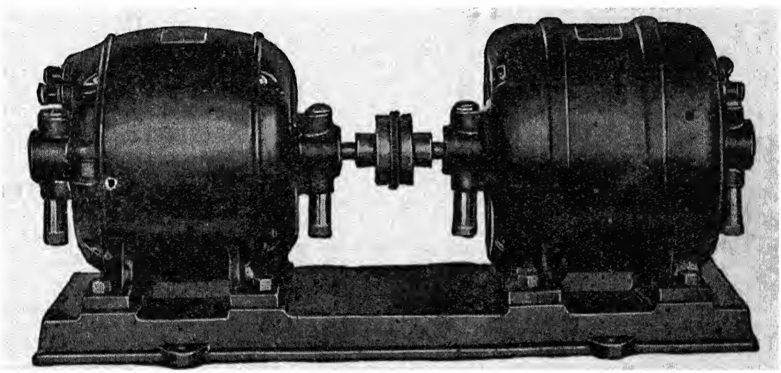


FIG 17—Small motor-generator. *(Central Scientific Co.)*

forms of energy, such as mechanical work, chemical change, "energy of a magnetic field," induced current in a neighboring circuit or in the secondary of a transformer.

The foregoing equation shows that a higher impressed e.m.f. requires a smaller current to transmit the same amount of power and that the power wasted in heat increases as the square of the current, from which it follows that it is more economical, by far, to use high e.m.fs. in the transmission of electric energy. Since the power supplied to a circuit is EI watts, the same power, 100,000 watts, for example, is supplied by 100,000 volts and 1 amp as by 1,000 volts and 100 amp. If the resistance in the circuit is 5 ohms, the power lost, RI^2, in the first case is 5 watts; in the other case, it is 50,000 watts. This shows clearly the necessity for using as high e.m.fs. as possible for long-distance

transmission. Those in general use vary from 60,000 to 260,000 volts. Larger voltages, at present, are not practical on account of the losses due to the corona (Art. XI–8) and to leakage over the insulators. The manner in which transmission lines are insulated from their supports is shown in Fig. 18.

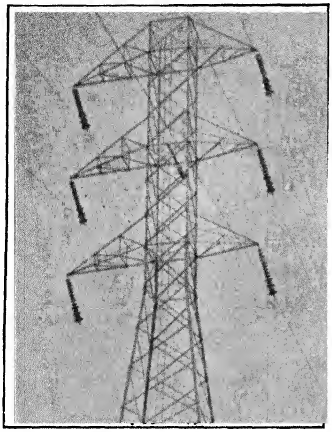


FIG. 18.—Section of a tower showing insulators supporting two 220,000-volt three-phase transmission lines. (*General Electric Co.*)

Since alternators generating more than 13,200 volts are not practical, a step-up transformer is used to obtain the desired high e.m.f. Transformers are usually immersed in oil and, because of the danger from such high voltages, are protected from easy access.

The fact that alternating e.m.fs. are so easily and economically transformed accounts for the use of alternating currents in preference to direct currents for the transmission of electric power.

In Fig. 19, a single-phase e.m.f. is first stepped up for long-distance transmission and then stepped down for safe use in factories and residences. Potentials above 110 volts are likely to be dangerous to life and usually are not led into buildings. Even less than 110 volts may be dangerous under proper contact conditions.

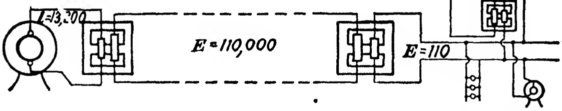


FIG. 19.—Transmission of electric power with step-up and step-down transformers.

It is customary, at the outskirts of a city, to step down the high-transmission e.m.f. to 13,200 volts. This then is led in underground conduits to convenient points within the city. At these points the e.m.f. again is stepped down to 2,300 volts, and finally, near the points where it is to be used, to 115 volts.

The arrangement of Fig. 20, however, is frequently used for the last transformation, which enables the user to employ an e.m.f. of 220 volts for power purposes.

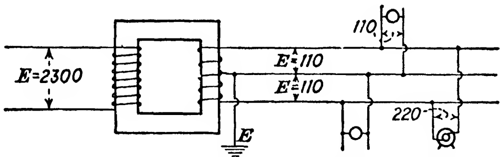


FIG. 20.—A three-wire system for distributing electric power.

The general practice, as already stated, is to employ the three-phase system in high-tension transmission lines. The mains,

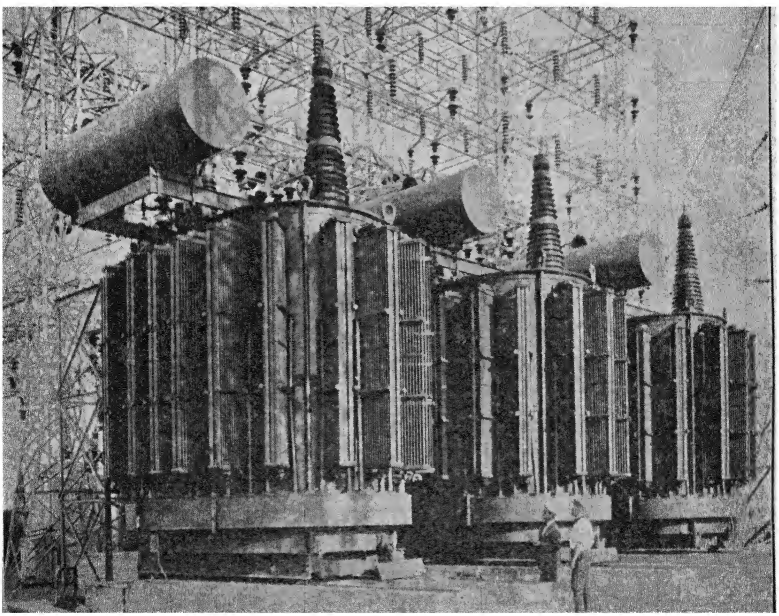


FIG. 21.—220,000-volt outdoor transformer substation showing three single-phase transformers for a three-phase transmission system; (note cooling radiators on the sides of the transformers and oil reservoirs on top). (*General Electric Co.*)

1, 2, 3, Fig. 22, have equal effective potential differences between any two of them, as described in Art. 3. Each of the primaries of three single-phase transformers or of one three-phase transformer is connected to two of the mains. Each secondary

then has impressed upon it a single-phase e.m.f., which is transmitted to the consumer. The connections, however, are often made as shown in Fig. 23, where A, B, C are the primaries of three single-phase transformers, and a, b, c the secondaries. The three mains from the secondaries supply a three-phase e.m.f. either to an extension of the mains or to the consumer. The three single-phase transformers of a large three-phase system are shown in Fig. 21.

The current in the principal mains is kept, as nearly as is practicable, in phase with the impressed e.m.f. This is accom-

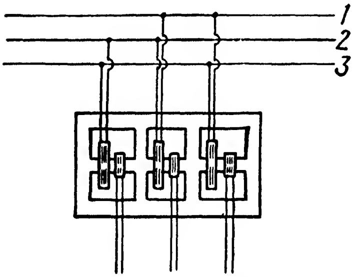


FIG. 22.—Three-phase transformer with connections to single-phase supply mains, showing six leads to the supply mains in place of the usual three.

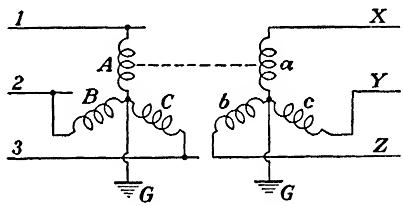


FIG. 23.—The primaries A, B, C of three transformers connected to the mains 1, 2, 3 by only three conductors, and the secondaries a,b,c which energize the three-phase mains X, Y, Z.

plished either by the introduction of capacitance across the mains or by the insertion of an overexcited synchronous motor carrying no load. Such a motor, when so used, is called a *synchronous condenser* because it takes the place of a condenser.

Questions

1. Derive the equation for the induced e.m f. when a loop is rotating in a uniform magnetic field. When a coil of N turns is rotating.

2. Why are the coils of the armature of a generator wound on an iron cylinder? Why are they set in slots? Why is the cylinder made of thin varnished disks? What is meant by hysteresis and eddy-current losses?

3. What is the direction of the eddy current in the armature of a generator?

4. Why is the field of a generator usually made strong enough to magnetize the iron well above the knee of the magnetization curve?

5. Give the five different methods of exciting the field of a direct-current generator.

6. Without the aid of a magnetizing current from the outside, how is it possible for an e.m.f. to be induced in a generator when the rotor first begins to turn?

7. What is an armature? A drum armature?

8. Since each coil in the armature of a direct-current generator produces a rectified pulsating current, how is the almost uniform current produced?

9. What is the highest voltage for which commercial direct-current generators are now constructed? How can a higher voltage be obtained from such units?

10. What factors cause the loss of energy in a generator?

11. Define the commercial efficiency of a generator. What is the efficiency of commercial generators?

12. Explain the construction and action of an alternating-current generator of (*a*) rotating-armature type and (*b*) stationary-armature type.

13. Why is the rotating field magnet made of many + and − poles?

14. What is the highest voltage now practical in commercial alternators?

15. What construction makes the generation of such high voltages practical?

16. What potential difference do the larger commercial alternators furnish?

17. How is this alternating p.d. raised to still higher values required for supplying power to transmission lines?

18. What are three-phase currents? How are they produced?

19. Explain how the three-phase currents supply energy by three wires in place of six. Is the effective potential difference between any two of the wires the same as that between any other two?

20. Explain the action of a direct-current motor. In what manner does it differ from a generator?

21. Explain how the counter e.m.f. in a motor is produced. How does it increase with the speed of rotation?

22. If there were no friction or other losses of energy in a motor, what would be the final magnitude of the counter e.m.f. when the motor is running without load? What would be the current and the energy used?

23. Develop the equation showing how the total electric energy furnished a series motor is distributed between the heating of the armature and field winding and the mechanical output. Of what factors is the mechanical output composed?

24. About how large is the resistance of an armature winding?

25. Why is a starting resistance necessary?

26. Explain how the speed of a direct-current motor is controlled, and give the theory involved.

27. At what efficiency do commercial motors usually operate?

28. Explain the action of a single-phase motor. Why are its field magnets laminated?

29. Explain how a rotating magnetic field is produced by means of two- or three-phase currents.

30. Explain the action of an induction motor.

31. Explain the action of a synchronous motor.

32. Explain the action of the motor generator and the rotary converter.

33. Explain fully why high e.m.fs. are used in transmitting electric power. How are these obtained? Why are not still higher ones used? Why are alternating e.m.fs. used?

34. Show how the high-tension alternating e.m.f. is changed to a low-tension e.m.f. and how that is carried to the consumer.

35. Draw diagram showing how the e.m.fs. in a three-phase system are transformed by the use of three single-phase transformers.

Problems

1. A direct-current generator is maintaining a potential difference of 115 volts between its terminals. The resistance of the two mains together to the point where a motor is taking the current is 0.5 ohms. (*a*) What is the power loss in the mains when the motor is taking 20 amp? (*b*) What is the power taken by the motor? (*c*) How much does it cost the consumer to operate the motor for 8 hr when electricity for power sells at 3 cts/kw-hr?

2. A 5.5-kw direct-current generator, operating at full load and maintaining a potential difference of 110 volts between the mains at the generator, has 20 ohms in its shunt field coils and 0.03 ohms in its armature windings including the brushes. (*a*) What is the current in the field coils? (*b*) What percentage of the generated power is expended in field excitation? (The 5.5 kw includes the power lost in the generator.)

3. A direct-current motor whose shunt-field winding has a resistance of 23 ohms and whose armature resistance is 0.03 ohms is using 100 amp from 115-volt mains. (*a*) What power is taken from the mains? (*b*) What is the counter e.m.f. in the motor? (*c*) What would be the current if the armature were at rest?

4. A step-up transformer, connected to 115-volt mains, has 150 turns in the primary and 1,500 in the secondary. (*a*) What is the potential difference between the terminals of the secondary coil? (*b*) When the current in the secondary circuit increases 2 amp, what is the increase in the current of the primary, neglecting losses in the transformer?

5. Two wires of long-distance transmission mains have a combined resistance of 20 ohms. The potential difference of 60,000 volts is in phase with the current of 100 amp. (*a*) What is the total power supplied at the transmitting station? (*b*) What percentage of power is lost in heating the mains? (*c*) Neglecting losses due to imperfect insulation, eddy currents, and hysteresis, how much power is being used? (*d*) What is the counter e.m.f. in the primary of the transformer? (*e*) What would be the power supplied at the transmitting station if the current lags 30° behind the impressed e.m.f.?

6. Assuming that the mains could withstand the heat generated in them, what would be the percentage of power loss due to the heating of the mains in Prob. 5 if the 6,000 kw were transmitted (*a*) when the potential difference is 220,000 volts? (*b*) When the p.d. is 110,000 volts? (*c*) When it is 12.000 volts?

Experiments

1. A coil attached to a segmented commutator shows the presence of a pulsating current when rotated in a magnetic field.

2. Faraday's disk used as a generator and as a motor.

3. Simple direct-current generator and motor.

4. Direct-current motor with starting resistance.

5. Rotating magnetic field produced by two-phase and by three-phase currents shown by means of a magnetic needle.

6. A rotating magnetic field due to these currents causes a metal beaker to rotate. Induction motor.

7. Various generators and motors shown.

CHAPTER XXIV

HEATING, LIGHTING, MEASUREMENT OF TEMPERATURE

1. Heating by Electricity.—The energy converted into heat when either a direct or an alternating current is energizing a circuit (Arts. IX–3, 6, 9; XVII–8) is

$$W_J = J_J H = RI^2 t \text{ joules,}$$

from which

$$H = \frac{RI^2 t}{J_J} = \frac{RI^2 t}{4.18} \text{ calories.}$$

This equation can be solved for the amount of heat generated in any part of a circuit or for the length of time required to generate any given quantity of heat in a given resistance when the current intensity is known.

Electricity usually sells at about 3 cts/kw-hr; but when used for heating purposes, its cost may be as low as 1 cent. Even at this low price it is too expensive for heating houses; however, electricity is used extensively for cooking. It is estimated that for a family of five persons the cost of cooking is as many dollars per month as electricity sells at cents per kilowatt-hour.

Electricity is used for heating flatirons, mangles, toasters, small heaters, soldering irons, etc., and in medical treatment it is used to heat tissues both by external heating and by passing or inducing high-frequency currents through any desired muscle.

Electricity is used for the production of high temperatures in the following various kinds of furnaces:

1. *Resistance Furnace.*—The resistance furnace consists of a coil of high-resistance alloy wound about a tube and protected by insulating material to lessen the loss of heat by conduction and radiation. Its use is practical for temperatures up to 1000°C but may be employed for somewhat higher temperatures. Nichrome (Ni, Fe, Cr, 60:24:16) is one of such alloys. Carbon

resistors are also used, from which temperatures over 2800°C may be attained in a vacuum.

2. *Arc Furnace.*—When two carbons are first brought into contact to close an electric circuit and then are separated, the gap between them becomes filled with ions of gases which, moving in opposite directions, maintain the current. The energy of the moving ions is transferred to the carbons by impact. The positive carbon is the hotter because of the greater energy of the lighter negative ions and attains a temperature of some 3500°C. The arc furnace is used for melting refractory substances and in certain chemical processes requiring temperatures up to 2800°C.

3. *Induction Furnace.*—High-frequency currents (Art. XXVIII–9) induce eddy currents in the body of a conductor and generate enough heat to melt the material. Nonconducting substances are melted by inducing currents in a conducting container. Temperatures of 3200°C are attained by this furnace.

The highest attainable temperatures are produced by discharging a large condenser through a short thin wire. The wire explodes and, from a study of the spectrum, temperatures of the order of 20,000°C appear to be produced.

2. Lighting by Electricity.—A body begins to emit red light at a temperature of about 525°C. The electrons in the outer energy levels (Art. XXVII–1) receive by the impacts of atoms the energy required to carry them into still more distant levels. The replacement electrons then falling from the higher levels emit their excess quanta of energy in the form of short electromagnetic waves that produce the sensation of light. When the temperature reaches 1200°C, wave lengths of many periods are emitted and produce the sensation of white light.

The amount of energy radiated by a hot body increases with temperature according to the following law:

$$w = \sigma T^4 \text{ ergs}\cdot\text{cm}^2\cdot\text{sec}^{-1}.$$

The energy w varies as the fourth power of the absolute temperature. The value of the constant σ is 5.74×10^{-5} for black-body radiation.

It is noticed that a body when heated gives off at first only radiant heat, then red light, then orange, and finally white. As the temperature increases, a greater and greater part of the

radiant energy appears in the shorter wave lengths. A spectrum of the radiation can be obtained by passing the radiant energy through a rock-salt prism. This prism, which does not absorb the heat rays, deviates the shorter wave lengths more than the longer ones. The spectrum, in which each part possesses the energy contained in waves of a particular wave length, may be thrown on a screen. A more uniform separation is produced by means of the diffraction grating. The spectrum may be studied by measuring the temperature in the different sections of it, including those that are invisible. The point in the spectrum having the greatest energy is determined for each of many spectra obtained from the radiations emitted by a body at different temperatures. The following relationship is found:

$$\lambda_m T = b \text{ cm}^\circ;$$

i.e., the product of the wave length at the maximum energy point of the spectrum and the absolute temperature of the radiating body is a constant. The magnitude of this constant is 0.2883.

The first radiation equation shows that the amount of radiant energy increases greatly with increase of temperature, and the second equation shows that the higher the temperature, the more the radiation shifts toward the shorter wave lengths. Both laws point to the advantages of using as high a temperature as is practicable for the commercial production of light by electricity.

3. Incandescent Lamp.—The incandescent lamp consists of a filament electrically heated to a high temperature in an evacuated bulb or in a bulb containing an inert gas (nitrogen or argon). Since all metals sublimate at high temperatures in a vacuum, it is impractical to let the temperature of the filament approach the melting point of its constituent metal. When the bulb is filled with an inert gas, this sublimation is retarded, making it practical to heat the filament to a higher temperature. Even though energy is lost by conduction and convection of the heat through the gas, the efficiency of the lamp is increased. Tungsten has been found to be the most practical material for the filament.

The relative efficiencies of the different types of lamps are shown in Fig. 1.

The nitrogen lamp (Mazda *C*) is by far the most efficient, but even in this form only 20 per cent of the radiant energy is light energy, and only 2 per cent of the energy of the coal used becomes that of the emitted light. Light is only a by-product of heat, 98 per cent of the energy of coal being wasted to obtain the 2 per cent in the form desired.

4. Arc Light.—The electric arc was considered in connection with the arc furnace (Art. 1). The positive carbon is heated to a high temperature and gives a great amount of light. The arc light had been used for lighting purposes. Its efficiency

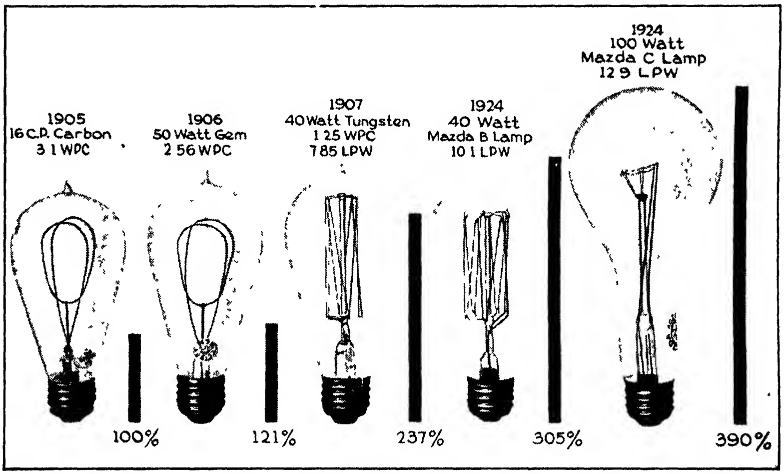


FIG. 1.—Relative efficiency of lamps. (*General Electric Co.*)

is practically that of the nitrogen lamp; but since the cost of maintenance is greater, it is now practically obsolete for general lighting. It is used in projection lamps because it gives an intense light that is localized, a condition desirable for good projection of pictures. The arc gives a large amount of ultra-violet light, which is of service in medicine.

5. Mercury-vapor Lamp.—The mercury-vapor lamp consists of an exhausted tube with mercury as one of the electrodes. When the lamp is tilted for an instant, the mercury connects the two electrodes. In flowing back, the mercury opens the circuit and thereby produces an instantaneous arc. This arc forms mercury vapor, which upon ionizing gives intense light and maintains the

flow between the electrodes. The vapor is replenished by the heat generated at the mercury electrode and the ionizing electrons are supposed to pour out of the heated mercury owing partly to the high potential gradient at the surface. This lamp gives an efficient light, which, however, lacks the red color. The light is rich in ultraviolet rays, which are transmitted when the lamp is made of quartz. This ultraviolet light is injurious to the eyes but is of service in medical treatment.

6. Thermocouple.—The action of the thermocouple was considered in Art. XV–8. The e.m.f. established in the circuit of a thermocouple per degree of temperature difference is of the order of 0.00004 volts. The currents produced, however, may be large enough so that a difference of 0.001°C in temperature produces an appreciable deflection on a sensitive galvanometer.

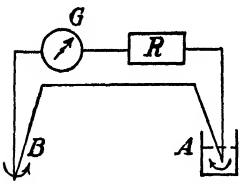


FIG. 2.—Thermocouple circuit.

If a galvanometer and a resistance box are placed into a thermoelectric circuit, Fig. 2, and the temperature of the junction A is kept at 0°C, the resistance R may be adjusted until the temperatures to be measured, at B, give deflections of the desired magnitude. The deflections for several known temperatures of the junction B are taken and plotted. The resulting curve gives the temperature of the junction B corresponding to any deflection. An unknown temperature, then, is determined by means of the curve from the deflection it produces. With the aid of this curve a scale may be drawn whose readings give temperatures directly.

It is found that the degree intervals on such a scale are not of the same length but change uniformly from division to division; therefore it is necessary to keep the junction A always at 0°C when readings are being taken. The same temperature difference, therefore, does not produce the same deflection if the temperature of junction A is changed.

7. Thermoelectric Diagram.—The characteristics of a thermocouple are best represented by means of a thermoelectric diagram as shown in Fig. 3 and explained in Art. XV–8. The e.m.f. per 1° of temperature difference developed in a circuit composed of

any particular metal coupled with lead is plotted on the axis of ordinates against the average temperature of the junctions. Let the metal B be coupled with lead and the observed e.m.f. per 1° difference in temperature at the temperature t_1 be represented by the distance ab. Let this be repeated for several temperatures, such as t_2, where the e.m.f. per 1° of temperature difference is represented by the line mn. A line B, drawn through all the points such as b and n, is found, in general, to be a straight line. It represents the Seebeck e.m.f. (Art. XV–8) per 1° difference in temperature for all temperatures when the metal B is coupled with lead. This is called the *thermoelectric diagram* for the metal B.

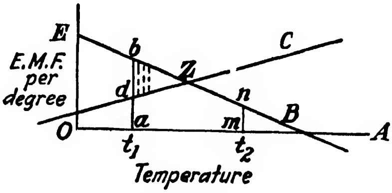


FIG. 3.—Construction of the thermoelectric diagram.

The diagram also gives the data for determining the e.m.f. for any temperature difference between the junctions. If it is desired to know the e.m.f. for the temperature difference $t_2 - t_1$, all that is necessary is to multiply this number of temperature degrees by the average e.m.f. per degree as given for those temperatures on the thermoelectric diagram. The area $abnm$ then represents the e.m.f. in the circuit when the temperatures of the junctions are t_1 and t_2.

Similarly, the lines C and A form the thermoelectric diagram for the metal C coupled with lead.

The thermoelectric diagram, in addition to showing the e.m.f. per degree of temperature difference for each of the metals with lead, shows the e.m.f. per degree for any two metals. For example, when the metals C and B form the thermoelectric circuit, the distance bd represents the e.m.f. per degree of temperature difference at t_1° and is called the *thermoelectric power* of the two metals at that temperature. The thermoelectric power of a metal coupled with lead is usually called *thermoelectric height.*

Let the broken lines to the right of bd be 1 temperature degree apart. The area each adjacent pair subtends between the boundaries of the lines B and C represents the e.m.f. in the circuit for 1° of temperature difference. If one junction remains at the

temperature t_1, and the other increases, each increment of 1° in temperature adds to the circuit an e.m.f. represented by the area of the space between the successive broken lines. In the illustrated case the successive increments per degree diminish

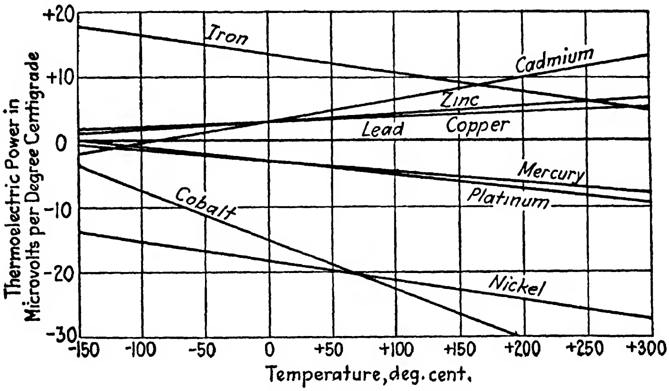


FIG. 4 —Thermoelectric diagram.

in magnitude until finally they become zero. Beyond that point each additional increase in temperature causes the e.m.f. in the circuit to decrease. Finally, the decrements are together sufficient to make the total e.m.f. in the circuit zero. Further increase of the temperature reverses the current as shown in the curve of Fig. XV–12. Metals that are used for the measurement of temperatures have the crossing of the lines at a temperature far beyond the range of temperatures to be measured. For many substances this point of crossing is beyond any attainable temperature. The thermoelectric diagram for several metals is given in Fig. 4.

8. Thermoelectric Scale.—The thermoelectric diagram for two metals gives the data for the construction of a scale for measuring the temperatures of the distant junction. The increment or decrement in the successive e.m.f. lines, drawn 1° apart in the thermoelectric diagram, is found from the diagram. The desired length of a division is arbitrarily determined for some part of the scale and the magnitude of the decrement or increment determined in terms of it. Successive scale divisions are then drawn, each differing in length from the preceding one by this amount.

It is a common practice to shift the variably graduated scale, when the circuit is open, to indicate the temperature of the "cold" junction, whose temperature is determined by a mercury thermometer. The circuit is then closed, and, if the instrument is being adjusted, the proper resistance is placed in the circuit so that the deflection reads the temperature of the distant junction. Thereafter, without further adjustment of the resistance, the galvanometer indicates correctly the distant temperature whatever it may be. The shifting of the scale does away with the necessity of keeping one junction at a constant temperature.

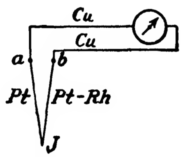


FIG. 5.—Three-metal thermocouple circuit.

The temperatures may be determined more accurately by measuring with a potentiometer the e.m.f. produced by the unknown temperature difference between the junctions. The temperature is then obtained from a calibration curve. The potentiometer may also have its scale marked to indicate temperatures directly.

9. Three-metal Thermocouple.—A three-metal thermocouple circuit usually is employed when the temperatures to be measured are higher than 300°C. For precision measurements, the two metals which form the hot junction are platinum and an alloy of platinum and rhodium (90:20). For commercial use cheaper heat-resisting alloys are employed. In either case it is impractical to continue either one of the materials throughout the whole circuit. The galvanometer, therefore, is connected by copper leads to the two wires that form the "hot" junction, as shown in Fig. 5. It is necessary to have the points at which the copper joins the two hot-junction wires kept at the same temperature. The e.m.f. in the circuit is then the same as if the circuit consisted of two junctions at the temperatures of the junctions *ab* and the junction *J*.

10. Practical Applications of the Thermocouple.—The thermocouple has been developed for the measurement of temperatures and is especially useful for the measurement of temperatures above and below those to which a mercury thermometer is applicable, for temperatures of points in inaccessible places, and for the measurement of small temperature differences.

Radiant energy coming from molten metal when focused on one of the junctions heats it and thereby produces a deflection on the galvanometer. This is the essence of the extensively used *radiation pyrometer* which can be made so sensitive as to measure the temperatures of different parts of even the spectra of stars.

The thermocouple is also employed to operate automatically a relay which controls a supply of heat.

For many purposes several junctions are placed in series in order to increase the e.m.f. An e.m.f. large enough to operate an electric bell may be readily obtained.

11. Resistance Thermometer.—The resistance of platinum changes nearly 0.4 per cent per degree centigrade. A properly protected coil of platinum wire, therefore, may be used for the measurement of temperatures. A temperature-resistance curve is drawn from the measured resistance of the coil at various standard temperatures. The unknown temperature is then determined by reference to this curve or by the use of an equivalent formula.

When the platinum resistance coil is placed in one arm of a Wheatstone bridge which contains an appropriately marked dial for varying a part of the resistance in one of the arms, the position of the dial, at balance, can be made to indicate the temperature of the platinum coil in degrees. The same change in temperature always requires the same alteration in the dial resistance.

Questions

1. Derive the expression for the heat generated in any part of an electric circuit.

2. Describe the resistance furnace, arc furnace, and the induction furnace.

3. Give the two radiation laws bearing on the efficiency of incandescent electric lamps. How does the radiant energy vary with the absolute temperature? How does the position of the region in the spectrum in which exists the maximum energy vary with the absolute temperature?

4. What factor other than the melting point limits the increasing of the temperature of the filament in an incandescent lamp?

5. Why is the nitrogen-tungsten lamp more efficient than the vacuum-tungsten lamp?

6. How much of the radiant energy emitted by the nitrogen-tungsten lamp is light energy? How much of the energy in coal, at present, can be converted into energy of light?

7. Describe the arc light; the mercury-vapor lamp.

8. What are the practical uses of the thermocouple?

9. Explain how a galvanometer scale may be made to indicate temperatures.

10. Does the thermoelectric current cause the galvanometer to indicate temperatures or temperature differences?

11. Explain the thermoelectric diagram, and state what information it gives.

12. Explain why in some thermocouples the current may increase with the temperature of the hot junction and then decrease and even reverse.

13. Explain how a scale for reading temperatures is constructed with the aid of a thermoelectric diagram.

14. Show how and why three metals are often used in a thermoelectric circuit.

15. How can the galvanometer be made to read temperatures directly without keeping one of the junctions at a fixed temperature?

16. Give some of the practical applications of the thermocouple.

17. Give the principle of the resistance thermometer.

Problems

1. It is desired to heat 5 liters of water 10°C. A coil of wire having a resistance of 10 ohms is inserted into the water and connected directly to 115-volt direct-current mains. How long will it take to heat the water?

2. The thermoelectric diagram for the two metals of a thermocouple gives for the point marked 20° a reading of 0 000040 volts/degree centigrade, and for 45°, 0.000036 volts. What is the e m.f. in the circuit when the junctions are one at each of these temperatures?

Experiments

1. Heating devices.

2. Carbon, tungsten, and nitrogen-tungsten lamps.

3. Carbons of arc light projected on screen.

4. Radiation pyrometer.

5. Commercial thermocouples.

6. Resistance thermometer.

CHAPTER XXV

PHENOMENA IN GASES AT LOW PRESSURES

1. Vacuum.—A gas at atmospheric pressure and 0°C contains 27×10^{18} molecules/cm³. The pressure on the walls of the containing vessel is due to the impacts of these molecules. When the pressure is reduced to one-millionth of an atmosphere, the "vacuum" still contains

$$n = \frac{N}{10^6} = \frac{27 \times 10^{18}}{10^6} = 27 \times 10^{12} \text{ molecules/cm}^3.$$

This number of molecules per cubic centimeter in such a vacuum is still more than 15,000 times the human population of the earth. The best vacuum attainable has a pressure of 5×10^{-9} mm of mercury and contains 177×10^6 molecules/cm³, which number is about one-tenth the human population of the earth.

2. Discharge of Electricity in Gases at Low Pressures.—The ions which always are present in any gas (Art. XI–7) are accelerated by an electric field and move an average distance of 7.6×10^{-6} cm before their acquired velocities are checked by collisions with neutral molecules. This distance is one-half that which the best microscope can resolve. After successive impacts, an individual ion finally reaches an electrode, the + ion having had an average velocity of 1.36 cm/sec for each volt per centimeter of potential gradient, and the − ion, 1.87 cm. The electrons which are freed in the process of natural ionization are, on account of their smaller mass, given a much greater velocity than the heavier ions and move in the same direction as the negative ions. When the potential gradient is 30,000 volts/cm, or more, each electron acquires between successive impacts with neutral molecules a velocity large enough to ionize some of the molecules through which it passes. Such a velocity is called an *ionizing velocity*. In this manner great numbers of ions are formed which, moving in reverse directions, cause a continuous

discharge of the electrodes. The air then conducts electricity like an electrolyte and as already stated (Art. XI–8) is said to be conducting.

If the gas pressure is diminished, the free electrons move longer distances before striking neutral molecules and therefore acquire a larger velocity between impacts. A smaller potential difference then gives them a velocity large enough to ionize the gas and to make it conducting. The greater the exhaustion of any tube, the smaller is the potential gradient required to produce an amount of ionization large enough for the passage of an appreciable current; but when the number of molecules becomes so small that not enough of them are properly struck to be ionized, the current begins to diminish.

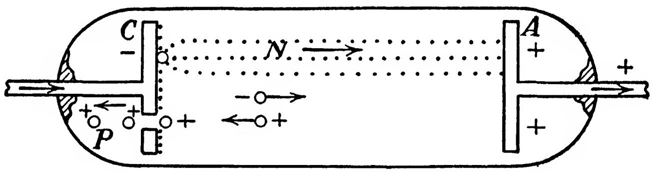


FIG. 1 —Production of cathode and canal rays.

An atom when ionized or recombining (Art. XI–7) is subject to a rearrangement of its orbital electrons. This transfer causes the electrons to send out electromagnetic waves (Art. XVIII–4) of the frequency of light. Ionization of a gas, therefore, when sufficient in amount, is accompanied by luminosity.

If the exhaustion is continued to 10^{-6} atmos, many of the ions move the whole distance between the electrodes without striking a molecule. The + ions on striking the negative electrode C, Fig. 1, knock out some of the excess electrons, which are then forced by the electric field to move across the space to the + electrode A. In this manner a stream of electrons N is produced in which the electrons acquire a great velocity before reaching the + electrode. In order that this action may continue, the exhaustion of the tube must not be too great, for a sufficient number of ions must continue to be produced by impacts to supply ions for the further ejection of more electrons. Such streams of electrons are called *cathode rays*. The negative electrode at which they are produced is called the *cathode*, and the positive electrode the *anode*.

If the negative plate is pierced by a hole, as shown in the figure, some of the attracted + ions in place of striking the plate pass through it into the space to the left. In such an apparatus a stream of electrons flows in one direction and a stream of positive ions (*canal rays*) in the other. A tube containing neon molecules has the paths of both streams plainly visible in different colors.

3. Cathode Rays.—The stream of electrons ejected from the negative electrode, as noted above, is called *cathode rays*. The following facts concerning these rays are observed:

1. When the amount of gas in the tube is still considerable (0.001 atmos), the large numbers of ions being formed by the bombarding electrons show as a luminous streak whose spectrum is that of the gas in the tube.

2. The electrons of cathode rays on striking paddles on one side of a wheel cause the wheel to rotate.

3. The electrons give part of their energy to the object they strike, causing a heating effect.

4. The impinging rays cause various minerals, salts, and glasses to fluoresce.

5. The electrons move in straight lines, casting clear "shadows" of objects placed in their path.

6. When the electrons strike an intercepting plate which is connected through the tube to an electroscope, they charge the plate and the electroscope with negative electricity.

7. When the tube is connected in series with a galvanometer, the stream of electrons completes the electric circuit and the galvanometer deflects in the appropriate direction.

8. When the stream of electrons passes through an electric or a magnetic field, the stream is deflected in a direction such as negatively charged moving particles would deflect.

9. When cathode rays impinge on a substance of high atomic weight, electromagnetic waves of high frequency (x-rays) are produced (Art. XXVII–2).

From some of the foregoing observations, the deduction is made that these rays consist of negatively charged particles having mass. The cathode rays are more efficiently produced by obtaining the necessary electrons from a hot filament (Arts. 5, 9).

4. Saturation Current.—If an electric circuit is broken by a gap between two parallel plates, Fig. 2(*a*), the ions naturally present in the air are too few to produce an appreciable current. The number of ions, however, may be greatly increased by exposing the space to x-rays or radium emanations.

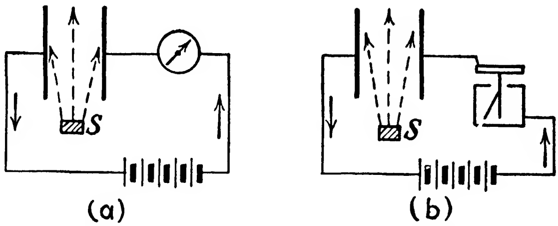


FIG 2.—Production of ionization current in air, the ionizing agent being placed at *S*.

With any given ionizing source, the current varies with the applied potential gradient as shown in Fig. 3. At first, when the rate of supply of ions is ample, the current varies as the potential gradient between the plates. The ions then are carried toward the plates too slowly to affect appreciably the

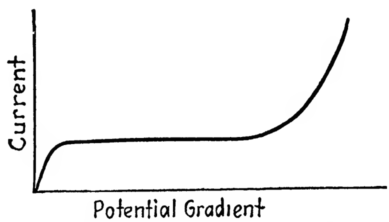


FIG 3.—Saturation current and ionization at high potential gradients.

condition of equilibrium at which the recombination of ions equals that of formation (Art. XI–7). With larger potential differences, the current does not change with the potential difference, because all the ions are drawn to the plates as fast as they are formed. The higher potentials cannot draw a larger number to the plates than is produced by the ionizing agent. [illegible] said to be a *saturation current.* At still higher potential differences, the current begins to increase very rapidly with increase of potential difference until finally a spark takes place. This increase in the current is due to the electrons

which are freed in the process of natural ionization acquiring a large enough velocity between impacts to ionize neutral molecules. The larger the number of ions formed in this manner, the greater is the increment to the saturation current.

The saturation currents are so small under ordinary conditions that it is necessary to replace the galvanometer with an electrometer or an electroscope as shown in Fig. 2(*b*). The current then is measured by the rate at which the electroscope discharges.

5. Emission of Electrons from Hot Bodies—Two-electrode Vacuum Tube.—If a wire near a charged electroscope is heated to incandescence by an electric current, ions of both kinds are formed in the neighborhood of the wire; one kind is attracted by the charge of the electroscope and discharges it. The exact manner in which these ions are formed is not understood.

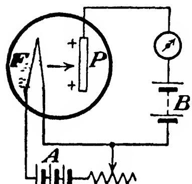


FIG. 4.—Two-electrode vacuum tube (thermionic tube).

At ordinary temperatures the free electrons in a metal, although assumed, for convenience, to be moving about like the atoms of a monatomic gas, do not escape from the metal (Art. II–4). At high temperatures, however, the fastest of these have sufficient velocity to carry them through the surface against the attraction of the equal opposite charges in their immediate neighborhood.

When a heated filament is in a vacuum, a cloud of electrons then pours out of it; and the space in its neighborhood becomes charged negatively, more and more, until finally further flow is prevented by the repellent action of the cloud. Such a cloud of electrons, called *thermions* to distinguish them from electrons originating from other sources, is shown about the filament *F*, Fig. 4. If a battery *B* is connected to the plate *P* as shown, the potential of the plate is higher than that of the hot wire. The electrons then flow toward the plate across the gap and thereby complete the electric circuit through the battery *B* and the galvanometer. The intensity of this electron flow increases with the increase of the applied potential difference between the filament and the plate until its magnitude becomes such that the electrons are drawn out of the neighborhood of the filament as soon as they appear. The current is then said to be saturated in the sense considered in Art. 4 and is the same in nature as that

of the cathode rays (Arts. 2, 3). Electrons flow much more readily out of a tungsten filament when it is coated with, or has incorporated in it, thorium oxide. When the emission current is saturated, its magnitude is

$$i = AT^2\epsilon^{-b/T}$$

where A and b are constants whose values depend on the material and nature of the surface, T = absolute temperature and $\epsilon = 2.718$.

An electron escaping from any metal acquires potential energy at the expense of kinetic. The magnitude of this potential energy in joules is

$$w_J = e\phi,$$

where e is the electron charge in coulombs, and ϕ the potential difference in volts against which the electrons force their way out of the metal. The magnitudes of ϕ for a few metals are

	Volts
Copper	4 0
Iron	3 7
Platinum	4 4
Tungsten	4.52

If the battery B is replaced by a source of alternating e.m.f., the current flows through the circuit only when the potential of the plate P is positive. The result is an intermittent current consisting of pulses in the same direction. These pulses are in such rapid succession that the galvanometer coil cannot follow them and therefore deflects an amount such as it would with the average value of this intermittent current.

This tube is known as a *vacuum-tube rectifier*. Two such rectifiers can be arranged to supply an almost uniform high-tension current and are used for supplying high-voltage direct currents for x-ray tubes.

6. Space Charge.—The electrons surrounding the hot filament, Fig. 4, constitute the so-called space charge of a vacuum tube. They lower the potential of that space and repel any electrons that tend to emerge from the filament; hence after the cloud is formed, the further outpour of electrons ceases. The cloud, tending to disperse (Art. IV–2), soon charges the walls of the

tube negatively, while the potential of the filament remains unchanged. When the battery B is attached as shown, the electrons of the cloud are accelerated toward the plate P and complete an electric circuit with the battery. If the e.m.f. of the battery is small, the electrons are carried away so slowly that only a comparatively small number flow out of the filament to replace those drawn to the plate. When the e.m.f. is increased, the outpour of the electrons is also increased. Finally when the plate potential reaches a certain value, the cloud is dissipated and the electrons are drawn from the filament as fast as they emerge. The current then has its maximum value for that filament temperature and is called the *saturation current* for that tempera-

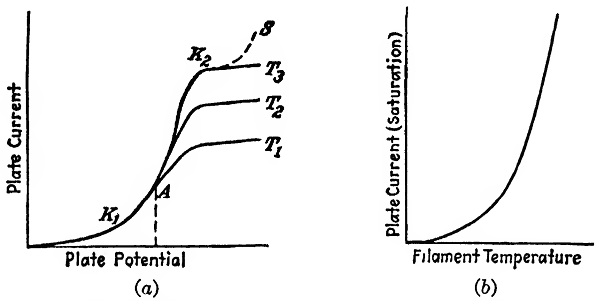


FIG. 5.—(*a*) Characteristic curves of the plate current for three different filament temperatures. (*b*) Saturation plate current for different filament temperatures.

ture. The characteristic curves showing the relation between the plate current and the plate potential for three different temperatures of the filament are shown in Fig. 5(*a*). The sharp bends K_1 and K_2 on a curve are known as the lower and upper *knees* of the curve.

If some air is present in the tube, the velocity of the electrons finally becomes sufficient to ionize it, and the additional current due to the moving ions is superposed on that of the electrons. This result is represented by the broken line S.

If the potential gradient remains constant, and the temperature of the filament changes, the plate current varies as represented in Fig. 5(*b*).

7. Three-electrode Vacuum Tube (Triode).—The three-electrode vacuum tube, Fig. 6, consists of a two-electrode tube, Fig. 4, with the addition of a grid G which consists of a wire

mesh placed between the filament and the plate. The grid and the plate are often in the form of concentric cylinders about the filament. The figure shows a tube connected to a tuned circuit S

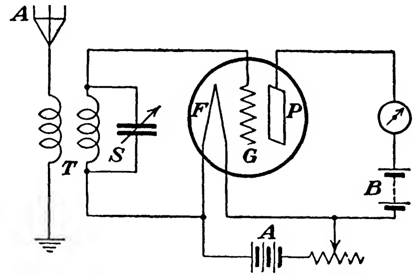


FIG. 6—Diagrammatic representation of a three-electrode vacuum tube and of its connections to the A and B batteries and to a source of alternating e m f.

which is energized by oscillations in the antenna A through the radio-frequency transformer T.

The curves of Fig. 5, which hold for this tube also, show the variation of the plate current with the plate potential. Figure 7

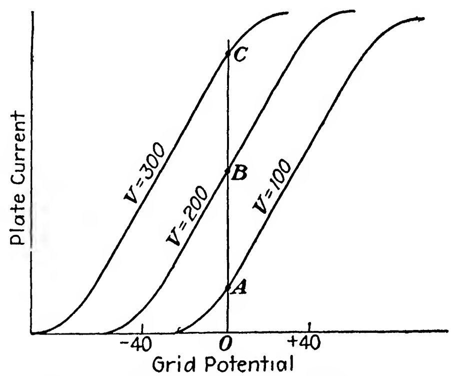


FIG. 7.—Characteristic curves showing the plate current for three different plate potentials (or B-battery adjustments) when using the same filament temperature and varying the grid potential drawn so as to show the relative positions of the knees when the "grid potential is zero."

shows the variation of the plate current with impressed grid potential for three different plate potentials. It should be noted that the three curves are similar and that they are shifted so that one crosses the zero grid-potential point at its lower knee,

one at the midway point, and one at its upper knee. This same shift may be obtained by placing a *C* battery (not shown) in the grid circuit. The steady potential it gives the grid has the effect of changing the plate potential as can be deduced by inspection of the curves in Fig. 7.

It is possible therefore by either one of the two means to adjust any tube circuit so that, when no alternating potential is energizing the grid, the characteristic curve has any desired one of the three standard positions *A*, *B*, *C* at which it crosses the "zero grid-potential" line of Fig. 7. If now an alternating potential is impressed on the grid as represented by *aa*, *bb* or *cc*, Fig. 8, the plate current has respectively the varying magnitudes A_1, B_1 and C_1 where the abscissas represent time.

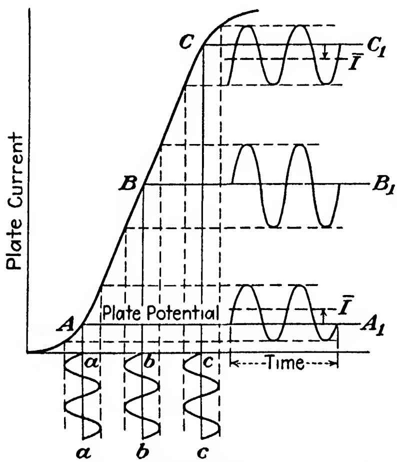


FIG. 8.—The three standard adjustments, Fig. 7, of a three-electrode tube are here represented together on the same curve by the points *A*, *B*, and *C*. In adjustment *A*, for example, an alternating e m f. represented by the curve *aa* when impressed on the grid results in an alternating plate current A_1 (for which the abscissas represent time).

When the grid *G* is charged to a + potential, it increases the strength of the electric field which gives the electrons their acceleration toward *P*. The plate current is increased thereby while the grid, being composed of fine wire with large interspaces, obstructs only a few of the fast-moving electrons in their motion toward the plate.

If the grid is charged negatively, it opposes the action of the plate *P* and diminishes or entirely stops the flow of electrons from the filament to the plate.

With the adjustment *B*, the increments and decrements to the plate current, due to an alternating e.m.f. impressed on the grid, are equal, so that the average value of the original current is unchanged. With the adjustment *A* the decrements are smaller than the increments, while with the adjustment *C*, they are larger, so that in one case the average value of the current is

increased and in the other decreased, as represented by the broken lines $\bar{I}$; the pulses then are said to be *distorted.*

When high-frequency alternations are impressed on the grid, the galvanometer in the plate circuit cannot follow the rapid fluctuations in the plate current but responds only to their average value. The telephone receiver responds to *audio frequencies* only, *i.e.*, up to about 10,000 oscillations/sec. Above that number the frequency usually is called *radio-frequency,* and neither the galvanometer nor the telephone responds to the individual pulses. The galvanometer and the telephone, however, both respond to a change in the average value of the current and therefore detect the presence of such alternating e.m.fs. impressed on the grid. The tube when adjusted for such response is called a *detector tube.* When the tube has the adjustment represented by B in Figs. 7, 8 where the pulses are not distorted, neither the galvanometer nor telephone responds to radio-frequencies; but if the detecting instruments are replaced by the primary of a high-frequency transformer, the pulsating current in the primary induces an alternating e.m.f. in the secondary which can produce a current with many times the energy of that energizing the grid. This induced e.m.f. may be applied in the same manner to a second tube and produce thereby another increase in energy. When such tubes are so used, they are called *amplifiers* from the fact that they give out more alternating-current energy than they receive. The extra energy, of course, is provided by the B battery. Radio-frequency transformers contain no iron because the magnetization of iron cannot follow radio-frequencies.

8. Current Rectifiers.—Current rectifiers are devices for transforming an alternating into a unidirectional current. The more important types are here described.

a. Mercury-arc rectifier consists of an evacuated glass or iron chamber, Fig. 9, with four electrodes. The cups containing the two lower electrodes contain mercury which, when the rectifier is tilted, closes and then breaks an electric circuit. The spark at the break vaporizes and ionizes a part of the mercury so that a continuous arc exists between the electrodes A and D. The figure shows the rectifier connected to a split secondary of a transformer and to a storage battery which is being charged by the rectified current. The polarity of the iron or carbon electrodes B and C

alternates so that each has first a higher and then a lower potential than electrode *A*. The arc between *A* and *D* probably heats a spot on the mercury surface to such a temperature that electrons pour out and are drawn to either of the electrodes *B* or *C*, and also a high potential gradient exists at the surface of the mercury which may extract electrons for the flow. These electrons are given high velocities and ionize the mercury vapor which ionized vapor then forms the main part of the flow through the tube. The electron flow is forced alternately through the circuits *X* and *Y*, but always in the same direction through the battery being charged or through any apparatus requiring a unidirectional current.

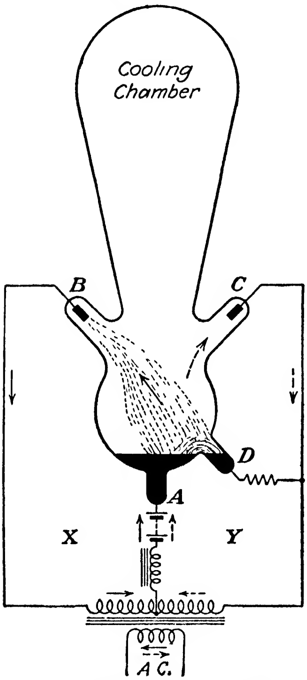


FIG. 9.—Mercury-arc rectifier.

b. The *vacuum-tube rectifier*, described in Art. 5, is used for obtaining high-voltage rectified currents. These rectifiers are made to carry currents as large as 10 amp.

c. The *tungar*, Fig. 10, consists of a bulb containing argon at a pressure of from 8 to 10 cm. The hot-tungsten filament electrode is heated by the transformer *T* and emits a cloud of electrons. These electrons are drawn toward the other electrode only whenever its potential is positive and on their way ionize the argon gas which then adds charged particles to the streams of the flow.

d. *Copper Oxide Crystal Rectifier*.—The contact surface between copper and the cone-shaped crystals of copper oxide conduct electrons many times more readily in one direction than in the other. Such crystals, therefore, can be employed as rectifiers of alternating currents.

To form good molecular contact between the crystals and the remainder of the circuit, an oxidized copper plate is treated to change the upper surface of the oxide to metallic copper. This

forms a plate of two layers of copper which protect and make electrical contact with a thin layer of copper oxide.

One such plate alone allows only one-half of the alternating-current pulses to pass. To rectify both pulses, four such plates

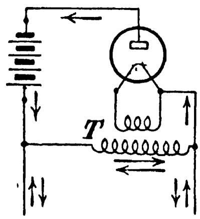


FIG. 10.—Tungar charging a storage battery.

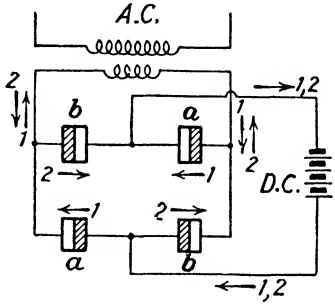


FIG. 11—Copper oxide rectifier The arrows indicate direction of electron flow.

are connected as shown in Fig. 11 and are faced so that the electrons can pass through them in the directions indicated by arrows. The alternating-current pulses flow in one direction through the plates *aa* and in the other through the plates *bb*,

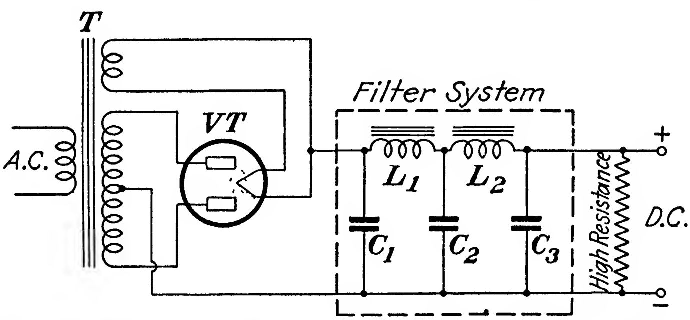


FIG. 12—Circuit for rectifier-filter system for obtaining a constant direct current from an alternating-current source.

both flowing in the same direction through the direct-current branch.

This form of rectifier is adapted for low potentials only and has been used with *A-battery eliminators* for supplying current to the filament of vacuum tubes but is now almost universally displaced there by an indirect-heating coil. The *B-battery*

eliminator, Fig. 12, contains a double high-vacuum two-electrode tube which is essentially two two-electrode tubes under one cover. These two component tubes are so connected, one in each branch of a divided circuit, as to rectify the alternating current. This rectified pulsating current passes through a *filter system* (combination of inductances and condensers) which "irons out" the undulations and thereby produces an almost perfectly uniform current or potential difference.

e. There are also various forms of mechanical and electrolytic rectifiers. The motor-generator and synchronous converter (Art. XXIII–6) are both rectifiers but usually, for obvious reasons, are not classified as such.

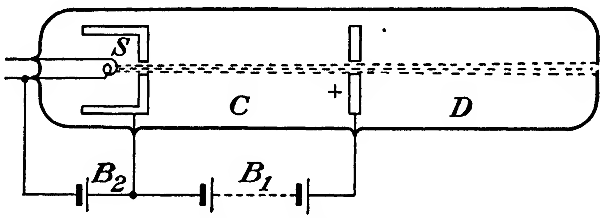


FIG. 13.—Electron gun.

9. Electron and Ion Guns.—The *electron gun* is an instrument for projecting a stream of electrons into an evacuated space with any desired velocity for experimental purposes. A simple form of the gun is shown in Fig. 13. The thorium-coated spiral S, in an evacuated chamber, is heated to incandescence by means of a battery (not shown). The emitted cloud of electrons finds its way into the compartment C across which the battery B_1 maintains a potential difference. The electric field in this compartment accelerates the electrons and thereby gives them a calculable velocity with which they are projected through an opening into the experimental compartment D.

The energy expended on each electron in giving it a velocity in the compartment C is

$$w = V''e'' = \frac{mv^2}{2}.$$

From which

$$v = \sqrt{\frac{2V''e''}{m}} \text{ cm/sec,} \tag{1}$$

where v is the velocity of the electrons as they are shot into the experimental compartment D. The apparent mass m of the electron varies with the velocity, so that at velocities greater than 0.1 that of light its normal value must be corrected by the relativity equation (Eq. XXVI–3).

The energy of a projected electron is measured by the p.d. through which the electron had been accelerated. An *electron volt* then represents the energy an electron possesses by virtue of having been accelerated through a p.d. of 1 volt. An electron volt $= 1.59 \times 10^{-12}$ ergs.

The *ion gun* differs from the electron gun only in the parts required to supply ions in place of electrons. A simple form of

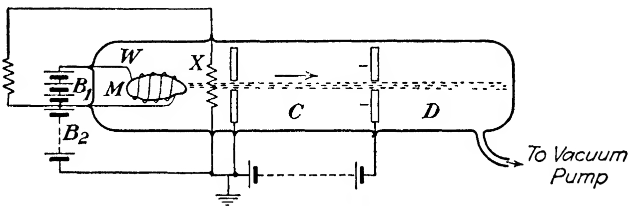


FIG. 14 —Ion gun.

such a gun is shown in Fig. 14. The metal M is vaporized by the heated part of the wire W. The part of the wire X within the evacuated vessel is thorium-coated tungsten and, when heated to incandescence, supplies a cloud of electrons. These electrons are given an ionizing velocity by the p.d. established between the wires X and W by the battery B_2 and ionize part of the metal vapor. The + ions of the metal enter, with negligible velocity, the accelerating compartment C, where they are given the large velocity with which they are projected into the experimental compartment D.

The electron gun is used for the production of x-rays and for the study of the atom outside the nucleus; the ion gun is employed chiefly for the study of isotopes and for the disintegration of the atomic nucleus.

An electron gun having an accelerating p.d. of 350,000 volts shoots electrons whose velocity is 150,000 mi./sec. These electrons can be made to penetrate an aluminum window for experimental purposes outside the tube.

Questions

1. What is the approximate number of molecules per cubic centimeter of a gas at N.P.T.? In the highest attainable vacuum?

2. Explain how the ions of the air between two electrodes finally reach the electrodes when the potential difference is small; when the potential difference is high enough to produce ionization.

3. Explain how cathode rays are produced, and give their physical nature. Give some experiments associated with them.

4. What are saturation currents?

5. Explain the production of a cloud of electrons about a heated filament in a vacuum.

6. Explain how a flow of electrons similar to cathode rays may be produced from heated filaments. What is the two-electrode vacuum tube?

7. Explain the dependence of the emission of electrons on the temperature of the filament.

8. Explain why currents can flow only in one direction between such a filament and a plate.

9. Explain how a pulsating current is obtained from an alternating current by the use of a two-electrode vacuum tube.

10. Describe the action of a three-electrode vacuum tube.

11. Explain the characteristic curve of a three-electrode tube and show by means of it how the tube is used as an amplifier and as a detector.

12. Describe the mercury-arc rectifier, the vacuum-tube rectifier, the tungar, and the copper-oxide crystal rectifier.

13. Describe the electron gun and the ion gun and develop the expression for the velocity of the projected particles.

Problems

1. If each molecule in 1 cc of air were a univalent ion, (*a*) to what potential in volts would these ions charge the earth? (The capacitance of the earth is 707 microfarads.) (*b*) How long would it take a current of 1 amp to carry off this charge?

2. With what velocity are electrons projected from the accelerating compartment of an electron gun when the accelerating p.d. is 30 volts?

Experiments

1. Long tube slowly evacuated showing the various electric discharge phenomena at low pressures.

2. Impact of cathode rays causes paddle wheel to rotate, platinum to heat, and various substances, including glass, to fluoresce.

3. Cathode rays move in straight lines, casting shadows of objects.

4. Cathode rays charge negatively a plate connected to an external electroscope.

5. Magnetic and electric fields deflect cathode rays.

6. Positive rays (canal rays). Knipp-Kunz tube.

7. Ionization current.

8. An electroscope discharged by placing a red-hot wire near it.

9. Current flows only in one direction through the space between the filament and the plate of a two-electrode vacuum tube.

CHAPTER XXVI

MEASUREMENT OF VELOCITY, CHARGE, AND MASS OF ELECTRONS, PROTONS, AND ISOTOPES

1. Intensity of Electric Field and Force Acting on Electron or Ion in an Electric Field.—The electric field between the two oppositely charged plates, Fig. 1, except at the edges, has a uniform intensity F''. In this field a statcoulomb of electricity is acted on with a force of F' dynes, and the work required to move a statcoulomb from one plate to the other measures the potential difference V'' between the plates.

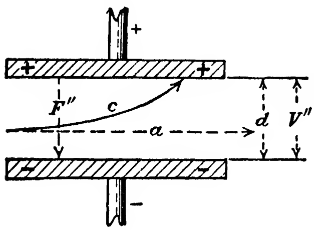


FIG. 1.—The path of a moving negative charge in an electric field

Then

$$w_1 = V'' = F''d.$$

From which

$$F'' = \frac{V''}{d} = \frac{V}{300d} \text{ e.s.u.} \tag{1}$$

If an electron or a proton has a charge e'', it is acted on in the field of intensity F'' with a force

$$f = F''e'' \text{ dynes.}$$

If a negatively charged particle is moving in the direction indicated by the broken line a, the uniform force due to the field causes it to take the path of a parabola, shown by the full-arrowed line c.

2. Velocity of Electrons and Ions.—If a stream of electrons or ions is subjected simultaneously to the action of an electric and a magnetic field at right angles to each other, and if the directions of these fields are such that they act on the electrons or ions in opposite directions, their intensities may be adjusted so that they

act with equal opposing forces. The path of the stream then is uninfluenced by the presence of the fields.

The directions of such electric and magnetic fields, acting in opposite directions on any charge moving at right angle to them, are represented by F'' and H, Fig. 2. The electric field is produced by charging oppositely the plates A and B contained in a vacuum tube (not shown). The magnetic field is that between the poles of an electromagnet into which the tube is so placed that the electric and magnetic fields are at right angles to each other.

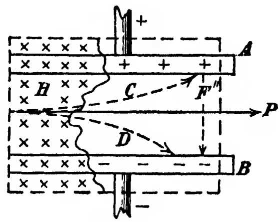


FIG. 2.—Action of crossed electric and magnetic fields on a stream of negative charges.

If with the fields off, for example, a stream of electrons takes the path P, it is forced to take the path C when the electric field alone is acting, and the path D in the magnetic field alone. When the two fields act together, and their relative intensities are properly adjusted, the charge again takes the original undeviated path P.

The equal opposing forces, from Eqs. VIII–7, III–3 are

$$f = Be'v = F''e'' = cF''e'.$$

From which

$$v = c\frac{F''}{B} \text{ cm/sec.} \tag{2}$$

The highest electron velocity that has been measured is 2.65×10^{10} cm/sec. To a stationary observer, an elemental charge having a great velocity appears to be appreciably flattened at right angles to the direction of motion. This brings the elements of the charge closer together and thereby increases the mass.

Theory and measurements of the mass of an electron or proton at large velocities both show that

$$m_v = \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}, \tag{3}$$

where m_v is the mass at the velocity v, m_0 the mass at rest, and c the velocity of light. The equation shows that the apparent change in mass is appreciable only when the velocity is greater than 0.1 that of light and increases slowly at first and then rapidly as the velocity of light is approached. When $v = c$, m_v is infinite, and, therefore, no velocity as large as c can exist.

3. Charge of an Electron and Proton.—Two insulated plates, Fig. 3, are attached to a battery and form opposite sides of one of two adjacent chambers which are connected by a small opening. Some droplets of oil sprayed by an atomizer into the upper

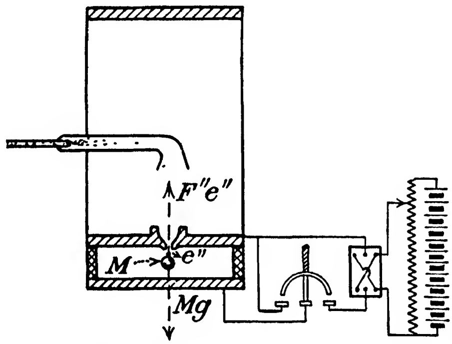


FIG. 3.—Apparatus for measuring the elemental charge.

chamber fall into the lower one through the opening. The terminal velocity in air of such a droplet is uniform and bears a known relation to the mass; therefore the mass, and thus the force $Mg - B$ acting downward on the particle, may be determined by observing the velocity with which the droplet falls when the plates are neutral. This is done by measuring the time it takes the droplet to fall a known distance between two parallel, horizontal cross lines as observed in a short-focus telescope. The term B represents the force with which the droplet is buoyed upward by the air.

The droplets of oil usually become electrically charged in the process of formation. The electric field between the plates acts on any charged droplet, and its direction may be made such as to urge the droplet upward. The intensity of the field may be adjusted until the two opposing forces acting on the droplet are equal; *i.e.*, the droplet remains suspended in the air. If now the air in the chamber is ionized by x-rays or by radium, the num-

ber of ions in the lower chamber is made large enough so that occasionally an ion attaches itself to the droplet. The mass of the ion is negligible compared with that of the droplet; so the mass of the droplet may be considered unchanged while the electric field acts on the added charge.

To simplify the computation, assume that an uncharged droplet to which one ion has attached itself has been balanced in an electric field. The droplet is pulled downward with a force $Mg - B$ and the attached charge is pulled upward with a force $F''e''$ dynes; *i.e.*,

$$f = F''e'' = Mg - B \text{ dynes,}$$

from which

$$e'' = \frac{Mg - B}{F''} = 4.770 \times 10^{-10} \text{ statcoulombs.} \qquad (4)$$

Several ions may attach themselves at the same time, but the magnitude of the measured charge is always some exact multiple of the smallest one. The smallest negative charge is found to be equal to the smallest positive charge. From these and other considerations this smallest charge is considered to be the elemental charge, *i.e.*, the smallest charge known to exist by itself.

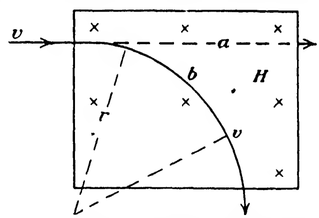


FIG. 4.—The path of a stream of negative charges in a magnetic field.

4. Ratio of the Charge to the Mass of a Moving Particle.—A stream of electrons shot from an electron gun (Art. XXV–9) when passing through a magnetic field H, Fig. 4, at right angles to the lines of force moves in the arc of a circle b in place of the straight path a. Each electron feels a force of $Be'v$ dynes (Art. VIII–8) which acts at right angles to the magnetic field in all parts of its path. If the field is uniform, the force has the same intensity at all points and each electron, therefore, acquires uniform circular motion in which the acceleration toward the center is

$$a = \frac{v^2}{r},$$

where v is the velocity of the particle and r the radius of the circular path. Then, since

$$f = ma,$$

$$f = Be'v = ma = m\frac{v^2}{r};$$

from which

$$\frac{e'}{m} = \frac{v}{Br}, \qquad \text{and} \qquad \frac{e''}{m} = \frac{cv}{Br}. \tag{5}$$

The path of a stream of electrons may be observed by passing it through a narrow slit from which it emerges as a wide, thin stream. This stream falls on a plane surface coated with fluorescent zinc sulphide and is so placed that at all points in the beam some of the electrons are striking the screen and causing it to fluoresce. The stream usually passes off the screen before making a complete circle, but the radius r may be obtained from the arc b of the circular path. The flux density of the magnetic field is measured (Art. XXI–20) and the velocity v determined (Art. 2). The ratio e''/m may be measured in this manner for any stream of electrons or ions and gives the mass of the individual particles when the charge on the particles is either measured or known.

5. Mass Associated with an Elemental Charge.—The mass of an electron may be determined by measuring the magnitude of e''/m. When $e''/m = a$ and e'' is known or measured, at low speeds

$$m_e = \frac{e''}{a} = 8.99 \times 10^{-28} \text{ grams.}$$

A stream of hydrogen ions, such as passes through the hole in the cathode of a vacuum tube (Art. XXV–1) which had been filled with hydrogen before exhaustion, may be deflected by a magnetic field, and e''/m measured. Then, from the knowledge that molecules of hydrogen are composed of two atoms, the mass of the hydrogen atom is determined. This is the smallest mass with which the elemental + charge is found normally to be associated and therefore is said to be the mass of the proton, which at low speeds is

$$m_p = 1.661 \times 10^{-24} \text{ grams.}$$

This gives the proton a mass 1,847 *times that of the electron, while the charges on both electron and proton have the same magnitude.*

With more elaborate apparatus, a stream of ionized hydrogen atoms is used in place of molecules. It also is seen that the mass of any ion may be measured in this manner. The mass of any charge at high speeds may be determined from that at low speeds by substitution in Eq. (3).

6. Isotopes.—The atomic masses of the chemical elements may be measured by the method of Arts. 4, 5. One practical form of the apparatus used is represented in Fig. 5. This consists of four evacuated chambers A, B, C, D, connected by small

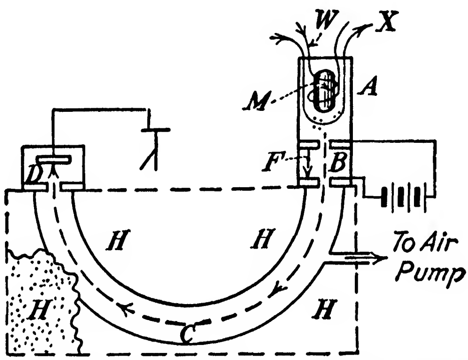


FIG. 5.—An apparatus for measuring the masses of individual atoms.

openings. The chambers A and B together form an ion gun (Art. XXV–9), from which + ions of the element are projected into the chamber C with any desired velocity (Eq. XXV–1).

The chamber C is semicircular and so placed between the poles of a magnet that the magnetic field H is at right angles to the direction of motion of the ions projected from the chamber B. The potential difference across the chamber B is adjusted until these ions, in the given magnetic field, move in an arc whose radius is that of the chamber itself. The ions then strike the opening that leads into the chamber D. If the velocity of the projected ions is too great or too small, their path is an arc whose radius differs from that of the chamber C. The ions then do not enter the chamber D.

When ions enter the chamber D, they charge a metal plate which is connected to an external electroscope.

Equations XXV–1, (5) give the following two relations between m and v:

$$v = \sqrt{\frac{2V''e''}{m}}. \qquad m = \frac{Be''r}{cv}.$$

Squaring both equations and substituting the $\frac{2V''e''}{m}$ of one for the v^2 of the other,

$$m = \frac{B^2r^2e''}{2c^2V''} \text{ grams.}$$

In this equation all the terms are known or easily measured. The only questions that may arise are whether e'' is one or more than one elemental charge, and whether m represents the mass of one or more atoms. The conditions of the experiment and the knowledge of atomic weights usually leave no doubt with regard to these questions in any particular case.

This method of measuring the mass of individual atoms has led to the important discovery of isotopes. The ions of a given element give several streams in the chamber C, and the electroscope responds to a corresponding number of adjustments of the potential difference in the chamber B. This shows that atoms of the same element differ in mass and that any particular atom belongs to one of several distinct groups which are called *isotopes* of the element. The atomic weights of isotopes are whole numbers; therefore the atomic weight of an element is the average atomic weight of all its atoms. All the isotopes of the same element have the same *atomic number.* This is the number of positive, unneutralized, elemental charges in the nucleus of the atom and therefore that of the orbital electrons. The following table gives the atomic numbers and the atomic weights of a few of the elements and the number of protons in each of their isotopes, the isotopes composing the greater proportion of the atoms being placed first in the list.

The atomic weight of the lightest atom, hydrogen, is 1.008, which is not an integral part of the atomic weights of the isotopes. It is believed, however, that each isotope is composed of a number of whole protons and electrons. The small loss of mass in the heavier atoms is attributed to the fact that the nucleus is com-

Element	Atomic number	Atomic weight	Proton numbers of isotopes
Li . .	3	6 94	7, 6
Mg	12	24 32	24, 25, 26
Cl .	17	35 46	37, 35
Cu ...	29	63 57	63, 65
Zn	30	65 37	64, 66, 68, 67, 70
Ag	47	107 88	107, 109
Hg	80	200 61	202, 200, 199, 201, 198, 204, 196

posed of several protons held together in part by a smaller number of electrons. The presence of the electrons in the immediate neighborhood of the protons diminishes their combined inertia (Art. XXVII–10). The arbitrary assignment of 16 for the atomic weight of oxygen, therefore, gives to hydrogen an atomic weight somewhat greater than unity.

Questions

1. Give the path taken by a stream of electrons in an electric field, and develop the expression for the force acting on a moving electron in such a field.

2. Describe how the velocity of electrons is determined, and derive the expression for it.

3. Explain how the charge of an electron is determined, and develop the equation. What is the charge of an electron? Of a proton?

4. Show that in a uniform magnetic field an electric charge, if properly projected, moves in a circle.

5. Show how the ratio of the charge to the mass of the electron is determined from this path.

6. Explain how the mass of an electron is measured, and derive the expression.

7. Can the velocities, masses, and charges of ions be measured in the same manner?

8. What is the ratio between the mass of a proton and that of an electron?

9. Show that the kinetic energy given to a charged particle by an electric field is proportional to the potential difference. Derive the expression for the velocity of the particle.

10. What are isotopes? Explain a method for measuring their masses.

Problems

1. (*a*) What is the strength of the electric field between two parallel plates 10 cm apart when the potential difference between them is 600 volts? (*b*) What force acts on an electron in that field?

2. The potential difference and the distance between the two electrodes of a vacuum tube are 30,000 volts and 15 cm, respectively. When electrons are moved through the whole distance between these electrodes, what is (*a*) their acceleration, (*b*) their final velocity, and (*c*) the kinetic energy of each electron at the moment of impact with the positive electrode?

3. (*a*) If the mass of a droplet of oil is 1.935×10^{-10} grams, what is its charge if an electric field of 200 e.s.u. just balances the force of gravity? (*b*) How many elemental units of charge is this? (The acceleration of gravity is 980.6 cm/sec. Neglect the buoyancy of the air.)

4. An electric field of 300 e.s.u. is at right angles to a magnetic field of 700 gauss. A stream of electrons moves through these crossed fields in a straight line. What is the velocity of the electrons in the stream?

5. The velocity of an ion whose charge is interpreted to be 2 elemental units is found to be 2.385×10^6 cm/sec. When passing through a magnetic field of 300 gauss, it moves in a circle of 20-cm radius. What is its mass?

Experiments

1. Deflection of a stream of electrons by an electric and by a magnetic field (repeated).

*

CHAPTER XXVII

X-RAYS, RADIOACTIVITY, MATTER

1. Orbital Electrons and Energy Levels.—The number of orbital electrons that surround the positively charged nucleus of a neutral atom is necessarily equal to the number of excess protons in the nucleus (Art. II–2), *i.e.*, the atomic number (Art. 12). These electrons are assumed to be rotating about the nucleus. But very little is known about the orbits with the possible exception of hydrogen, for which there is a fair agreement between mathematical calculations and observed facts. The orbital electrons, therefore, are now tentatively assumed to be located about the nucleus *only in certain definite energy levels*, and no thought is necessarily given to the form or even to the existence of orbits. The electrons possess the least amount of energy in the innermost or lowest level (nearest the nucleus) and the greatest amount in the highest level. The lowest level, called the *K level*, is followed in succession by the levels L, M, N, etc., as represented in Fig. 1. Energy from outside the atom must be given to an electron to force it into a higher level. The atom, then, is said to be in an *excited state* and is not in proper equilibrium until an electron falls from some higher level into the vacated space. There is only an interchange of electrons between the different levels, except in the case in which the ejected electron is forced entirely out of the atom. One of the electrons receives external energy, and the other electron parts with its energy excess by generating electromagnetic waves. At each interchange one electron receives and another radiates a definite amount or *quantum* of energy. The amount of energy in any particular quantum depends on the level or levels through

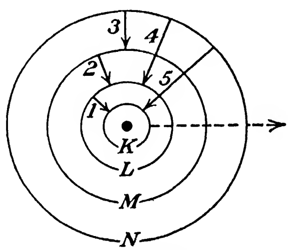


FIG. 1.—Diagram of an atom showing four innermost orbits about the nucleus.

which the electron emitting the quantum fell. The greatest energy difference, of course, exists between the innermost levels because of the greater intensity of the electric field in the neighborhood of the nucleus.

Assume that an external electron in passing through an atom expels an orbital electron from level K entirely out of the atom, as represented by the broken-line arrow, Fig. 1. The atom then, as in any transfer of an electron to a higher level, is said to be in an excited state. Another orbital electron immediately falls from a higher level to satisfy the normal condition that each of the lowest levels be occupied by some definite number of electrons. This can take place by electrons falling in succession, each through one level, as represented by arrows 1, 2, 3; or an electron may fall through more than one level as represented by arrows 4 and 5. The described atom is called the *Bohr atom* and is hypothesized to account for the line spectra of the elements.

The emitted quanta differ in energy content which depends on the element and the level or levels through which the radiating atom falls. The energy emitted, however, is expressed for all quanta by

$$w = h\nu \text{ ergs,} \tag{1}$$

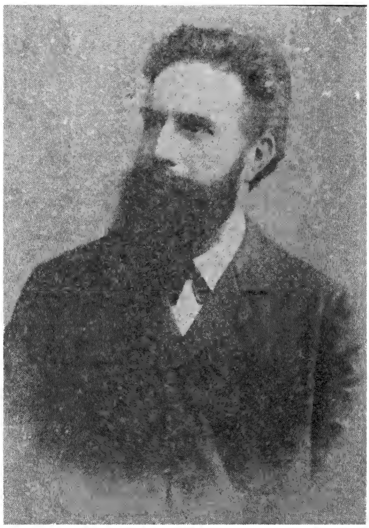
where h is a universal constant, called *Planck's constant* or *quantum of action*, whose magnitude is 6.547×10^{-27} erg sec and ν is the frequency of the oscillations that produce the radiated quantum. The frequency ν is seen to vary directly as the quantum energy w, and therefore, since the energy differences are greatest between the innermost levels, both the vibration frequency and the energy of individual quanta originating there are also the greatest. The energy of each individual quantum of the longest light rays ($\nu = 3.9 \times 10^{14}$) is 2.5×10^{-12} ergs and that of hard x-rays ($\nu = 1.8 \times 10^{19}$) is 1.2×10^{-7} ergs.

Although the electron probably oscillates, the mechanism involved in radiation by quanta is a matter of speculation. Ionization phenomena seem to indicate that quanta travel as particles or localized "bundles" of pulses which do not spread during propagation through space. Interference phenomena, in which a large number of quanta always is involved, show that such quanta spread and interfere in a manner attributed to wave

propagation. In other words, quanta have the properties of both particles and waves.

The quanta of radiant energy are now usually called *photons*. A photon, therefore, is a discrete particle of radiation the magnitude of whose energy is $h\nu$ ergs. All radiant energy emanating from atoms consists of these discrete particles.

2. Production and Nature of X-rays.—When cathode rays impinge on any material target, especially on a heavy element such as tungsten, platinum, or molybdenum, the target emits a



William Conrad Rontgen (1845–1923), professor of physics in succession at Wurzburg and Munich, Germany, discoverer of x-rays (1895).

penetrating radiation called *x-rays* or *Rontgen rays*. Interference phenomena, observed on reflection of the rays from crystals, show that these rays have the wave property of electromagnetic waves of extremely short wave length (10^{-7} to 10^{-9} cm, corresponding to frequencies of 3×10^{17} to 3×10^{19} cycles/sec). These frequencies are from 300 to 30,000 times the highest frequency producing visible light.

There are two types of x-rays. Those which are produced at the point of collision of the electrons with the target are called *general x-rays* and comprise the main part of the radiation. The spectrum of these rays is continuous like that of a noise-pulse

which contains all sound frequencies. The rays of the other type are emitted in discrete quanta (photons) from the innermost orbital levels of the atoms as described in Art. 1. These are called *characteristic rays* because they give a line spectrum which is characteristic of the element composing the target. Those having the longer wave lengths are emitted from the higher energy levels by comparatively slow electron velocities while others, emitted from the *K* level, require, for example in tungsten, an electron velocity of about 59,000 electron volts.

The spectrum of the general x-rays extends into lower and lower wave lengths with the increase of the applied p.d., so that the quality of the rays changes with the applied

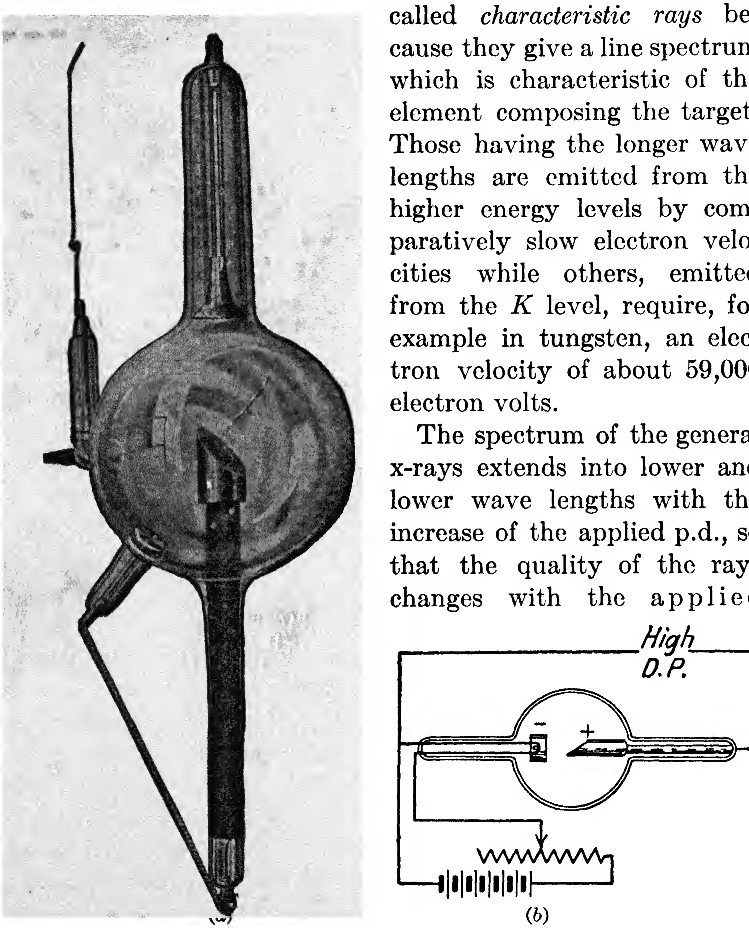


FIG. 2.—(*a*) X-ray tube in which electrons are splashed from the cathode by ions. (*b*) Coolidge x-ray tube in which the electrons are emitted by an incandescent spiral.

potentials. The rays of shorter wave lengths have the greater penetrating power and are called *hard x-rays* to distinguish them from the less penetrating *soft x-rays*. The general x-rays play the important part in x-ray photography and medical treatment, while the characteristic rays are of the greater service in research.

There are also two types of x-ray tubes. In Fig. 2(*a*) is shown the type in which electrons for the cathode rays are splashed from the cathode by positive ions, as explained in Art. XXV–2. The lower of the two main electrodes is the target (anode) which is bombarded by the electrons from the upper electrode (cathode). The upper of the secondary electrodes (on the left side of the tube) regulates the vacuum and the lower one steadies the focal point of the impinging cathode rays by partly discharging the negative charge which collects on the inner surface of the tube.

In Fig. 2(*b*) is shown the Coolidge x-ray tube in which the electrons for the cathode rays pour out of an incandescent tungsten wire located at the negative electrode, as explained in Art. XXV–5. The Coolidge x-ray tube is now generally used because both the supply of electrons and the potential difference can be varied at will; hence the intensity as well as the hardness of the rays is under perfect control. Potential differences as high as 800,000 volts are commonly used.

3. X-rays in Research.—*a*. Characteristic x-rays, when passed through a crystal or when reflected from the surface of it in an x-ray spectrograph, produce diffraction photographs and photographic spectra. From these, when the atomic spacing of the crystal is known, the wave length of the x-rays is calculated, and conversely, when the wave length is known, the applicable equation gives the unknown distance between the spacings.

b. The characteristic line spectra of the bombarded elements are similar but differ with respect to the frequency which the corresponding lines represent. It is found that the corresponding frequencies are to each other as the squares of the atomic numbers (Art. 12) of the elements. The observed relationship of frequencies to atomic numbers shows that the inner structure of all atoms is similar except that in the heavier atoms there are additional higher energy levels. It also calls attention to the basic importance of atomic numbers.

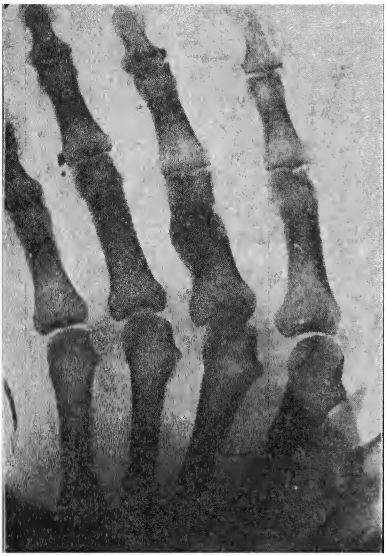
c. A study of scattered x-rays led to the concept that the impacts between photons and free electrons can be treated like those between elastic solids. A reflected photon, losing a part of its energy by the impact, has its frequency decreased proportionately to conform with, Eq. (1), $w = h\nu$. The photon, although having the properties of a particle, consists of electro-

magnetic pulses which must continue to move with undiminished velocity. It therefore can be changed only in its energy content and in vibration frequency.

d. The ionizing and penetrating powers of x-rays make them of service in many directions.

4. X-rays in Medical Practice.—X-rays are invisible but affect photographic plates and cause certain substances such as barium platinocyanide, fluorescent zinc sulphide, and calcium tungstate to fluoresce.

To observe the bones of the hand, for example, the hand is placed on the back of a screen the front of which is covered with one of the foregoing salts. The x-rays pass more readily through the flesh than through the bones and less readily through metals that may be imbedded in the flesh. They pass on to the salt, in which they excite fluorescence in varying degrees corresponding to the various remnants of the original rays. A "shadow" picture of the bones and of any imbedded metal or glass appears on the screen. Decreasing the hardness of the rays increases the visibility of the flesh. With proper hardness of the rays such large muscles as the heart may be seen and the beating of the heart observed. In order to study the intestines, barium sulphate (specially purified for internal use for x-ray purposes) is taken mixed with food, which usually is buttermilk.



(*a*) (*b*)

FIG. 3.—X-ray photographs of (*a*) healed break in a finger bone and (*b*) abscess under tooth (*Dr. W. H. Ude, Minneapolis.*)

Since this compound is opaque to x-rays, it enables the shape of the intestinal tract to be seen or to be photographed.

An x-ray photograph is taken by placing the screen face down on the photographic plate. The x-rays and the fluorescent screen both affect the plate, which is then developed in the ordinary manner.

X-rays are of great service in medicine. They discover foreign bodies, dislocations, and fractures, they aid in diagnosing diseases

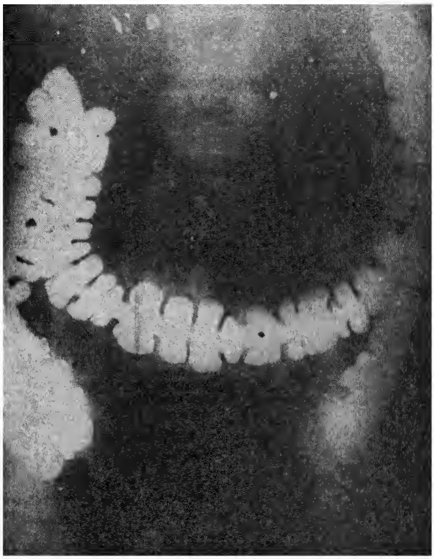


FIG. 4.—X-ray photograph of colon.

of internal organs and the teeth and are useful in treating diseases of the skin and other disorders. The curative properties are based mainly on the fact that physiologically young and abnormal tissues are injured by x-rays more easily than normal tissues. Some x-ray photographs of pathological cases are shown in Figs. 3, 4, 5.

X-rays must be employed with care as an undue exposure destroys the normal tissues and produces deep "burns." A succession of moderate exposures injures the tissues enough to develop cancer. Reproductive tissues and cells are injured about four times as easily as other body tissues, and it is possible that

evên heredity may be affected, as it has been shown to be in some insects. The operator is protected by enclosing the x-ray tube in lead glass or a lead box which allows the rays to pass only through an opening. X-rays, however, generate secondary x-rays which emanate from the patient and in case of hard rays may be sufficiently intense to be injurious.

FIG. 5.—X-ray photograph of stomach and gall stones. The gall stones appear at right of lower side of photographed plate number. (*Dr. W. H. Ude, Minneapolis.*)

Electromagnetic pulses of high frequency are produced by great accelerations, and therefore their electric components have great intensities. These intensities, it has been noted in Art. XI–7, are sufficient to ionize air by forcing electrons out of atoms. This ionization within a substance accounts for the observed chemical changes produced by x-rays and therefore for their physiological effects.

5. Radioactivity.—The nuclei especially of certain heavy elements are subject to explosions. An explosion of such a nucleus ejects an electron (*beta particle*) or a helium nucleus (*alpha particle*), or both, out of the atom, and at the same time a photon of high frequency (*gamma ray*) is radiated into space.

If a radioactive substance, including its series of disintegration products, be enclosed in a lead vessel A, Fig. 6, the emitted particles and photons move in a straight line through the opening B. The position of this stream may be studied by means of an ionizing chamber (not shown). The particles and photons enter this chamber through a fine hole or slit and pass between two electrodes which are connected in circuit with a battery and a galvanometer. The particles and photons ionize the air, and the ionization current (Art. XXV–4) is detected or measured by the galvanometer (or electrometer) deflection. The intensity either of radioactivity or of x-rays is measured in this manner by the magnitude of the ionization current. The particles and photons also affect a photographic plate and may be studied by means of it.

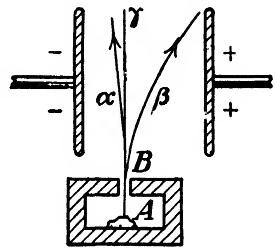


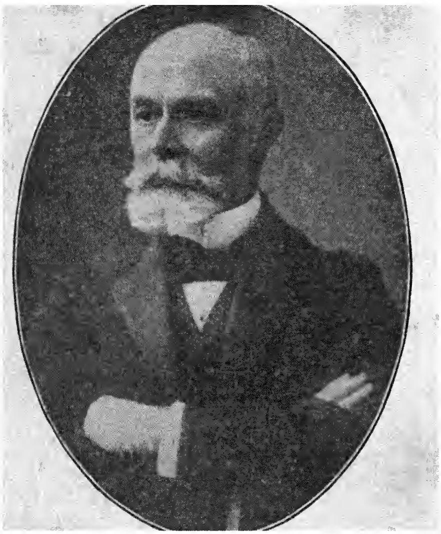
FIG. 6.—Separation of the three "rays" of radioactive substances by an electric field.

If either an electric or a magnetic field be placed in the path of the stream, the single stream is found to divide into three. In an electric field the positive α-particles are bent toward the negative plate, the negative β-particles are bent toward the positive plate, and the γ-rays (photons) are undeviated. The velocity, charge, and mass of the α- and β-particles may be determined, therefore, in the manner employed for electrons and isotopes (Chap. XXVI). The nature of the γ-rays is found by a study of the spectra obtained by reflecting the rays from crystals, as in the case of x-rays (Art. 3). The γ-rays are found to consist of electromagnetic waves (or photons) of various high frequencies, some of which are greater than those of the hardest x-rays. The wave length ranges from 41.4×10^{-9} to 0.17×10^{-9} cm. The spectrum of the β-particles, due to a magnetic field, indicates that in the nucleii of heavy radioactive elements there are definite energy levels; but very little is known about them. The velocity of α-particles is about 0.1 that of light; while the velocity of the β-particles is about 0.9 that of light.

The ejection of an electron from the nucleus of the radioactive substance adds to the positive nuclear charge, Arts. 12, II–2, resulting in the acquisition of an additional orbital electron.

The atomic weight is not changed, but the new atom has different chemical properties because these properties depend only on the number of the orbital electrons.

When the bivalent helium nucleus (α-particle) is ejected, the atomic weight of the atom is diminished by four, and its positive nuclear charge by two. The neutral atom then is also a new



Antoine Henri Becquerel (1852–1908), professor of physics, École Polytechnique, Paris; discoverer of radioactivity (1896).

substance, the number of its orbital electrons being diminished by two.

The helium nucleus appears to be a very stable combination. The nucleus of an element of high atomic weight appears to be mainly a collection of such helium nuclei.

Some elements such as boron, carbon, sodium and aluminum become radioactive after being bombarded with α-particles, protons or other missiles. This is known as *artificial radioactivity*. The half-life period of this radioactivity varies with the element and for sodium is 15.5 hr.

6. Radioactive Substances.—Nearly all the observed radioactivity is associated with uranium, actinium, thorium, and their

disintegration products. They are the three elements of the highest atomic weights. The heaviest of these, uranium, has a nucleus consisting of 238 protons and 146 electrons and has a positive charge of 92 elemental units. The neutral atom, therefore, must have 92 orbital electrons. This complex system is broken into simpler systems by successive ejections of helium nuclei and nuclear electrons.

The following table gives the history of the atoms of uranium through their successive explosions. The "atomic number" (Art. 12) is the nuclear charge in elemental units, and the "proton number" the number of protons in the nucleus of the principal isotope.

RADIOACTIVE TRANSFORMATIONS OF THE URANIUM SERIES

Name	Atomic number	Proton number	Typical rays	Half-value period
Uranium I	92	238	α	4 4 $\times$ 10^9 years
Uranium X_1	90	234	β	24 5 days
Uranium X_2	91	234	β	1 14 min
Uranium Z	91	234	β	6 7 hr
[1]Uranium II	92	234	α	3 $\times$ 10^5
Ionium	90	230	α	8.3 $\times$ 10^4 years
Radium	88	226	α	1 59 $\times$ 10^3 years
Radon	86	222	α	3 825 days
Radium A	84	218	α	3 05 min
Radium B	82	214	β	26 8 min
Radium C	83	214	β, α	19 7 min
Radium C′	84	214	α	10^{-6} sec
Radium C″	81	210	β	1 32 min
Radium D	82	210	β	22 years
Radium E	83	210	β	4 9 days
Radium F	84	210	α	140 days
Radium G (Lead)	82	206		

[1] Uranium II also transforms to uranium Y from which arises the actinium series of transformations not shown in this text.

The Greek letter α indicates emission of alpha particles (helium nuclei), and the letter β beta particles (electrons). These particles are called the *typical rays* emitted in the transformation of the given element into the one that follows on the list. The "half-value period" means the time required for one-half of the atoms of any given mass of the element to explode. The exceptional cases of double transformation are indicated by arrows. The γ-rays which are emitted with different degrees of intensity and hardness at all of the transformations are not listed in the table.

Thorium and actinium have their transformations represented by similar tables. The transformations of both of these elements also end in lead, the isotope of lead from thorium having an atomic weight of 208.0 in place of 206.0.

Potassium and rubidium are the only elements not included in these three series that definitely show traces of radioactivity.

The table on page 401 shows that the amount of lead and helium in any given uranium rock depends on the age of the rock. The age of the oldest uranium rock is found, in this manner, to be 1,852 million years.

7. Radium and Radon.—Radium is one of the transformation products of uranium, as is seen in the table of Art. 6. Although its half-value period is a large number of years, it is short enough so that large numbers of particles are emitted. The immediate transformation products are short lived and are always associated with the radium. They contribute much to its apparent radioactivity.

The first transformation product of radium is *radon*, formerly called *radium emanation.* Radon is collected as a gas, usually above a solution of $RaBr_2$ or $RaCl_2$ in dilute HCl. The newly collected gas is comparatively weak, but in about an hour the solid transformation products, deposited on the walls of the containing tube, contribute the bulk of the emanations.

The curative properties of radium are the same as those of hard x-rays. The radon alone is usually used. It is placed in a hollow steel needle through which only the γ-rays penetrate.

The α- and β-particles do not penetrate the flesh to any considerable depth, but the β-particles are sometimes used for surface effects.

It is dangerous to handle large quantities of radium, for not only do the γ-rays produce the same effects as x-rays but the bombardment by the α- and β-particles destroys the surface tissues. The red corpuscles of the blood are also destroyed by frequent exposure to relatively small amounts of the radiation.

Radium is always warmer than the surrounding atmosphere because every nuclear explosion produces molecular vibrations.

Madame Marie Sklodowska Curie (1867–1934), professor of physics, University of Paris, discoverer of radium, in collaboration with her husband, M. Pierre Curie, and G. Bremont (1898).

One hundred calories of heat are produced per gram of radium per hour. In changing from radium to lead, 1 kg of radium generates an amount of heat energy equal to 1,800 horsepower acting for a year.

The presence of radium may be detected by means of a charged electroscope which, because of the ionized air, discharges when brought into the neighborhood of radium.

8. Detection of Individual α- and β-particles and Photons.— A funnel-shaped vessel, Fig. 7, having a window *W* on one side and a rubber bulb *B* on the other, is partly filled with water

up to the level shown. The lower side of the window is covered with transparent, conducting gelatine which is connected to the battery B as shown. The battery maintains a p.d. across the cloud or Wilson chamber, *i.e.*, across the space between the water and the window. Compressing and releasing the bulb raises and lowers the level of the water and thereby subjects the moist, dust-free air in the cloud chamber to compressions and expansions. The air becomes supersaturated, at each expansion, and deposits a visible fog on the ions which are formed in the paths of individual α- and β-particles that are projected into the chamber from the radioactive substance in R. The electric field across the chamber pulls the ions out of the chamber quickly, so that during the compression it is cleared of fog for the following expansion.

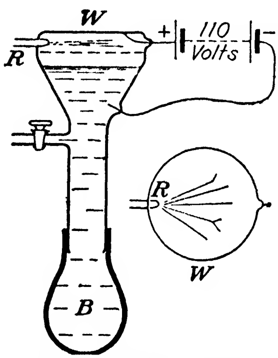


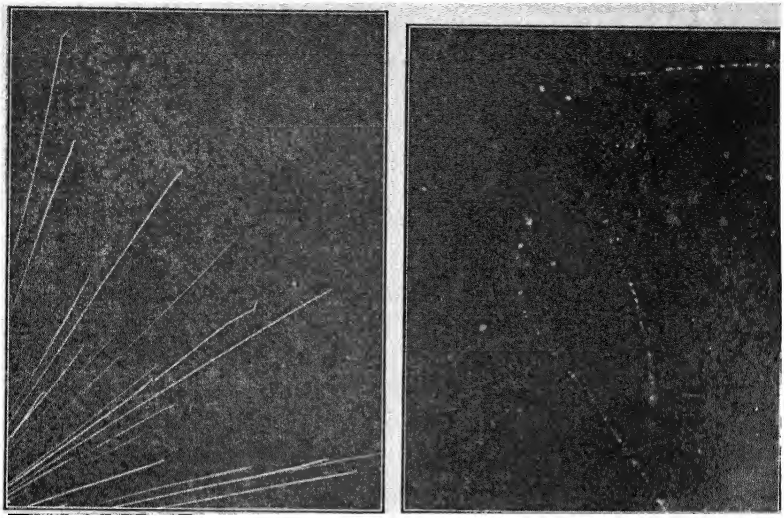
FIG. 7.—Diagram of apparatus for observing the paths of individual projected particles and photons.

Figure 8(a) shows the paths of α-particles as they appear through the window W, and Fig. 8(b) the (enlarged) paths of β-particles. The paths of the β-particles are thinner because the particles produce fewer ions on account of their velocity being much greater than that of the orbital electrons, and they are more crooked because their smaller mass is more easily deflected in passing by atomic nuclei.

Figure 9 shows the many curved paths of electrons ejected by the photons in a beam of x-rays. The blotches are due to electrons which are forced out of the atoms by impacts of the electrons initially ejected by the x-rays.

9. Neutrons and Positrons.—When beryllium and boron are bombarded by α-particles, an emanation comes from these elements which consists of neutral particles called *neutrons*. It is assumed that when an α-particle strikes a beryllium nucleus it adheres to it. The resulting explosion of the now unstable nucleus ejects the neutron, which, because of not having a charge, produces no ionization. The neutron, therefore, cannot be detected in the ordinary manner; but when it makes a direct

hit with the nucleus of an atom, the recoil of the nucleus is seen in the Wilson chamber. The neutron may stick to the



(a) (b)

FIG 8.—(a) Paths of α-particles. (b) Paths of β-particles (enlarged) showing water droplets on individual ions. (*Photographs by C. T. R. Wilson.*)

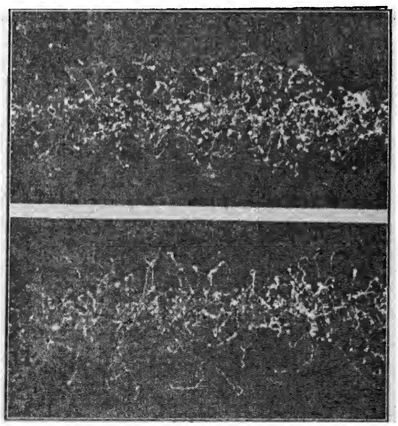


FIG. 9.—Ionization in paths of x-rays. The paths of electrons ejected by the x-rays are clearly shown. (*Photographs by C. T. R. Wilson.*)

impinged nucleus, which then explodes as is shown by the divergent tracks in Fig. 10. The experimental evidence for the existence of the neutron, therefore, lies in the sudden appearance

of tracks in the middle of the Wilson chamber. Regardless of the kind of nucleus involved, the same mass value is calculated for the impinging particle. This could not be the case if the paths were due to disruptions produced by x-rays or γ-rays. The mass of the neutron is found to be approximately that of the proton. The approximate equality of the masses leads some to conjecture that the neutron may be a close combination of a proton and an electron.

FIG. 10.—Tracks due to two parts of a nitrogen nucleus whose disintegration was produced by a neutron. (*Photograph by Harkins, Gans and Newson*)

The existence of positively charged particles whose mass is that of an electron was demonstrated by the photograph of Fig. 11. A beam of γ-rays, impinged on a piece of lead from above, ejected an electron and a positron. The paths show equal intensities in a magnetic field and have equal radii. These conditions can exist together only if the two particles have equal masses and equal charges. The reverse curvatures show that the charges of the ejected particles differ in sign. Theoretical considerations also point to the existence of a positron.

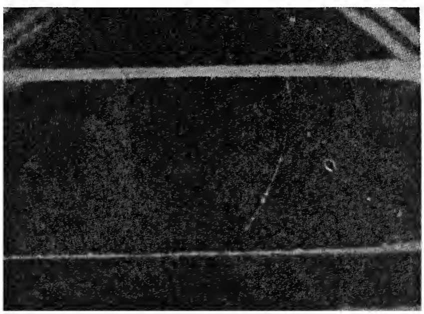


FIG. 11.—Tracks of electron and positron produced together by γ-rays, showing equal intensities of ionization and equal radii of curvature. (*Photograph by C. D. Anderson.*)

10. Elemental Masses Due to Electric Charges.—The dots on the surface of the sphere, Fig. 12, represent elements of an

elemental charge. If the charge is accelerated, any element *a* is the source of an electromagnetic pulse which urges all the other elements of the charge, such as *b*, in the reverse direction (Law C). This opposing force acting on *b* represents inertia. If the charge in motion is being retarded, the electromagnetic pulse from each element of the charge urges the neighboring elements in the direction in which they are moving. The force evoked by the retardation, then, opposes the force producing the retardation. This effect again represents inertia.

If the same charge is concentrated on a smaller sphere, the elements of the charge are closer; and when each charge is accelerated or retarded, it exerts a larger force on its neighbors. The same charge on a smaller sphere, consequently, has the greater electric inertia. The electric inertia is independent of the kind of charge but depends on the extent of the surface over which the charge is distributed. The inertia of electrons and protons (assumed to include positrons and neutrons), and therefore of matter, is at least partly electrical and is now assumed to be so entirely. It is not necessary to assume the existence of any other medium or substance than electricity to explain the observed inertia of matter. The inertia (mass) of electrons and protons, or of matter, may be briefly ascribed to self-induction (Art. XVII–5). On this assumption and from the known mass, the radius of the electron and of a positron is calculated to be 1.88×10^{-13} cm. If it is assumed that the proton is an elemental particle and not an uncoalesced union of a positron and a neutron, the proton has a radius of 1.03×10^{-16} cm. The radii vary inversely as the masses.

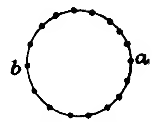


FIG 12 — Elements of an elemental charge.

If an electron and positron are imagined to coalesce so that the surface, Fig. 12, can be imagined to be covered with elements of the two charges, equal in number and symmetrically spaced, the coalesced particle would have no inertia and therefore no mass. The forces producing inertia, because of the elements of reverse sign, would annul one another.

From the foregoing considerations it may be conjectured that the mass of a neutron is that of a large equal number of uncoalesced electrons and positrons united to form one neutral particle.

11. Transformation of Mass and Energy.—Equation XXVI–3 shows that in the retarding of the motion of a fast-moving particle is involved the loss of mass. This does not mean that the particle is partly destroyed but means that a part of the energy which was assigned by us to mass effect becomes energy in some other form. Mass, as we use the term, therefore appears to change into energy; and, likewise, increasing the velocity involves a change of energy into mass.

The tendency of an electron and a positron to unite and, in so doing, to produce a photon is conjectured by some to involve the "transformation of matter (mass) into energy." The two charges could so coalesce (Art. 10) that their combined mass (inertia) would be zero. Likewise it can be conjectured that, when a photon ejects, from lead, an electron and a positron with equal velocities, "energy has been transformed into matter." A photon separating such coalesced charges would necessarily eject them with equal velocities and the separation would endow them with their original masses.

The concept of "energy changing into matter and of matter into energy" does not necessarily assume the destruction of any really basic entities in nature but is only the conceptual consequence of our definitions of mass and energy.

12. Nucleus and the Atomic Number.—α-particles can penetrate several hundredths of a millimeter of gold. It is found that those which are reflected and scattered as well as those which pass through the gold have almost the same velocity as the bombarding particles. The number of reflected or scattered particles is small. To be reflected the particles must strike something elastic to make them rebound. The elastic body can be a positive charge, which repels the positive α-particles. From the observed facts it is inferred that the elastic body (the nucleus of the atom) contains the bulk of the atomic mass and that the nucleus is concentrated in a very small volume (Art. II–2) compared with that of the atom. The relative nuclear charges of the elements are determined from the relative amounts of the scattering and also from the photographic study of x-ray spectra. For the lighter atoms this charge is found to be numerically equal to one-half the atomic weight of the atom. If the atomic weight is an odd number, the next lower even number is taken and

divided by two. These statements do not hold for the heavier atoms. The atomic weight of the principal isotope of gold, for example, is 197, and the atomic charge is 79 elemental units. There are in this nucleus, therefore, 197 protons held in equilibrium by 118 nuclear electrons. Some of these protons and electrons, no doubt, are in stable groups of four, known as *helium nuclei* or *α-particles*, and others may be in the form of neutrons.

The observations, mentioned here only, firmly establish the existence of a positively charged nucleus, the magnitude of whose charge is known. All the elements, therefore, can be classified by the magnitude of their nuclear charge. The *atomic number* is the term used to designate the number of elemental units of charge in the nucleus and the various atoms are tabulated in the order of their atomic number. All the 92 elements whose atomic numbers range from 1 to 92 have been discovered. The atomic number is a more significant constant than atomic weight.

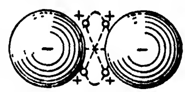


FIG. 13.—Nucleus of the helium atom.

13. Mass of a Nucleus.—The mass of a proton is found to be that of a hydrogen atom, which then is pictured as consisting of one proton with one electron located in some one of the several possible energy levels (normally the innermost level) and revolving about the common center of gravity. The mass of the electron, as far as present measurements are concerned, is negligible compared with that of the proton. The electron in the hydrogen atom is so distant that its charge does not appreciably affect the electric inertia of the proton. The mass of the hydrogen atom, then, is practically equal to the sum of the masses of its two constituents.

Since the atomic weight of helium is 4 and the atomic number 2, the nucleus of the helium atom must consist of 4 protons and 2 electrons which are possibly distributed in some such manner as shown in Fig. 13. The 4 protons necessarily repel one another but are drawn together by the electrons. These electric forces, together with the magnetic forces evoked by motions of the charges, act to form the stable combination of the nucleus.

When 4 protons are close together, any force attempting to accelerate them meets with more than four times the resistance that it would encounter from 1 proton. Each accelerating proton

pushes the others in the reverse direction for the same reason as do the elements of an elemental charge (Art. 10). However, the 2 electrons of the nucleus are urged in the same direction as the accelerating protons and thereby diminish the mass (inertia) of the system. The net result is that the mass of the nucleus is slightly less than four times that of one proton.

The nuclei of atoms other than helium are similarly composed of protons and of binding electrons which diminish the mass of the aggregation. This phenomenon explains why the atomic weight of hydrogen (1.008) is greater than unity. The nucleus of an oxygen atom has 16 protons, but its 8 binding electrons diminish the total mass to less than sixteen times that of 1 proton. The total mass (including the orbital electrons) of the atom then is less than sixteen times that of the hydrogen atom. Since the atomic weight of oxygen is arbitrarily assigned the number 16, that of hydrogen then must be slightly greater than unity. The atomic weights of all isotopes other than hydrogen are practically whole numbers.

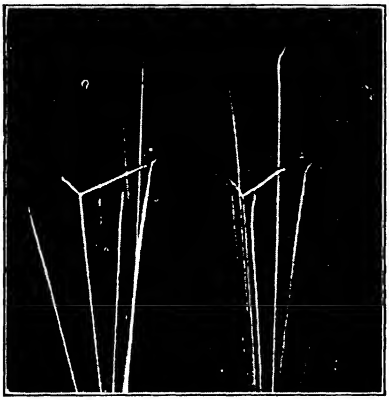


FIG. 14—Double photograph of a collision of an α-particle with an oxygen atom. (*Blackett*)

14. Artificial Transformation of Atomic Nuclei.—Observations of the visible paths (Art. 8) of α-particles passing through hydrogen gas show them to have a definite penetrating distance, but occasionally the path divides into two parts, one of which has a larger penetrating distance than that which is normal for the α-particle. The relative magnitudes of these distances is such as is given by calculation for an elastic impact of the α-particle with a hydrogen nucleus. The lighter body is given a greater velocity than that of the impinging heavier particle and therefore penetrates the air farther before its velocity is checked sufficiently to stop ionization.

A zinc sulphide screen placed in the hydrogen gas, therefore, scintillates up to a certain distance because of the impact of α-particles, and beyond that distance because of the few hydrogen nuclei that have been hit and driven forward.

When nitrogen gas is bombarded, similar long-distance scintillations or split paths are observed which are interpreted to be due to hydrogen nuclei, *i.e.*, protons. Each of these protons is believed to have been forced out of the nitrogen nucleus by the impact of an α-particle. The protons, in a similar manner, have

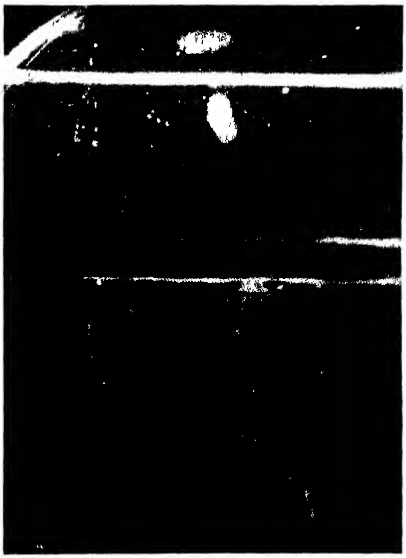


Fig. 15.—Secondary effects of a cosmic-ray particle in a Wilson chamber.

been forced out of the nuclei of other light atoms. A double photograph of the split path of an α-particle after impact with an oxygen atom is shown in Fig. 14. This ejection of protons constitutes what is termed the *artificial disintegration of the elements.* Atoms of smaller mass are produced by bombarding nuclei with α-particles and similar missiles.

It is believed to be possible to synthesize atoms. A proton or an α-particle hurled into a nucleus could become a part of it to form a heavier atom.

15. Cosmic Rays.—A very penetrating radiation coming with about equal intensity from all parts of space floods the earth.

In air at the earth's surface it produces 1.4 ions of each kind per cubic centimeter per second and probably is the cause of much of the less penetrating secondary radiation coming from the upper atmosphere. These rays have a penetrating power about 100 times that of the hardest γ-rays and can be detected after penetrating 18 ft. of lead.

The physical nature of these rays has not been fully established. Observations in progress indicate that probably they are nuclei of atoms. The great velocity they must possess may be produced by electric fields in cosmic space or by the earth's field. A cosmic-ray particle produces visible paths due only to secondary effects which give no clue as to the nature of the rays. Figure 15 shows these effects especially at the point where the particle emerges from the lower of the two lead strips which partition the chamber. The nature of the rays, therefore, is being deduced from the study of the distribution and absorption of the rays in the earth's atmosphere and magnetic field, at all elevations and in all parts of the world and also from observations on the ionization of air in containers submerged to great depths at sea. The earth's magnetic field appears to deflect the rays slightly, and, at great depths at sea, the diminishing ionization begins to increase at 200 meters and drops to zero at 275 meters. At these great depths it is now inferred that the velocity of the cosmic-ray particles has diminished sufficiently to enable the particles to produce direct ionization.

Questions

1. What are the energy levels of an atom? What is a quantum of energy? What circumstance causes the quantum of energy to be radiated? Give the expression for the energy of a quantum What is a photon?

2. Explain how the stream of cathode rays is produced in each of the two types of the x-ray tube.

3. How are x-rays produced? What is their physical nature? Their wave length? Distinguish between hard and soft x-rays. Between characteristic x-rays and those which give a continuous spectrum.

4. Show how they are contributing to basic research.

5. How can the bones of the hand be made visible by means of x-rays?

6. How is a photograph taken with x-rays?

7. Why do x-rays ionize gases and produce chemical changes in organic substances?

8. What are the medical uses of x-rays?

9. What are the dangers from exposure to x-rays?

10. How are the intensity and the hardness of x-rays regulated in a Coolidge tube?

11. Describe radioactivity Name the primary radioactive elements

12. What are the particles and rays emitted by radioactive substances?

13. How are these separated and studied to determine their physical nature?

14. What is meant by the half-life period of a radioactive element?

15. Give, in general terms, the history of a uranium atom until it becomes lead.

16. What changes take place in an atom when an electron is ejected from its nucleus? When a proton is ejected? When an electron and a proton are ejected simultaneously?

17. Why are radium and radon known better than other radioactive substances?

18. Name two ways of detecting the presence of α-particles.

19. Why are γ-rays used in place of the emitted particles in medical practice? How are the α- and β-particles excluded?

20. What are the dangers from exposure to radium?

21. How much heat is produced in radioactive transformations? How are the paths of α- and β-particles and of γ-rays made visible?

22. What are neutrons and positrons and how is their presence detected?

23. Explain how the elemental masses are due to their electric charges

24. Explain how the combined mass of an electron and positron could be zero. Also how a photon could give masses to these particles.

25. Give the expression for mass at great velocities, and show how mass is converted into energy and energy into mass

26. How was the nucleus of atoms discovered? What is its structure?

27. What is the atomic number, and how is it determined? What relation has it to the atomic weight, the nuclear charge, and the number of orbital electrons? Why is it a more significant constant than atomic weight?

28. Why is the mass of the nucleus less than that of its constituent protons?

29. Explain how a hydrogen nucleus is driven out of certain of the lighter atoms

30. Give the main known facts concerning cosmic rays

Experiments

1. Various types of x-ray tubes shown
2. X-ray tube in operation, showing washer in box and bones in the arm.
3. Fluoroscope
4. X-ray plates shown and slides projected.
5. Lead apron.
6. Air ionized by x-rays or by radium discharges electroscope.
7. Spinthariscope.
8. Model of spinthariscope (projected).
9. Paths of α-rays shown in a Wilson cloud chamber.
10. Slides of various effects bearing on the disintegration and synthesis of matter.

*

CHAPTER XXVIII

OSCILLATING CIRCUITS—PRODUCTION AND DETECTION OF ELECTROMAGNETIC WAVES

1. Electric Oscillations in a Wire.—Imagine the free electrons in the wire, Fig. 1(*a*), to have been displaced to the left and to have caused the two ends to become charged oppositely as shown. Then imagine the displacing force to be suddenly removed. The potential difference between the ends of the wire causes the electrons to be accelerated toward the right. When the two unlike charges finally become neutralized, the accelerating force ceases; but the moving electrons continue to flow beyond the point of electric equilibrium until their kinetic energy has been

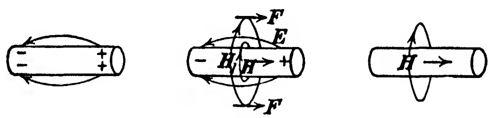


FIG 1.—Oscillations in a wire: (*a*) Electrostatic field about a wire whose ends are charged oppositely; (*b*) when electrons are accelerating to the right, the magnetic field H is being established by the electromagnetic pulse FH_1, and the intensity of the electrostatic field E is diminishing, (*c*) magnetic field in the immediate neighborhood of the wire at the instant the electron velocity is constant.

expended in a second separation of the charges. The two ends of the wire then are again charged oppositely, but now in the reverse direction. The process repeats itself so that the electrons oscillate. The original potential energy of the separated charges is converted into the kinetic energy of their motion and the kinetic energy again into potential in a manner analogous to that of an oscillating pendulum. If no energy were lost, the electrons would oscillate indefinitely; but energy is lost through the heating of the wire and through the electromagnetic waves which emanate from all oscillating circuits.

It should be observed that in the foregoing process the energy of the electric field repeatedly is converted into that of the mag-

netic and the energy of the magnetic field into that of the electric, as represented in Figs. 1(a, b, c).

2. Electric Oscillations in a Closed Circuit Containing Capacitance and Self-inductance.—If the electrons are displaced in a closed circuit containing a condenser, Fig. 2, the electrons oscillate from one side to the other as in the case of a straight wire, the only difference being that for the same potential difference a larger quantity of electricity oscillates. It is seen that the greater the capacitance of the condenser, the greater is the quantity of electricity to be moved. The time required to displace the electrons from one plate to the other is then increased; hence the period of the oscillation is prolonged. If also the self-inductance in the circuit is larger, the e.m.f. of self-induction is greater, the current is reduced (Arts. XVIII–9, XVIII–14), and therefore the time required for the same charge to be displaced is increased. The period of the oscillations then varies directly as some functions of the capacitance and self-inductance. It is also seen that this period must increase with the resistance of the wire connecting the condenser plates.

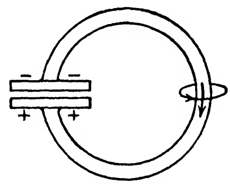


FIG. 2.—Oscillations in a closed circuit containing self-inductance and capacitance.

Equation XVIII–10,

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}},$$

for alternating currents is likewise applicable to oscillating currents. It is seen that the current in an oscillating circuit must at all times be the maximum that R, L, and C permit. With high frequencies the resistance R is necessarily small, so that L and C are the chief limiting factors. In any case the maximum current exists when

$$\omega L = \frac{1}{\omega C},$$

from which

$$\omega = \sqrt{\frac{1}{LC}}.$$

If the resistance R is so small that its effect on the frequency can be neglected, then f in the expression $\omega = 2\pi f$ is the frequency of the oscillating current when limited by L and C only. Then from the foregoing equations

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}. \tag{1a}$$

If, however, the resistance is large enough to influence the period appreciably, it can be shown (not readily) that

$$f = \frac{\sqrt{LC - \frac{1}{4}R^2C^2}}{2\pi LC} \tag{1b}$$

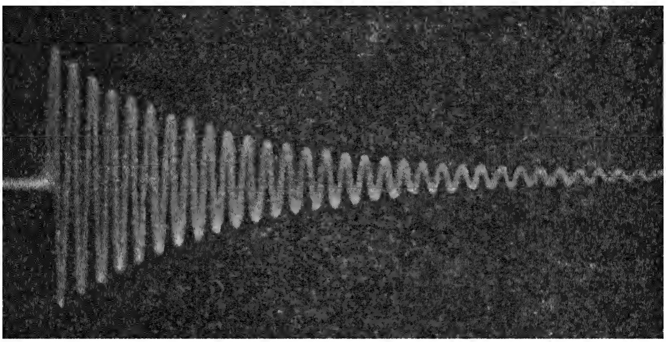


FIG. 3—Oscillogram of a damped oscillatory discharge of a condenser. (*J. Zenneck.*)

The frequency becomes zero, *i.e.*, the charge does not oscillate. when the numerator of this expression has zero value. Then

$$\frac{1}{4}R^2C^2 = LC,$$

from which

$$R = \sqrt{\frac{4L}{C}}.$$

Then in order that the charge may oscillate,

$$R < \sqrt{\frac{4L}{C}}. \tag{2}$$

The oscillations in the electric circuit correspond to those of a pendulum in a viscous fluid.

Figure 3 is an oscillogram showing the current intensity in such oscillations, taken by means of a cathode-ray oscillograph (Art. XXI–8). The frequency in the illustrated case was 250 oscillations/sec. Such oscillations are called *damped oscillations.* Their energy soon disappears, being dissipated, as already stated, in the production of heat and electromagnetic waves.

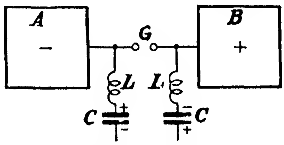


FIG. 4.—Hertz oscillator for the production of high-frequency oscillations.

3. Practical Production of Damped Oscillations—Inductive Coupling—Resonance.—To produce oscillations of the order of a hundred million per second, the capacitance and self-inductance must both be small. In order to have an appreciable amount of energy, the potential, therefore, must be high. The method of charging also must be such that the oscillating charge is choked from the energizing apparatus.

Heinrich Rudolph Hertz (1857–1894), professor of physics, University of Bonn, Germany, proved experimentally (1888) the existence of electromagnetic waves thereby verifying the deductions of Clerk Maxwell. Maxwell's deductions and Hertz's experimental verification are the basis of radio communication.

The plates *A* and *B*, Fig. 4, form the two plates of a low-capacitance condenser and produce such high-frequency oscillations. When the plates are charged to a potential difference sufficiently large to produce a disruptive discharge (Art. XI–11)

through the gap G, the heated ionized gases make the gap conducting and of negligible resistance for an instant. During this instant the discharge is oscillatory and the frequency is that given by Eq. (1).

The charges are supplied to the plates A and B through the inductances LL from the condensers CC which are charged by means of an induction coil or a transformer. The coils LL, which have comparatively large self-inductances, prolong the time of charging but prevent the oscillating current from reaching the condensers (choking effect, Art. XVIII–14). The charges then oscillate between A and B as though they were disconnected from the condensers. Such an apparatus is known as the *Hertz oscillator.*

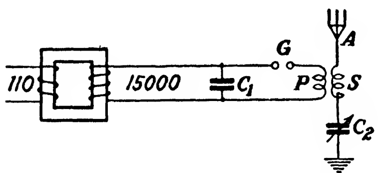


FIG. 5.—Production of damped oscillations in a closed- and in an open-circuit oscillator—Tesla coil.

A more practical apparatus for producing damped oscillations is the Tesla coil, represented in Fig. 5. The high-tension transformer charges the condenser C_1 twice during each cycle of the alternating current. During each half cycle the potential difference between the plates becomes large enough to produce a spark through the gap G. The condenser discharge is oscillatory and completed so quickly that the comparatively slow-changing impressed e.m.f. plays no part in it.

This high-frequency current gives some of its energy to a neighboring open circuit by means of the mutual induction between the coils P and S, which act as a high-frequency transformer. Such a high-frequency transformer consists of the two coils without an iron core. The oscillations in the wire A, then, are mainly in one plane, as in the Hertz apparatus of Fig. 4, and are called *open-circuit oscillations.* Such oscillations send out electromagnetic waves without interference, as explained in Art. XVI–3. The oscillation frequency in the secondary circuit is controlled by means of the adjustable condenser C_2 or by changing the self-inductance of the coil S.

When the secondary circuit is adjusted to have the same natural frequency as the primary, the successive induced charges are superposed on what is left of the previous charges still

oscillating. The magnitude of the oscillating charge is then greatly increased. The secondary is then said to be *tuned to* or *in resonance with* the primary circuit.

The oscillating current in the secondary S induces in the primary P a current which is not exactly superposed upon the original. This lack of exact superposition produces two peaks, slightly separated, where there should be only one in each half cycle. This condition again reacts on the secondary, making sharp tuning impossible. To reduce this effect to the minimum the secondary is placed at some distance from the primary, receiving less energy, but permitting sharper tuning. This is called *loose inductive coupling.*

The oscillations in the secondary circuit damp out quickly because, in addition to the heat generated and the energy

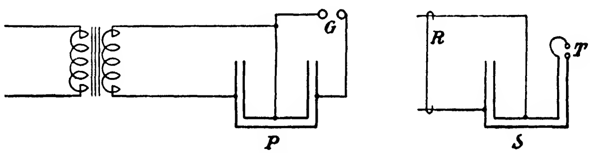


FIG. 6.—Resonance jars.

radiated some of the energy is returned to the primary circuit by induction.

Resonance in inductive circuits may be illustrated by means of two Leyden jars with their coatings connected to rods forming practically closed circuits of equal period, as shown in Fig. 6.

The primary jar P is energized by means of an induction coil and at each oscillatory spark sends out electromagnetic waves each pulse of which induces an e.m.f. in the circuit of the secondary jar S. The natural frequency of jar S is adjusted by changing the position of the rod R. When this period is the same as that in the jar P, and only then, the induced surges in the jar S are sufficiently intense to cause a spark through the small gap T.

4. Electromagnetic Waves.—Electromagnetic waves (Arts. XVI–2, XVIII–4) consist of alternating electromagnetic pulses which are composed of alternating nonconservative electric fields associated with moving magnetic fields and are produced by an alternating acceleration of electric charges. These alternating fields propagate through space with the velocity of light and

produce alternating e.m.fs. in conductors or circuits through which they pass.

The space in the immediate neighborhood of an open-circuit oscillator, through which these electromagnetic waves pass, also contains an alternating electrostatic field due to the alternating displacement of the charges. This alternating electrostatic field impresses an alternating potential difference on conductors. Any conductor in that space, therefore, is affected by both the alternating electrostatic field and the electromagnetic pulses.

Figure 7(a) represents the space about two uncharged spheres of a Hertz oscillator. The proton and electron fields of each conductor "neutralize" but do not destroy each other. When

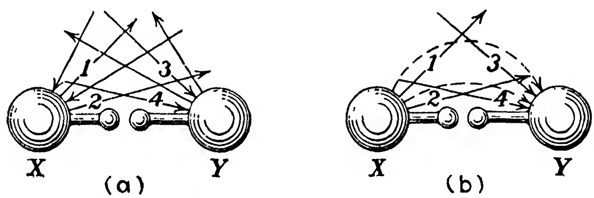


FIG. 7.—(a) Superposed proton and electron fields about an uncharged oscillator, (b) superposed proton and electron fields about a charged oscillator. The lines 1, 2, 3, 4 are identical with those in (a) and have not moved during the transfer of electrons from X to Y. The broken-arrowed lines represent the resultant or the observed field.

electrons are transferred from X to Y during an oscillation, the unnumbered electron lines about the sphere X are transferred with them to the sphere Y, neutralizing the unnumbered proton lines associated with that sphere. The result of the transfer is shown in Fig. 7(b). The lines 1, 2, 3, 4 of the two superposed fields are identical with those so numbered in Fig. 7(a) and have not moved during the transfer. The resultant or observed field, represented by the broken-arrowed lines, was in the same place before the transfer and has not moved. The transfer only eliminated an equal opposing field. The changing electrostatic field about the spheres, therefore, appears and disappears without moving and consequently has no magnetic field associated with it and being a conservative field can produce no e.m.f. in a circuit. Since the two spheres form an electric doublet (Art. VII–8), the intensity of the electrostatic field varies inversely as the cube

of the distance and becomes inappreciable at comparatively short distances.

The conductor C, Fig. 8, is shown in the neighborhood of the two charged plates A and B of a Hertz oscillator. The electrons in C are shown displaced to the right by the electrostatic field E of the charges on A and B; hence when those in A begin to flow

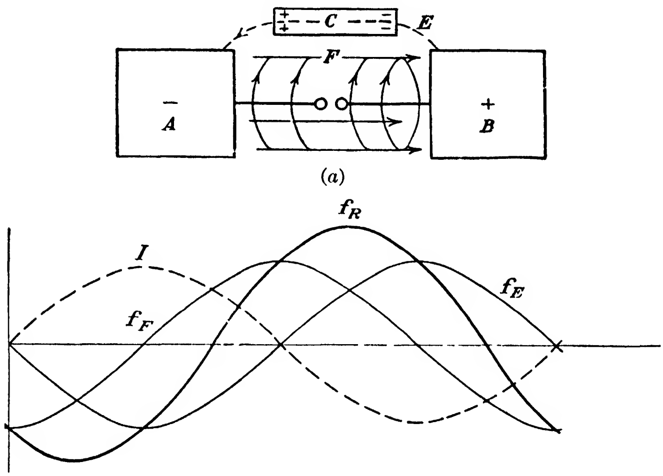


FIG. 8.—(a) Combined effect on a conductor C in the neighborhood of a Hertz oscillator, produced by the electric component F of the electromagnetic pulse and the unbalancing of the electrostatic forces. (b) Phase relationship of the flow (I) in the Hertz oscillator to the force (f_F acting on an electron in the neighboring conductor because of the electric component F and the force (f_E) acting on the same electron because of the unbalanced electrostatic forces. The resultant of the two forces is represented by f_R.

(accelerate) toward B and thereby diminish the intensity of this electrostatic field, the displaced electrons in C, on account of the diminution of the force separating them, move from right to left (in a direction reverse to that of the acceleration of those between the plates). The electric component F of the electromagnetic pulse cuts through the conductor C at the same time and also urges the electrons from right to left at the represented instant.

The intensity of the electric component of the electromagnetic pulse varies as $1/d$ because the surface of a cylindrical pulse varies directly as the radius. But it has been shown (Art. VII-8) that at some distance d from the oscillator, the intensity of the electrostatic field varies as $1/d^3$. The latter then becomes

negligible in comparison with the former except in the immediate neighborhood of the oscillator.

The rate of change of the electrostatic field E is the maximum when the current between the plates has its maximum magnitude, because the quantity of electricity on the plates is then changing at the greatest rate; while the electric component of the electromagnetic pulse at that instant has zero intensity because the electron acceleration then is zero. The curves representing the forces acting on an electron at all points of a cycle therefore differ 90° in phase, as shown in Fig. 8(*b*).

All alternating and oscillating currents produce electromagnetic waves whose wave length depends on the oscillation frequency. Ionized vibrating molecules of a hot body emit electromagnetic waves called radiant heat. Orbital electrons falling from one energy level into another emit photons which produce heat, light, ultraviolet rays, and x-rays, all of which display the interference phenomena of wave motion (Art. XXVII–1). The γ-rays differ only in that they are photons emitted from atomic nuclei. All these display interference phenomena which give them definite characteristic wave lengths. These wave lengths are given in the following table:

Kind of waves	Wave length, centimeters	Source	Method of measurement
γ-rays	$(1 \text{ to } 100) \times 10^{-10}$	Nuclear explosions	Magnetic deflection of secondary β-rays
X-rays	$(5.7 \text{ to } 100{,}000) \times 10^{-10}$	Orbital electrons	Crystal diffraction
Ultraviolet	$(1 \text{ to } 390) \times 10^{-7}$	Orbital electrons	Photoelectric effect and grating
Light	$(3.8 \text{ to } 7.6) \times 10^{-5}$	Valence electrons	Grating
Heat	$(7.6 \text{ to } 1{,}000) \times 10^{-5}$	Atomic and molecular vibrations	Grating and interferometer
Short electromagnetic	.01 to 30	High-frequency discharges	Resonance
Radio	$30 \text{ to } (25 \times 10^5)$	Oscillators	Resonating circuit
Long electromagnetic	$(25 \times 10^5) \text{ to } \infty$	Alternating currents	Oscillograph

The velocity c of electromagnetic waves is 3×10^{10} cm/sec, and the wave length

$$\lambda = \frac{c}{f} = \frac{c}{\nu} \text{ cm,} \tag{3}$$

where f or ν is the frequency in cycles per second. The frequency of radio waves is usually given in *kilocycles* (1 kilocycle = 1,000 cycles).

5. Radiation from Closed and Open Oscillating Circuits.— The condenser circuit, including the primary coil P, Fig. 5, is a closed oscillating circuit and the current at all times is flowing in opposite directions on its opposite sides. The electromagnetic waves in any one direction come from both sides of the circuit; and since the lengths of the waves are usually much greater than the distance between the sides, there is almost complete interference, as explained in Art. XVI–3. Such circuits radiate but little energy and can produce large effects only in their immediate neighborhood.

The secondary circuit S is an open-circuit oscillator, the electrons oscillating in one plane only. Because there is no interference such as exists in the closed circuit, this type of circuit is always used for generating electromagnetic waves. The oscillations are produced in a closed circuit and then are transformed into open-circuit oscillations which radiate the energy.

6. Absorption of Rays—Radiation Pressure.—When electromagnetic waves pass through a conductor, they induce an alternating current in it (Art. XVI–5). The production of this current takes energy from the waves; hence the energy of the waves beyond the wire is lessened by the amount absorbed. If the wire is tuned (Art. 3) to respond by resonance to the electromagnetic waves, a much larger amount of energy is absorbed.

When short electromagnetic waves, those of light, for example, pass through a body, they are diminished but slightly in intensity if no charges within the substance respond by resonance or are forced into higher energy levels. When the body is opaque, there are systems of charges within the substance that respond by resonance or electrons that are forced into higher levels. The

free electrons also are given velocities and absorb energy, most of which is ordinarily converted into heat.

Electromagnetic waves of all periods exert radiation pressure on all substances that absorb their wave energy, *i.e.*, that are opaque to them. This follows from the fact (Art. V–9) that a moving electric field and its accompanying magnetic field urge both positive and negative charges in the direction of motion of the fields. Bright sunlight exerts a pressure of 0.4 milligrams on each square meter of a black surface.

7. Resonance Circuit.—A resonance circuit, Fig. 9(*a*), is a special case of a divided circuit and consists of a condenser in one of the two branches and an inductive coil of negligible (or low) ohmic resistance in the other. The branches have reactances of such magnitudes as, for the particular frequency of the impressed e.m.f., cause the current in the divided circuit to oscillate in resonance with the impressed e.m.f.

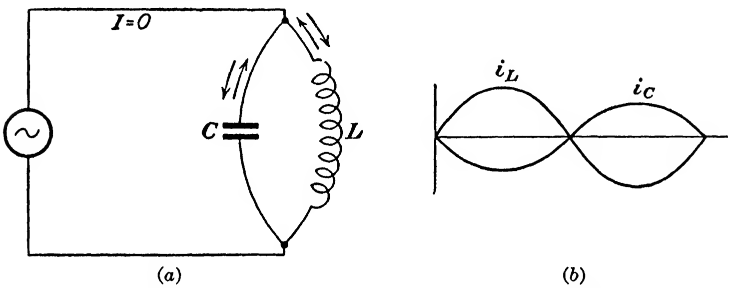


FIG. 9.—(*a*) Resonance circuit; (*b*) phase relation of the currents in the two branches.

From Eq. XVIII–10, the e.m.f. impressed on the inductive branch is $E_L = \omega L I_L$ and that impressed on the capacitive branch is $E_c = I_c/\omega C$. But since $E_L = E_c$, both being the same impressed e.m.f.,

$$\omega L I_L = \frac{I_c}{\omega C}.$$

Because the current in the inductive branch lags 90° behind the impressed e.m.f. and that in the capacitive branch leads by the same amount (Art. XVIII–11), the two currents are exactly opposite in phase as represented in Fig. 9(*b*). From this it follows that, since the current in the line must be equal to the vector

sum of the currents in the two branches, the magnitude of the current in the line can be made zero by the proper adjustment of L or C. Then the two equal currents in the branches continue to flow as one current around the divided circuit and thereby charge and discharge the condenser with the frequency of the impressed e.m.f.

When the currents in the two branches are equal, the preceding equation becomes

$$\omega L = \frac{1}{\omega C},$$

from which

$$\omega = 2\pi f = \frac{1}{\sqrt{LC}}$$

and the frequency

$$f = \frac{1}{2\pi\sqrt{LC}}.$$

This is the expression (Art. 2) for the natural frequency of an oscillating current in the LC circuit. Since this is also the frequency of the impressed e.m.f., either the frequency of the e.m.f. for resonance in any given circuit or the magnitude of LC required for any given resonance frequency can be calculated.

8. Practical Production of Undamped Oscillations.—Since oscillating electric charges heat the conductor and radiate energy, they are necessarily damped as represented in Fig. 3. In order to produce undamped oscillations, energy must be supplied to the circuit in the same manner that the main spring supplies energy to the pendulum of a clock. The resonance circuit of Fig. 9 has energy supplied to it in this manner and the oscillations in it therefore are undamped. This form of oscillator is used generally as a part of radio circuits.

The *thermionic* or *vacuum-tube oscillator*, one form of which is shown in Fig. 10, is supplanting other methods for the production of undamped oscillations and especially for the production of carrier waves in radio. The operation of this oscillator may be explained as follows:

1. The condensers C_P and C_T become charged at the instant the B-battery circuit is closed. The sudden rush of electrons from one plate of C_T to the other causes it to become overcharged because of electron inertia. Such a condition, as explained in Art. 2, produces oscillations which in this case are in the so-called closed *tank circuit* C_TL_T, represented by the heavier lines. The frequency of the oscillations depends on the capacitance and self-inductance in that circuit and, in the illustrated case, may be adjusted by changing the capacitance.

2. The coil L_T is composed of two parts, L_P and L_G; and when a current is flowing through it, a potential difference exists between the extremities of L_G. These extremities are connected, directly

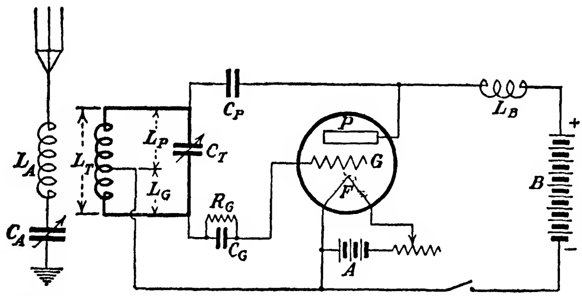


FIG. 10.—Three-electrode vacuum-tube oscillator coupled to an antenna for the generation of undamped electromagnetic or carrier waves.

or indirectly, one to the filament F and the other to the grid G of the three-electrode vacuum tube. The potential difference between the filament and the grid then alternates in unison with the oscillations in the tank circuit.

3. These alternating potentials of the grid alternately increase and decrease the plate current. This varying current repeatedly charges the condenser C_P and thereby impresses potential variations on the condenser C_T in the same manner as the original charge. In-phase impulses therefore are superposed on those in the tank circuit and contribute to the energy of the oscillations. This increased energy in turn increases the variation in the plate current, etc. The energy of the oscillations grows until it is limited by the supply of electrons from the filament. The oscillations in the tank circuit are called *undamped oscillations* and are of uniform intensity. They take place automatically

and continue until the circuit is opened, the energy being supplied by the battery B.

4. The open-antenna circuit is inductively coupled to the tank circuit and may be tuned to resonance with it by means of either a condenser C_A or the inductance L_A. It then receives the maximum possible amount of energy from the tank circuit and radiates a part of it in electromagnetic waves.

High-frequency generators are also employed in the production of long electromagnetic waves. Frequencies as high as 100,000 cycles/sec have been obtained with them.

9. High-frequency Currents.—The current oscillating in the condenser circuit containing the spark gap G, Fig. 5, is known as a *high-frequency* current. Although the e.m.f. in this circuit is high, that of the secondary circuit is made many times greater by the use of a step-up high-frequency transformer PS (without iron) and by adjusting the circuits to resonance. Many interesting experiments may be performed with this higher e.m.f., among which are the corona, large sparks, lighting an electric lamp with the current flowing through the body, setting wood on fire, exciting a Geissler tube placed at a distance from the circuit, etc.

When an e.m.f. is applied to a circuit, the electrons start to accelerate, but because of self-induction each one pushes its neighbors in the reverse direction (Law C). The acceleration is retarded more within the wire than on the surface, because the surface electrons are being retarded only by neighboring electrons on one side while the inner ones are being retarded by electrons on all sides. Because the rapid oscillations of high-frequency currents do not allow time for the retarded flow to build up, the outer electrons are set in oscillation while the inner ones remain practically at rest. The oscillating current, therefore, exists mainly on the surface of the conductor. This phenomenon is called the *skin effect* of high-frequency currents.

In the case of an electrolyte (Art. XI–1) and that of the human body the current is carried by ions instead of electrons. The masses of these ions are comparatively large and the acceleration is proportionately small. The skin effect, therefore, is comparatively slight, so that the high-frequency current passes through the whole mass.

A high-frequency current does not produce the injurious effects of a low-frequency current because the rapid alternations do not permit a large enough displacement of the ions. However, since the displacements are sufficient to heat the tissues, high-frequency potential differences and currents are used for that purpose in medical treatment. The currents are used also in surgery where the operating electrode has the form of a needle or knife. The intense current produces molecular disruption of the tissue along the line of application and a coagulation, so that there is but little, if any, bleeding.

The current in the primary closed circuit P, Fig. 5, contains more energy than that in the secondary open circuit, and with it the following experiments may be performed: (1) If the circuit contains a small coil of a few turns of wire, and a piece of metal is held within it by an insulating handle, the metal soon becomes red hot because of the large eddy currents induced within it at such high frequency. This is the principle of the *induction furnace.* (2) A partially exhausted tube becomes brilliantly luminous when held within a coil of a few turns of wire in such a circuit. The free electrons and the natural ions (Art. XI–7) are forced to oscillate by the rapidly oscillating field, and the electrons attain such velocities as to ionize the gas. This is known as *electrodeless discharge.*

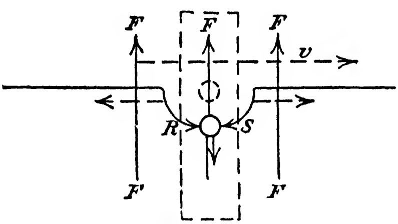


FIG. 11.—Change of phase of electromagnetic waves by reflection.

Great care must be exercised in connection with the current in the primary or condenser circuit. Placing the hands across the gap G applies the two terminals of the transformer directly to the body and produces fatal results.

10. Reflection of Electromagnetic Waves from Conducting Surfaces.—Electromagnetic waves impinging on a conductor cause its free electrons to oscillate and send out electromagnetic waves in all directions which are opposite in phase to the impinging waves, as may be seen by inspection of Fig. 11. The electron is shown being accelerated by the electric component F of the impinging wave. The electric lines of force associated with the

electron are shown distorted by the acceleration. These distortions are the pulse of an electromagnetic wave which emanates from the electron in all directions.

The part R of the electromagnetic pulse is the reflected beam whose phase is the reverse of that of the impinging field; *i.e.*, the phase of a beam is changed 180° by reflection. The part S of the electromagnetic pulse travels with the transmitted field and, disregarding absorption, diminishes it by an amount equal to the energy in the reflected field and without changing its phase.

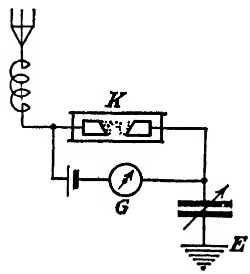


FIG. 12.—Coherer.

Conducting layers (Kennelly-Heaviside layers, Art. XXX–6) exist in the atmosphere at heights varying from 60 to 400 km from the surface of the earth. Electromagnetic waves are reflected from them as well as from the surface of the earth and are propagated outward between these two surfaces. The intensity of the waves diminishes far less with distance than it would if these reflecting surfaces were not present.

Electromagnetic waves of any length are reflected, refracted, and polarized in the same manner as light waves.

11. Detectors of the Longer Electromagnetic Waves.—The following are the chief methods used for detecting electromagnetic waves:

1. *Resonating Circuit.*—A loop of wire adjusted to have the same period of oscillation as the oscillating circuit, if near it, responds by resonance sufficiently to produce small sparks. The tuned Leyden jars of Fig. 6 are a more complex example of such circuits.

2. *Coherer.*—A mass of metal filings has a large electric resistance because of adhered gases. When electromagnetic waves induce an e.m.f. in the circuit, Fig. 12, the metallic particles of the coherer K "cohere." It is believed that small sparks break down the insulating layer and cause the metal surfaces to come in direct contact. When the particles cohere, they cause a current to flow through the galvanometer G. Tapping the tube decoheres the particles; however, the decohering may be made automatically by replacing the galvanometer with a bell which, when ringing, jars the tube. The coherer is practically of historic interest only.

3. *Crystal Detector.*—Neither a sensitive galvanometer nor a telephone receiver can respond to a radio-frequency oscillating

current; therefore some method must be employed to change such a current to one consisting of unidirectional pulsations. This change may be accomplished by means of various crystals, such as those of carborundum, lead sulphide, and molybdenum,

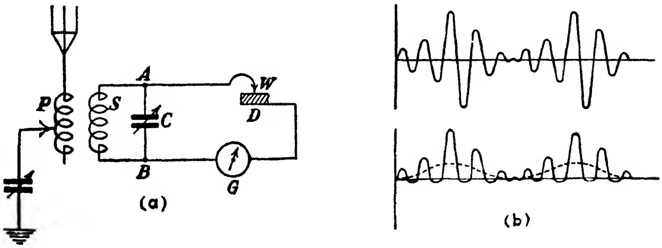


FIG 13.—(a) Crystal detector, (b) unrectified and rectified waves.

which when placed in contact with a fine wire as shown at D, Fig. 13(a), allow the current to flow more readily in one direction than in the other (Art. XXV–8d).

The coupled condenser circuit, $ASBC$, is a resonance circuit, tuned to the frequency of the antenna, and its induced oscillations produce alternating potential differences between the points A and B which are greatly increased in magnitude by resonance. These potential differences are impressed on the crystal and cause an almost unidirectional pulsating current to flow which deflects the galvanometer coil or causes a click in a telephone.

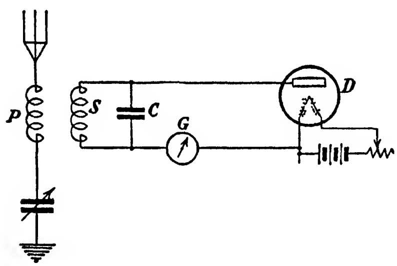


FIG. 14.—Two-electrode vacuum-tube detector.

Figure 13(b) illustrates a modulated incoming wave of radio-frequency and the same wave after being "rectified" by the crystal detector. The broken line indicates the audio-frequency modulation to which the telephone responds (Art. XXIX–5).

4. *Two-electrode Vacuum-tube Detector.*—If the crystal detector is replaced by a two-electrode vacuum tube D, Fig. 14, the oscillating potential applied to the plate causes the current to flow in one direction only, accomplishing more fully than the crystal detector the function of a valve. The three-electrode

vacuum tube, however, accomplishes this purpose in a still more satisfactory manner because it has also other desirable characteristics.

5. *Three-electrode Vacuum Tube Detector and Amplifier.*—The three-electrode vacuum tube has already been described in Art. XXV–7. The resonant circuit there has the same function as that in the crystal-detector circuit of Fig. 13.

12. Detection of Electromagnetic Waves of all Periods.—All electromagnetic waves whose frequency is greater than 0.3×10^{12} cycles/sec may be detected and measured by means of resonant circuits. Oscillations of higher frequency are produced, as already explained, by oscillating individual ions or electrons. In order to detect the waves from these oscillations, individual ions or electrons must respond by resonance. The methods of detecting and measuring the various types of waves are listed in the table of Art. 4.

The energy of electromagnetic waves of all frequencies is ultimately changed into heat.

Questions

1. Explain how electric oscillations are produced in a wire.

2. Explain how the energy of the electrostatic field changes into that of the magnetic field and *vice versa*. Give two points of view.

3. Explain why the oscillations in a circuit are damped. What becomes of the energy?

4. Explain why an increase either in capacitance or in self-inductance increases the period of oscillation.

5. Give the period of oscillation in terms of L and C.

6. Why must the resistance be small in order that the discharge may be oscillatory?

7. Describe the Hertz oscillator. How is the oscillating current kept from the circuit supplying the electricity to the plates? Are these closed- or open-circuit oscillations? Damped or undamped oscillations?

8. Explain how damped oscillations may be produced in a closed circuit (Tesla coil).

9. How can open-circuit oscillations be induced by means of closed-circuit oscillations?

10. Explain resonance and the action of resonance jars.

11. Explain how the production of the electrostatic field about a Hertz oscillator may be regarded so that its appearance and disappearance do not involve the field's motion.

12. Describe how the changing electrostatic field about the oscillator affects the electrons in a neighboring conductor compared with the effect on

them produced by the electric component of the electromagnetic pulses. Give the phase relationship.

13. Why does the effect of the electrostatic field become negligible at some distance from the oscillator?

14. Describe the production of electromagnetic waves by oscillating charges.

15. Explain why closed-circuit oscillators radiate less energy than open-circuit oscillators.

16. Explain how the short electromagnetic waves, known as radiant heat, light, x-rays, etc., are produced

17. Explain one form of a resonance circuit and the required condition for resonance.

18. Explain how undamped oscillations are produced by means of a vacuum-tube oscillator.

19. What are high-frequency currents? Why do they flow only on the surface of metallic conductors? How do they flow through electrolytes? Through the human body?

20. What two types of oscillating circuits are represented in the Tesla coil? Which of the circuits is dangerous and why?

21. Explain why the reflected and the impinging waves differ 180° in phase.

22. Explain the different methods of detecting electromagnetic waves: resonating circuit, coherer, crystal detector, two-electrode vacuum tube, three-electrode vacuum tube.

23. Name the different types of electromagnetic waves, and state how each type is detected and measured

Problems

1. A condenser whose capacitance is 500 microfarads is in an oscillating circuit whose self-inductance is 0.002 henrys and whose resistance is negligible. What are (*a*) the period, (*b*) the number of oscillations per second, and (*c*) the wave length of the radiated electromagnetic waves?

2. A Leyden jar whose capacitance is 0.02 microfarads is a part of an electric circuit whose self-inductance is 0.025 henrys and whose resistance is negligible. (*a*) What is the natural frequency of that circuit? (*b*) What is the length? (*c*) Would this be an oscillatory circuit if its total resistance were 300 ohms?

Experiments

1. Natural period of oscillation—colored water oscillating in a U tube—pendulum oscillating.

2. Resonance top to illustrate principle of resonance.

3. Hertz oscillator and loop detector (Neon tube in gap of detector).

4. Resonance of jars (tuned).

5. Tesla coil.

1. Corona.
2. Discharge (*a*) into space (*b*) to earth.
3. Discharge to person on an insulated stool. (*a*) Spark to person. (*b*) Lighting incandescent lamp. (*c*) Setting wood on fire.
4. Geissler tube operating at a distance without connections.

6. High-frequency currents in primary of Tesla coil. (*a*) Electrodeless discharge. (*b*) Induction furnace principle (metal heated to redness).

7. Resonance circuit.

8. Vacuum-tube oscillator (giving audible tones).

9. Various detectors of electromagnetic waves.

CHAPTER XXIX

COMMUNICATION BY ELECTRICITY

1. Electric Bell.—The *electric bell,* Fig. 1, consists of an electromagnet M, which is energized by a battery or a transformer when the circuit is closed through the push-button K, and of an iron armature A, which normally is held at some distance by means of the spring. When the armature is attracted, the electric circuit is broken at the point P, and the electromagnet is demagnetized. The armature then springs back to the original position where the circuit is again closed. This operation

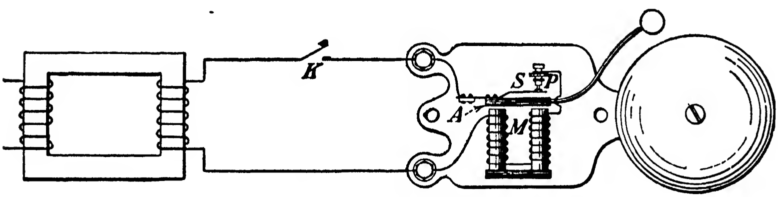


FIG. 1.—Electric bell.

repeats itself indefinitely as long as electric contact is being made at the push button. A step-down transformer connected to the lighting system gives better satisfaction than a voltaic cell which requires attention.

2. Telegraph.—The *telegraph* is another application of the electromagnet. In its simplest form the telegraph consists of a *key* at the sending station and a *sounder,* Fig. 2, at the receiving station. The sounder is an electromagnet whose armature makes an audible click both on being attracted and on being released. The interval between these clicks indicates whether a dot or a dash of the code is being sent. The *relay* is similar to, but more sensitive than, the sounder; instead of producing a sound it closes and opens a local battery circuit which furnishes a new supply of energy to operate the local sounder. In ordinary telegraphy the earth is used as the return circuit.

Telegraph instruments may be constructed and connected so as to allow a message to be sent in both directions at the same time or two messages each way at the same time. These systems are known as *duplex* and *quadruplex* systems.

The *rotary multiplex system of telegraphy* employs two electric motors, one at each end of the line, which are kept rotating in exact synchronism by means of tuning-fork controls. Each motor rotates an arm at high speed; this arm makes a contact in succession with a multiplicity of lugs, each of which is connected to a sending or a receiving instrument. For an instant during each revolution each sending instrument is connected through a single common transmission line to a particular mated

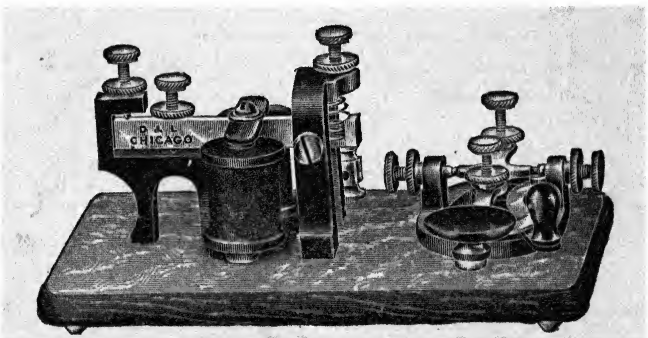


FIG. 2.—Telegraph key and sounder. *(Central Scientific Co.)*

receiving instrument at the receiving station. These connections are made so frequently that any one circuit operates without sensible interruption. Ten messages may be sent over the same wire at the same time; and by using the duplex systems that many messages may be sent simultaneously in each direction. The large capacitance of long transmission lines, however, does not usually permit the simultaneous sending of more than five messages in each direction.

The *multiplex page-printing telegraph* consists of five electromagnets and in its simplest form would require five wires between stations. The message is sent by means of a typewriter which operates in the ordinary manner except that holes, a combination of which corresponds to a given letter, are punched in a paper tape. Each letter is represented by one hole in each of one or more of five lanes in which the holes may be punched.

This punched-paper tape passes through a mechanism which completes electric circuits as the holes pass certain points. The holes in each lane operate one of the five electromagnets at the receiving station. The lever that prints any particular letter is released only when the proper combination of electromagnets is energized at the same time. For example, for the letter *a* the sending typewriter punches one hole in each of two particular lanes. At the receiving station the two corresponding electromagnets are energized and together release the lever that prints the letter *a*.

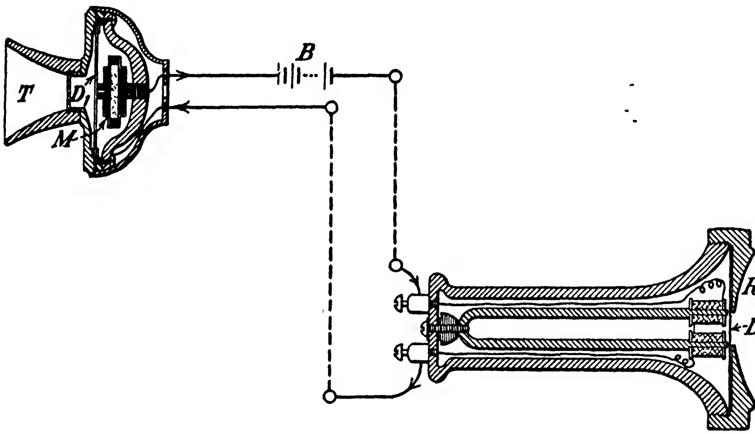


FIG. 3.—Principle of the telephone.

The multiplex page-printing telegraph system is now generally used with the *rotary multiplex system,* whose 10 lugs enable two such sending instruments to transmit messages over the same wire at the same time; and if the duplex system is added, two messages can be sent in each direction.

3. Telephone.—If one speaks into the transmitter *T*, Fig. 3, the thin iron plate or diaphragm D_1 is set into vibration in unison with the sound waves. This vibration increases and decreases the pressure on the nearer side of the *microphone M*, which consists of carbon granules between two plates. These granules offer a large electric resistance which decreases rapidly with pressure. Changes of pressure due to the vibrating diaphragm then produce variations in the current of the transmitting

circuit which correspond to the vibrations produced by the speech.

At the receiving end the variable current passes through a double solenoid wound on soft-iron pole-pieces which are attached to the poles of a horseshoe magnet. The current and the magnet are of such strength as to magnetize the soft-iron pieces to a point halfway between the knees of the magnetization curve. The change of the magnetization is then proportional to the fluctuations in the magnetizing current, and the diaphragm D_2 of the receiver vibrates in unison with the diaphragm D_1 of the transmitter and thereby reproduces the sound.

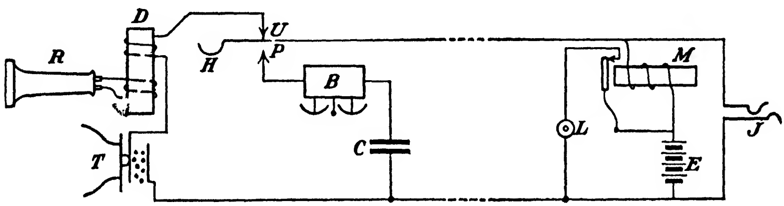


Fig. 4.—Subscriber's connections to central station.

The connections to the central station are shown in Fig. 4. When the receiver R is on the hook H, the arm makes contact through P to the bell B and the paper condenser C. The battery E cannot force a current through the condenser, but the operator can attach an intermittent source of alternating current (not shown) which by repeatedly charging the condenser in reverse directions rings the bell. When the receiver is taken from the hook, the arm springs up (making contact at U as shown) and completes the electric circuit, which includes the battery E, the transmitter T, and the electromagnet M. The electromagnet attracts an armature which closes the circuit of the lamp L, notifying the operator that the subscriber is on the line. The operator makes the desired connection to the called subscriber through the jack J.

4. Long-distance Telephony.—In long-distance transmission, the fluctuating potential difference across the primary of a step-up transformer, Fig. 5, is transformed in the secondary into a high-tension alternating e.m.f. whose wave form corresponds to the diaphragm vibrations of the transmitter. At the receiving end,

the high-tension e.m.f. is stepped down by means of a second transformer which then energizes the receiver.

In uncompensated transmission lines the effect of capacitance on phase difference exceeds that of self-inductance, so that in lines of considerable length the phase lead of the current is appreciable and differs in magnitude with the frequency as is seen by inspection of Eq. XVIII–12. The parts of the current pulsations due to the various frequencies composing speech (or music), therefore, are distorted unequally and change the form of the pulses so that the reproduced speech is unlike the original and may even be unintelligible. This distortion is practically eliminated by placing self-inductances called *loading coils* (Fig. 5)

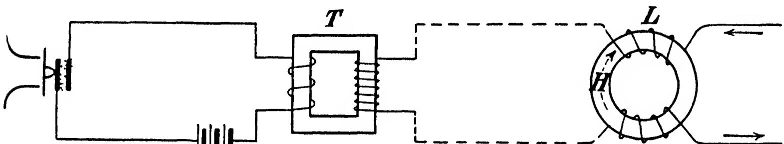


FIG 5.—Loading coil L in circuit with a transformer coupled to a telephone transmitter.

into the circuit at regular intervals. The coils are distributed so as to balance out the distributed line capacitance as much as possible. The coils are usually 6,000 ft apart, but in high-quality radio-program transmission lines the distance is only 3,000 ft. The inductances produce a phase lag which neutralizes the lead caused by the line capacitance. Two loading coils, L, are wound on the same core of permalloy (Art. XX–7) whose permeability at low degrees of magnetization is about one hundred times that of iron (Arts. XX–7, 8).

In a submarine cable the loading coils are replaced by an insulated wire of permalloy wound spirally about the cable and encased with it.

In transcontinental telephony the distances are so great that thermionic relays, Fig. 6, must be employed at intervals to add energy to the circuit. The distance between two adjacent relays in the case of open wires is about 300 miles, while that in the case of 19-gage cable circuits is 50 miles. The fluctuating current from the telephone comes to the relay by the line wires L_1 and varies the potential of the grid G. This variation causes the

battery B to vary the current in the outgoing circuit L_2 and thereby to duplicate therein, with increased energy, the varying current of the incoming circuit. The inertia of the electrons is so small that no distortion of the sound is produced. The energy in the outgoing circuit comes entirely from the B battery of the relay circuit. The service of the original fluctuating current is to vary the potential of the grid in the tube, which then functions as an amplifier (Art. XXV–7). In practice a more complex arrangement with two tubes is found to be more efficient and is used.

A recent development in the art of telephony employs high-frequency "carrier currents" to "carry" the sound frequencies over the transmission line. The apparatus used for each

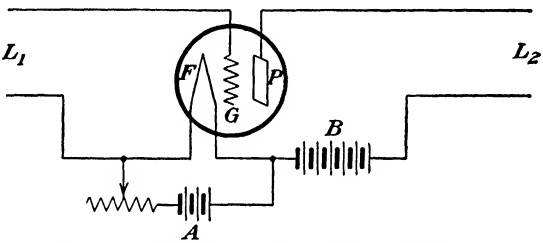


FIG. 6 —Thermionic relay repeater station.

communicating set consists of a microphone transmitter, a high-frequency oscillator to generate the carrier current, apparatus for modulating the carrier with the sounds to be transmitted, a filter system to keep the carrier in its proper place without interfering with the other instruments, a transmission line, another filter system to sort the carrier to the proper detector (demodulator), and a telephone receiver. It is thereby possible to transmit several telephone messages at the same time over a single pair of wires. These messages may be, and are, sent without interference over lines which are already in use transmitting telegraph messages by the rotary multiplex system. In this manner the cost of erecting and maintaining transmission lines is greatly reduced.

The telephone receiver is an exceedingly sensitive instrument and, for that reason, it is necessary to have a complete metallic circuit with the line wires twisted tightly together in order to prevent inductive disturbances from neighboring circuits. The

ordinary telephone transmitter functions well only within the four octaves of ordinary speech. For the transmission of music it is necessary to employ a transmitter or microphone which responds uniformly to all the audible frequencies in music. Among the types fulfilling this requirement are the condenser microphone, the ribbon microphone, the dynamic microphone, the double-button microphone, and the piezoelectric microphone. Only the first-mentioned will be discussed. The condenser microphone consists of an air condenser, one of whose plates is a tightly stretched duralumin diaphragm at a distance of about

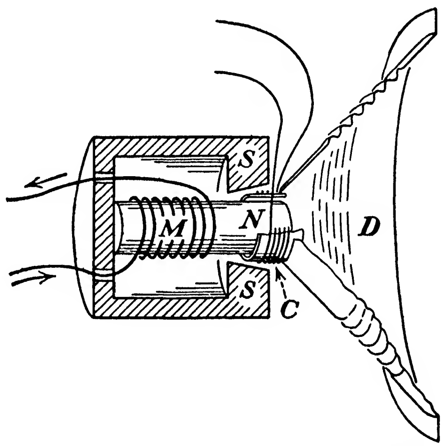


FIG. 7.—Section view of dynamic speaker.

0.03 cm from the other plate. The vibrations impressed on the diaphragm change the capacitance of the condenser and thereby the potential difference due to its charge. This p.d. and the e.m.f. of the charging battery normally are in equilibrium, so that the p.d. variations produce a current in the circuit whose intensity variations correspond to the sound vibrations.

It is also necessary to employ a special receiver or reproducer which responds to all the octaves of music. One type of reproducer fulfilling this requirement is known as the *electrodynamic reproducer* or *dynamic speaker* (Fig. 7). It consists of a cone-shaped parchment diaphragm D, to the apex of which is attached a coil C suspended in the field of a strong electromagnet M. The modulated current flowing through this coil causes it and the

diaphragm to vibrate, thereby reproducing the transmitted sound.

5. Continuous-wave Radio Telegraphy.—A transmitting station may send out electromagnetic waves from an open oscillating

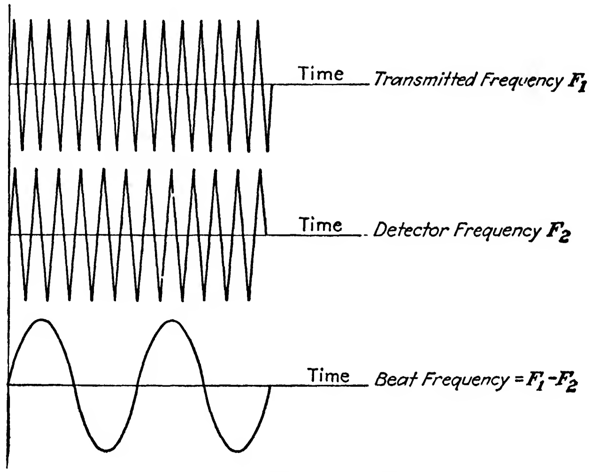


FIG. 8.—Beat frequency.

circuit (Art. XXVIII–8) energized by a vacuum-tube oscillator or a high-frequency generator. The operator of the station interrupts the outgoing wave in accordance with some code by means

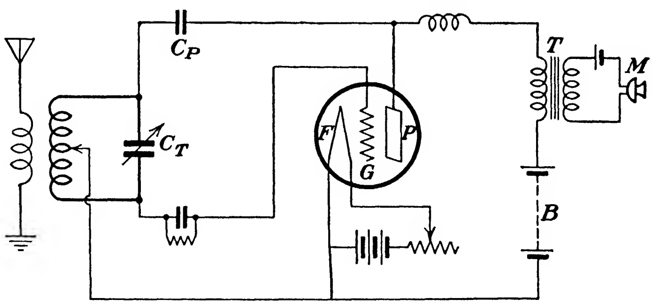


FIG. 9.—Modulation of the carrier waves in a transmitting set.

of a key inserted in a vital part of the circuit. These waves are received by employing an oscillating detector tube the frequency of which forms a beat note with the transmitted frequency. The frequency of the beat note is equal to the difference between the transmitted frequency and the frequency at which the detec-

tor is oscillating (Fig. 8). If this difference is made not more than 10,000, the result is an audible beat note, thus making the signals interpretable.

6. Wireless Telephony.—In order to telephone by means of electromagnetic waves it is necessary to vary the intensity of the carrier waves emanating from an oscillating circuit (Art.

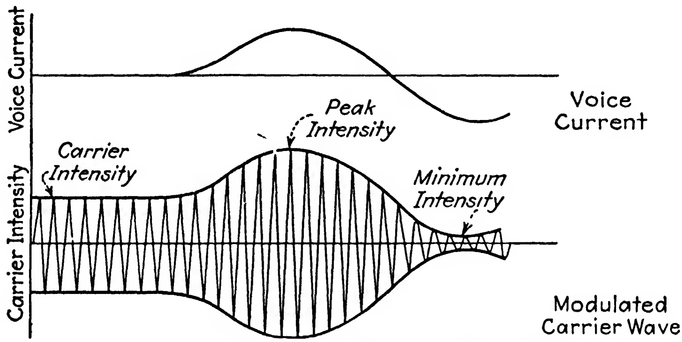


FIG. 10.—Intensity modulation of carrier waves.

XXVIII–8), to correspond with the sound waves. This may be accomplished by a method shown in Fig. 9, which in the main is a reproduction of the vacuum-tube oscillator of Fig. XXVIII–10. Talking into transmitter *M* produces audio-frequency current variations in the primary of transformer *T*. These variations

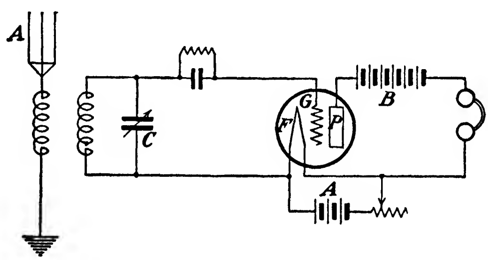


FIG. 11.—Three-electrode vacuum-tube detector circuit.

act inductively on the secondary, thereby inducing a corresponding alternating e.m.f. which alternately assists and opposes the e.m.f. of battery *B*. Since the carrier intensity varies as the square of the applied voltage *B*, it follows that these audio undulations produce corresponding variations in the intensity of the carrier wave (Fig. 10).

Modulated waves are received by a device the essentials of which are an antenna system to transform the electromagnetic waves to electric impulses, a tuned circuit resonating at the frequency of the transmitter, a detector (Arts. XXV–7, XXVIII–11) to change the modulated radio-frequency currents into audio-frequency currents, and a telephone to transform the audio currents into sound waves. Of the many possible types the one shown in Fig. 11 is practicable.

7. Radio-frequency and Audio-frequency Amplification.—Figure 12 illustrates radio-frequency amplification, the function of the detector tube, and audio-frequency amplification and shows the type of oscillations in each of the several circuits.

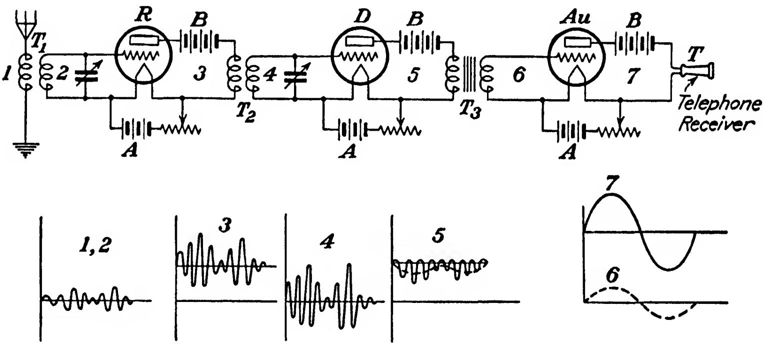


FIG. 12.—Modulated radio-frequency oscillations changed to audio-frequency oscillations: Circuits 1, 2, 3, and 4 have radio-frequency oscillations; circuit 5 is a detector circuit; and circuits 6 and 7 have audio-frequency oscillations.

For the purpose of enabling the different circuits to be more easily traced, each of the three tubes is provided with its own A and B batteries. In practice one A and one B battery, or their substitutes (Art. XXV–8), supply energy for all the tubes. The tube R is a radio-frequency amplifier, and D a detector. If a telephone receiver were placed in the plate circuit of D, the receiver would respond, and the set would then be using one stage of radio-frequency amplification and one detector. The plate circuit of D, however, is shown containing the primary of an audio-frequency transformer T_3. The audio-frequency transformer, on account of its iron core, cannot respond to the individual radio-frequency pulses; hence the current induced in the secondary is such as would be produced by the average current

value of the adjacent pulses. This average value is shown by the broken-line curve in curve 5. The tube *Au* then has audio-frequency pulses impressed upon it, and its amplified oscillations affect the telephone receiver.

It is necessary to use only one detector regardless of the number of stages of radio and audio amplification that may be employed.

These illustrations of radio telephony only indicate the principles involved. Many schemes are employed in radio building, but they all center around the vacuum tube.

Only a few of the many modifications and improvements are here mentioned:

a. The microphone and reproducer which respond uniformly to all the frequencies in music are necessary for satisfactory transmission and reproduction of music. These were discussed in Art. 4.

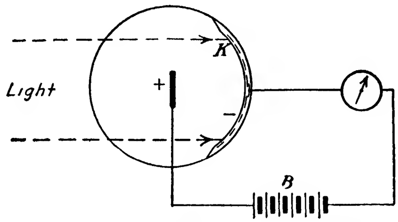
b. If a coil is inserted into the plate circuit of the detector tube of a receiving set and so placed that it acts inductively on a coil in the grid or input circuit of the tube, the impulses are thus sent back through the tube and reamplified. Such a circuit arrangement produces a much greater amplification and is called a *regenerative circuit.*

c. The *superheterodyne* receiving set has a detector circuit tuned to the incoming frequency and an oscillating circuit to a lower fequency by about 175 kilocycles/sec. The two frequencies produce a beat the frequency of which is independent of the incoming frequency because the oscillator is tuned relative to it. This beat frequency, which still contains the original modulations, is then amplified and then redetected by a second detector. One of the main advantages of the superheterodyne is that the beat frequency of 175 kilocycles/sec can be amplified with greater efficiency than can the commercial broadcast frequencies of 550 to 1500 kilocycles/sec.

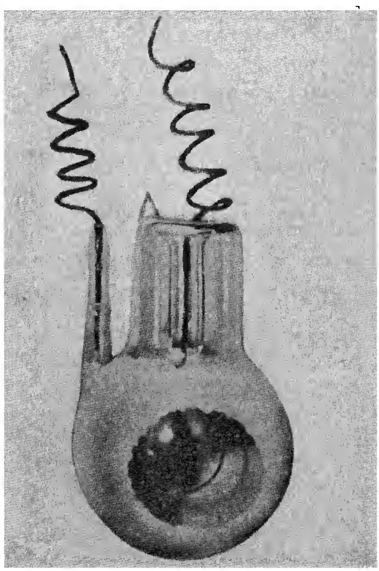
d. The construction of most modern vacuum tubes permits alternating current to be used in heating the filament. This is made possible by either of two methods. In the first the filament heats a cylindrical cathode which is coated with barium and strontium oxides. A tube with such an arrangement is called a *heater-type tube.* The second employs merely a heavy filament which draws a large current. The thermal lag of such a filament

is sufficiently great to prevent an appreciable temperature variation throughout the alternating-current cycle.

e. The use of a rectifier filter system to convert alternating to direct current for use in place of *B* batteries was considered in Art. XXV–8.



(*a*)



(*b*)

FIG. 13.—(*a*) Photoelectric cell circuit; (*b*) a commercial form of the cell.

f. Vacuum tubes with more than three electrodes have, in many cases, supplanted the three-electrode tube.

The *four-electrode* or *screen-grid tube* contains a fourth electrode which is so connected as to nullify the electrostatic field due to the plate in the region between the grid and the plate. The grid

potential then alone controls the electron flow in that region. The amplification factor of such a tube is about 400 as compared with less than 10 for most triodes.

The *five-electrode tube* or *pentode* is similar to the four-electrode tube except that it has an additional screen between the fourth electrode and the plate the purpose of which is to divert back to the plate the electrons which are "splashed" off by the bombarding electrons. Such a tube may have an amplification factor as high as 1,500.

Some tubes have more than one set of elements in a single glass envelope. These are called *multi-unit tubes.*

8. Selenium Cell.—Transparent crystals of selenium that have a resistance of 15,000 ohms in the dark have a resistance of only 500 ohms in bright sunlight. Such crystals, when made a part of an electric circuit, cause the current to vary with the intensity of the light, but not in exact proportion to the intensity of the light falling upon the crystals. When these crystals are placed in a proper protecting case, they form what is called a *selenium cell.* This is being replaced by the photoelectric cell.

9. Photoelectric Cell.—The photoelectric cell (Figs. 13*a*, *b*) consists of an evacuated glass or quartz bulb and two electrodes, one of which connects with a film of photoactive material such as properly treated metallic potassium or sodium which covers about one-half of the inside surface of the bulb. This electrode is represented by K in the figure. A battery of about 100 volts is connected to the terminals and charges the active metal negatively. Light which falls on the negative electrode ejects electrons from it in numbers which are proportional to the intensity of the light. These ejected electrons move to the positive electrode and complete an electric circuit in which the current due to these ejected electrons is accurately proportional to the quantity of the impinging light. The strength of the current is only a few microamperes but, when desired, it can be increased by vacuum-tube amplification. The photoelectric cell finds wide application in photometry, radiotelegraphy, talking moving pictures, and a variety of miscellaneous applications.

10. Transmission of Pictures and Scenes by Electricity.—The negative of the photograph to be sent is bent into the form of a cylinder and attached to a mechanism which rotates it and

at the same time gives it progressive motion to one side. A thin beam of light is focused on the revolving film so that every part of the picture will have been successively illuminated while the cylinder was progressed sidewise the width of the picture. The beam of light passes more or less through the picture depending on the transparency or darkness of the part of the film being illuminated. Inside the cylinder this beam falls on a photoelectric cell and causes the current in the cell circuit to increase and decrease corresponding to the shading on the picture. This varying current may be amplified and then transmitted any distance either by means of a telephone circuit or by radio.

At the receiving station there is a similar mechanism which rotates in synchronism with that at the sending station. A

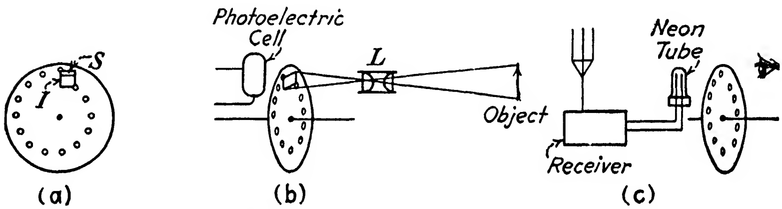


FIG. 14.—(a) Scanning disk; (b) transmitting station; (c) receiving station.

photographic film bent in the form of a cylinder rotates and progresses to one side at exactly the same rate as that of the photograph at the sending station. A beam of light is focused in a manner similar to that described above; however, this beam is more or less obstructed by a narrow-ribbon shutter in a magnetic field. The motion of this shutter is controlled by the current received from the sending station and is due to the reaction between the current and the magnetic field. The intensity of the light which falls on the film at any instant is thereby made to correspond to that of the light passing through the picture at the transmitter; hence the film areas are properly exposed to light and are in form to reproduce the picture being sent. The photographic film is developed in the ordinary manner.

Another method employed in the transmission of pictures or scenes uses a rotating scanning disk, Fig. 14(a), to scan the scene being transmitted. This disk contains a spiral row of holes as shown. The highly illuminated picture or object is

focused by the lens L, Fig. 14(b), upon the scanning disk in such a position that the image I lies between the first and last holes of the disk and is as wide as the distance between any two consecutive holes. When the first hole passes over a narrow strip S of the image, the photoelectric cell, which is placed behind the disk, receives a succession of small spots of light which represent the light passing from the image through the hole in the disk as the hole is cutting across the small uppermost part of the image. The intensity of the light of each individual spot that is transmitted through the hole is determined by the character of the picture or object (*i.e.*, a white surface reflects a greater amount of light than a dark one); hence the current in the circuit of the photoelectric cell varies with the intensity of light of successive spots. The variations in the photoelectric-cell circuit are amplified and used to modulate the carrier current of the transmitting station.

Just as the first hole leaves the image surface, the second begins directly under it and transmits the light on its assigned area to the photoelectric cell. The remaining holes in succession perform the same operation, and the result is equivalent materially to cutting the object into horizontal strips and transmitting the strips in rapid succession as they are cut.

The variations in the current of the photoelectric cell circuit then correspond to the variations in the light intensity of the succession of spots. The light values of all parts of the picture or object have been transformed into current values which are transmitted to distant stations either through a telephone system or by radio.

In order to reproduce the picture from its transmitted current values it is necessary to change the current variations to light variations. This operation is accomplished in a manner which is closely related to the method of transmission; in fact, it is the process of transmission reversed. For example, the transmitted signals carrying the light values of the picture are picked up by the receiving antenna and amplified, Fig. 14(c). From the amplifier the signals, in the form of potential differences, are applied to a neon tube which replaces the loud speaker of a radio receiver. The neon tube consists of two rectangular square plates in a glass tube containing a small amount of neon gas. If

a potential difference in excess of from 90 to 180 volts, depending upon the tube, is applied to the two plates, the negative plate is covered with a pink or reddish glow. For highest electrical and optical efficiency the glow should be limited to the outer surface of the plate. This is accomplished by placing the plates of the tube very close together. If the potential difference applied to the tube is varied, the intensity of glow varies in direct proportion. This last property is what makes the neon tube transform the current values back into light values. If the tube is placed behind a scanning disk which rotates in exact synchronism with the transmitting disk and is supplied with the amplified signals, it glows, as already stated, in direct proportion to the *intensity of the individual spots* of light coming from the picture at the transmitter. The holes in the rotating disk transmit to the eye of the observer one spot of light at a time, in orderly succession. Because the persistence of vision causes any spot of light to be visible about one-eighth of a second, the whole of the picture, which is covered in about one-half of that time, is visible simultaneously.

The system described is only one of several and gives promise that it may be used, after further developments, for the transmission of large scenes.

Scenes in their natural colors are transmitted by means of three carrier currents which are produced simultaneously. Each current is modulated in the same manner as that in the transmission of monochromatic scenes except that the light also passes through an appropriate color filter. Each current then carries the color values of the scene in one of three fundamental colors (red, green, or blue).

At the receiving station the current carrying the red colors controls the luminosity of a neon tube, and each of the other two controls the luminosity of an argon tube, which emits both blue and green colors. With proper color filters the three beams of light can be obtained, each varying in intensity as the color value of the appropriate filtered beam at the sending station. These three beams are superposed by reflection and transmitted through semitransparent mirrors to form one beam which contains, in rapid succession, the color values of all the points in the scene. This beam, when viewed through the scanning disk, reproduces the scene in its natural colors.

Questions

1. Explain the action of the electric bell and of the telegraph sounder.
2. Explain the function of a relay in telegraphy.
3. How is it possible to use only one wire in telegraphy?
4. What is meant by duplex and quadruplex telegraphy?
5. Explain the rotary multiplex system of telegraphy.
6. Explain, in general terms, the action of the multiplex page-printing telegraph.
7. How many telegraphic messages may be sent over one wire at the same time?
8. Explain the action of a simple telephone system employing a microphone transmitter.
9. How does the capacitance of the transmission line affect long-distance telephony? What are loading coils? What is permalloy?
10. Explain the action of a simple thermionic relay.
11. Explain how undamped oscillations are modulated by sound vibrations.
12. Explain how undamped waves are used in wireless telephony.
13. Draw diagram of a vacuum-tube oscillator including the modulating circuit.
14. Draw a diagram of a receiving station containing one stage of radio-frequency and one stage of audio-frequency amplification, and represent the oscillations in each circuit.
15. Explain why the telephone does not respond to radio-frequency currents.
16. Explain why the average value of a current is not changed in an amplifier as it is in a detector.
17. Explain how the radio-frequencies of modulated waves are changed to audio-frequencies of the modulation.
18. Describe the action of the condenser microphone; the dynamic speaker; the superheterodyne.
19. What is the four-electrode tube?
20. Describe the battery eliminators.
21. Describe the action of a selenium cell; photoelectric cell.
22. Explain one method by which pictures are transmitted by electricity.
23. How are actual scenes transmitted and viewed or projected as moving ictures on a screen?
24. How are scenes in their natural color transmitted?

Experiments

1. Electric bell. Instruments used in telegraphy.
2. Telephone transmitter and receiver.
3. Loading coil (shown). Telephone cables.
4. Sending and receiving signals by wireless.

5. Photoelectric cell.

6. Principle of the scanning disk.

7. Luminosity of the gas film about a plate in a properly exhausted tube shown to vary with the potential of the plate.

8. Neon and Argon vacuum tubes, and the three-colored beams obtained by filters from them.

CHAPTER XXX

ELECTRICITY OF THE ATMOSPHERE

1. Magnetism of the Earth.—Since magnetic fields appear to be associated only with moving electric charges, either on a large scale or with individual atoms, the magnetism of the earth is no doubt produced in the same manner. It is certain that in some localities the magnetic field is modified by the presence of iron ore that is permanently magnetized. The main part of the magnetic field, however, may be assumed to be due to some cause that is equivalent to electrons moving relative to the earth from west to east.

When an iron sphere is rotated in a space in which the earth's magnetism has been completely neutralized, the sphere becomes a magnet, whose magnetic poles have positions corresponding to those of the earth, except that they are exactly in the line of the axis of rotation. This is explained by assuming that the electron orbits become oriented by the rotation as should be expected by taking into consideration the inertia of the electrons. The atomic axes are forced into positions where they are parallel to the axis of rotation of the sphere and together produce the magnetic effect of charges moving around the sphere, relative to it, in the direction of rotation. The atomic magnets are equivalent to one large magnet. The earth contains much iron and other more or less magnetic elements, so that the main part of its magnetism, also, may be attributed to its rotation. The displacement of the poles from the line of the axis of rotation may be due to the secular changes in the position of the axis.

The intensity of the magnetic field is continually changing, as are its horizontal and vertical components. The daily variations of declination amount to about 8 min of arc; the horizontal component changes about 0.00035 oersteds, or about 0.2 per cent. These variations are regular, the north pole of a suspended magnet moving from east to west during the day and from west to east during the night.

The regular annual variations (from the summer of one year to that of the next, for example) have about the same magnitude as the daily variations.

The secular variation is much greater. If the magnetic needle is imagined to be suspended so as to turn freely in all directions, the north pole moves about 33° from west to east and about 8° up and down in 470 years, making a complete circuit in about 940 years.

Irregular variations of the magnetic field are associated with the activity of the sun spots and are related to them in some manner.

At Minneapolis (1930), the horizontal intensity of the earth's magnetic field was 0.161 oersteds and the dip 74°53′ from the horizontal. The total field intensity was 0.617 oersteds.

2. Ions of the Atmosphere.—The presence of ions in the atmosphere was considered in Art. XI–7. The γ-rays of radioactive substances in the ground and in the air, ultraviolet light, and cosmic rays are electromagnetic waves that produce ionization in the atmosphere. Radioactive substances in the soil and in the air also are bombarding the air molecules with helium nuclei and electrons. From these sources ions are continually being formed but disappear partly by recombination and partly by being carried up or down in the earth's electric field (Art. 5). The number of each kind of ions formed per second at the surface of the earth, as stated before, is about 10 per second—3 by radiations and bombardments from the earth, 6 from the air above, and 1.4 by direct cosmic radiation. The cosmic rays in passing through the atmosphere produce less penetrating radiations which account for some of the 6 ions that are caused by radiations from the air above.

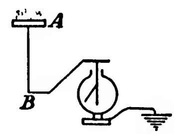


FIG. 1.—Apparatus for measuring the potential at any point in the atmosphere.

Equilibrium is established when the same number of ions recombine as are produced. The number of ions at the surface of the earth is of the order of 800 positive ions and 680 negative ions per cubic centimeter. The number of positive ions is larger because the electric field of the atmosphere is drawing them toward the earth. The amount of ionization decreases with elevation up to 1.5 km and then increases. At an elevation of

9 km it is several times that at the surface. At a height of about 60 km the air is a good conductor for a distance and again at several intervals (Art. 6).

3. Measurement of the Potential at Any Point in the Atmosphere.—An insulated metal plate A, Fig. 1, is placed at the point at which the potential of the atmosphere is to be measured, and an electroscope, with its outer casing grounded, is metallically connected to it. Imagine the plate to have been grounded and to have zero potential. There exists, then, a potential difference between the surrounding air and the plate. The few ions normally present in the air do not appreciably affect the system; but if radium F is placed on the plate, the α-particles it emits, having a range of only 3.92 cm, ionize the air only in the immediate neighborhood of A. The positive ions that are formed move from the higher potential of the space to the plate and charge it and the connected electroscope positively. When finally the potential of the plate together with that of the electroscope reaches that of the space immediately above the plate, the charging stops. The deflection of the calibrated electroscope then gives the potential of the space just above the plate.

4. Potential Gradient of the Atmosphere.—The magnitude of the potential gradient (Art. III–6) of the atmosphere is obtained from the measured potentials (Art. 3) of two points located at a known vertical distance from each other. It is customary to express the potential gradient in volts per meter.

For some unknown reason the potential gradient in any locality varies greatly with the time of day and with the season.

The average potential gradient near the surface of the earth is about 100 volts/meter but may be as high as 250 volts.

The potential gradient decreases rapidly with elevation as shown in the following table

Elevation, Kilometers	Potential Gradient, Volts per Meter
0 0	100
1.5	25
4.0	10
6.0	8

5. Flow of Atmospheric Ions.—The existence of a potential gradient in the atmosphere means the existence of an electric field. The direction of this field is toward the earth, indicating

the existence of a positive charge in the upper regions of the atmosphere and a negative charge on the surface of the earth. The lower region of the atmosphere separates the two charges and is the dielectric of a huge condenser. The positive ions of the atmosphere are given a velocity downward, and the negative ions upward. These velocities are those of small particles in air, which do not accelerate after they acquire a certain terminal velocity unless the intensity of the field changes.

When the potential gradient is 100 volts/meter, the velocity of the + ions near the surface of the earth is 1.36 cm/sec, and that of the − ions 1.87 cm/sec, but it increases with the decrease of atmospheric pressure and of humidity. The flow of ions of opposite sign in opposite directions is equivalent (Art. VIII–1) to an electric current and to a flow of electrons from the earth to the upper conducting layer.

The electric current between the earth and the upper conducting layer is 2×10^{-16} amp/cm^2 of the earth's surface, or a total current of about 1,000 amp.

6. Kennelly-Heaviside Layer.—That several conducting layers exist in the upper atmosphere is shown by the reflection therefrom of electromagnetic waves of radio-frequency. All conducting surfaces, including those of ionized air, reflect these waves (Art. XXVIII–10). The shorter waves, however, require a greater concentration of the atmospheric ions for reflection than the longer waves. Reflection also takes place wherever the concentration of the ions changes rapidly.

This upper conducting region in the atmosphere is called the *Kennelly-Heaviside layer*. It is a varyingly ionized region having at least three more or less distinct "planes" of reflection, Fig. 2. The long waves (5,000 meters) are reflected from a region apparently about 60 km above the surface of the earth; the radio waves (300 to 400 meters), from about 100 to 120 km; and the short waves (70 meters) from a region that changes from about 240 km in the daytime to varying higher values at night.

These reflecting surfaces vary in regularity from time to time and account, to some extent at least, for the differences in the ease of transmitting radio messages at different times. That sufficient energy to affect instruments can be sent such long distances by radio is due to the fact that the electromagnetic

waves are kept, by reflection, between two conducting surfaces—the earth and the Kennelly-Heaviside layer, as already stated in Art. XXVIII–10.

7. Charge of the Earth.—The negative charge on the earth, as calculated from the measured potential gradient, is 450,000 coulombs. That, alone, would give the earth a negative potential of 630×10^6 volts. The positive conducting layer of the atmosphere, however, contributes a positive potential that more or less completely neutralizes the negative.

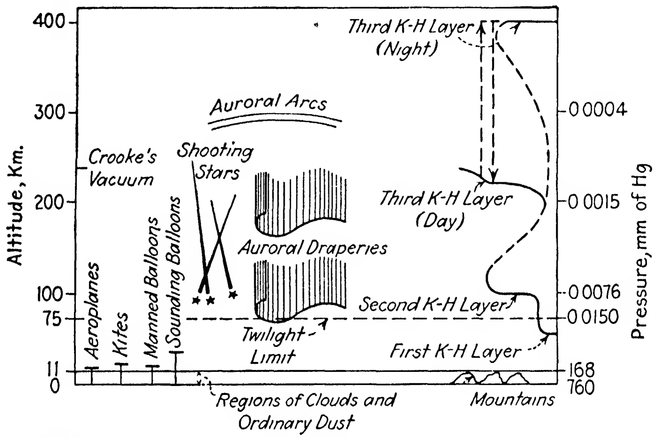


FIG. 2.—Heights of the Kennelly-Heaviside layers and of the northern lights. (*Modification of Fig 12, Humphrey's Physics of the Air.*)

The current of 1,000 amp that is flowing between the earth and the conducting layer would discharge the earth in a few minutes. Therefore there must be some source of supply both for the negative electricity of the earth and for the positive electricity in the upper layer.

The northern lights (Art. 8) are believed to be due to streams of electrons ejected from the sun. It is possible that a sufficient number of them have a velocity large enough to penetrate the whole atmosphere, charging the earth negatively. The sun, then, would have to be expelling positive charges also. If the earth received negative charges alone, it would become so highly charged negatively that it would prevent more such charges entering. The + charges, as helium nuclei, ejected by the sun and stopped in the upper atmosphere give at least a tentative

explanation for the positive charge in that region. There is, however, no direct experimental evidence of solar charges penetrating to the surface of the earth.

If the foregoing is the correct explanation for the charges, the earth is receiving about 72 grams of matter per hour from the sun in the form of positive nuclei and electrons.

Another tentative explanation for at least a part of both the + charge of the upper atmosphere and the − charge of the earth is that the cosmic-ray particles in their motion through the atmosphere make many direct hits with orbital electrons. These electrons are driven from the atoms with velocities large enough to carry them to the earth.

8. Northern Lights (Aurora Borealis).—The *northern lights* appear at night as a curtain of streamers, isolated streaks, or green and pink draperies in the northern heavens and are seen at their best in high latitudes only. Their distance from the surface of the earth varies from 75 to 300 km, but they appear most frequently and with the greatest brilliance at a height of about 100 km, at which height the density of the air is 10^{-5} that at the earth's surface or about 0.01 that in an ordinary Geissler tube.

The spectrum of the northern lights contain the characteristic green line, 5577Å, which in nature is emitted by all parts of the night sky. This line is also emitted by dissociated atoms of oxygen when they pass from one metastable state to another. That such metastable states exist in the upper regions of the atmosphere is probable because the ultraviolet rays of solar radiation are known to dissociate oxygen molecules.

The α- and β-particles emitted by the sun, on entering the magnetic field near the poles of the earth, are forced to move in curves and to enter the earth's atmosphere on the dark side. It is assumed that streams of such particles are emitted and that they produce the northern lights by impact with the excited atoms of oxygen which are in a condition to emit the characteristic light.

9. Lightning.—A drop of water when falling increases in size because of the continued condensation of moisture upon it. It finally reaches a size and velocity such that the surface tension cannot hold the drop together. Usually two smaller drops, both

charged negatively, break off and leave the greater part of the original drop charged positively. The larger drop falls with a larger terminal velocity than the smaller ones and becomes separated from them. The separation establishes a potential difference between the two groups of drops.

FIG. 3.—Streak lightning taken with a stationary camera. (*B. Walter.*)

During a thunderstorm the warm air at the surface of the earth rises, expands, and cools. It becomes supersaturated, and moisture collects on all dust particles and ions. The uprushing air finally disrupts the growing drops and produces, as described above, negative charges on the fine spray and positive charges on the larger droplets. The negatively charged spray is carried upward by the air; and the positively charged droplets continue falling, their charge and potential increasing through repeated disruptions and by coalescence with other drops.

In this manner the upper side of a cloud becomes negatively charged, and the lower positively. The two parts may even separate to form two oppositely charged clouds.

The large number of small charges produce large potentials, for each contributes its share to the potential at any point in

Fig. 4.—Streak lightning of Fig. 3 taken with a rotating camera. The order of the discharges is from right to left. (*B. Walter.*)

the cloud. This potential due to all the charges in the neighborhood, Art. III–9, is

$$E'' = \sum \frac{q''}{r}.$$

The greater the number and the magnitude of the charges, the larger is the potential.

When the potential difference becomes large enough to ionize the air, a brush or a disruptive discharge (Arts. XI–9, 11) takes place between different clouds (see frontispiece), parts of the same cloud, or between a cloud and the earth, Fig. 3. The luminous streak marking the path of the discharge is called *lightning*, and the sound produced by the sudden expansion of the heated air *thunder*. The zigzag nature of the path is caused by the unequal distribution of ions which makes the longer path the path of least resistance.

When photographs of lightning are taken simultaneously with a stationary and a rotating camera, the single flash, Fig. 3, taken with the stationary camera is resolved by the rotating camera into the several flashes of Fig. 4.

Recent photographs taken with a rotating-lens camera, and having a much greater resolving power, show the first of the series of strokes along the same path to differ in character from the others. A faint luminosity ("leader" or electron "avalanche") about 200 ft long moves by jerky steps from the cloud to the earth and is immediately followed by a bright luminosity moving from the earth to the cloud. The velocity of the faint leaders varies from 800 to 12,000 mi/sec and that of the main flash from 15,000 to 68,000 mi/sec.

The time of a discharge varies from 0.0001 to 0.001 sec, while that of a set of discharges, such as shown in Fig. 4, may be as much as 0.3 sec. The quantity in a discharge varies from 10 to 50 coulombs while the current may have a magnitude of 100,000 amp. The p.d. is variously estimated as being from 50×10^6 to 10^7 volts, which, because of the space charge, is much less than the usual 30,000 volts/cm. The estimated energy in a usual stroke is 3,000 kw-hr.

The incompleteness and revolutionary nature of the recent observations preclude the giving of any theory at this time.

Other discharges in and between clouds are from 6 to 12 miles in length. These usually appear forked and may take an appreciable time in elongating in the direction of the thinning fork. These discharges may be explained by assuming the electrified drops in a cloud of high potential gradient to coalesce, because of the churning motion of the rising air, into thin strings of highly charged water. Each string is a conductor at whose ends the

charge density is sufficient (Art. XI–9) to ionize the air. The neighboring strings A and B, Fig. 5, with the ionized air between them suddenly become one conductor. If the potential in the region X is higher than that in the region Y, the potential of the string A is higher than that of B, and a transfer of the charges takes place between A and B. This transfer lowers the potential of A and thereby causes it to receive an additional charge from the points of higher potential in its neighborhood. Part of this charge it transmits to B. The two strings together then form a longer conductor with a greater potential difference between it and the next succeeding string C. They give part of their charge to C and in turn receive more from the neighboring space. In this manner electricity is conducted forward from string to

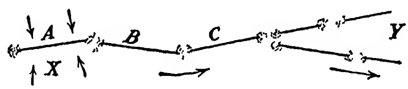


FIG. 5.—Coalesced charged strings of water, showing ionized air at their extremities, in a cloud in which there is a large potential gradient.

string and forks out as represented. The luminosity accompanying the transfer marks the path of the forked or chain lightning.

During a thunder shower the rain water is highly electrified, and the charge is often as high as 6 statcoulombs/cm³ of water. Hot weather favors thunderstorms because of (1) the stronger upward air currents, (2) more moisture in the air, and (3) warm drops disrupting more easily and producing larger charges at each disruption.

There are about 1,800 thunder storms in progress at any one time in different parts of the world, totaling on the average about 100 flashes/sec.

10. Protection against Lightning.—Lightning strikes the electrically nearest point between the cloud and the earth. Any raised grounded conductor or an ionized gas, such as comes from a chimney, makes the nonconducting distance between itself and the cloud less than that of other points in the neighborhood. When lightning strikes in that region, it strikes this point which is electrically nearest the cloud. It strikes the tall conductor or the ionized air far above the chimney. The ions conduct the charge through the chimney to the earth. The sudden expan-

sion of the heated air within the chimney causes that to explode. The high conductor may be a water pipe or a gas pipe within the building. The disruptive discharge, then, "passes" to it through the wooden roof. The potential gradient within the wood is such that the ions of opposite sign are urged in opposite directions with such force as to destroy the fiber of the wood. So much heat may be generated as to produce an explosion of the gases formed, resulting in a shattered hole splintered outward on both sides. Lightning also may induce dangerously high voltages in neighboring conductors although these may be at a considerable distance from the main surge.

The obvious method of protection against lightning is to have a high grounded conductor near the house. The lightning rod is such a conductor. It consists of a metal rod with one or more points and is placed on the roof, preferably near the chimney. A strong metal conductor passing over the outside of the house connects the rod to the moist earth. When a large potential difference exists between a cloud and the earth, the earth becomes charged by induction, and the density of the charge at the upper end of the lightning rod becomes so great that the air in its neighborhood becomes ionized (Art. XI–9). The ions flow in opposite directions and the repelled kind form a conducting path toward the cloud through which the cloud may discharge to the earth.

A person in a cave or inside a metal vessel is protected from lightning. A grounded metal roof together with metal lath form the best protection. One is comparatively safe in a deep narrow ravine and safer in the house than in open space. One should keep away from wire fences, tops of hills, isolated trees, stoves, fireplaces, and walls.

A conductor of height l protects a conical region whose radius is $4l$.

If two oppositely charged clouds are close together, their charges are "bound," and practically no potential gradient exists between them and the earth. One cloud neutralizes the effect of the other, and no stream of ions is formed between the cloud and lightning rod. If air currents then carry another positively charged cloud into the neighborhood, close enough to the upper negative one to discharge it, the potential due to the charge of

the lower cloud suddenly becomes effective. Lightning then strikes whatever is the highest projection (usually the hot gas issuing from a chimney). Not enough time is given for a stream of ions partly to span the distance from the rod to the cloud.

The ordinary lightning and lightning rod are illustrated in the laboratory experiment of Fig. 6(a). The electrostatic machine is charging the cloud C positively. The grounded plate E represents the earth. When the potential of the cloud reaches the proper magnitude, lightning strikes the knob H which represents a house. If, however, the sharp point of equal height, representing a lightning rod, is placed near the house, it dis-

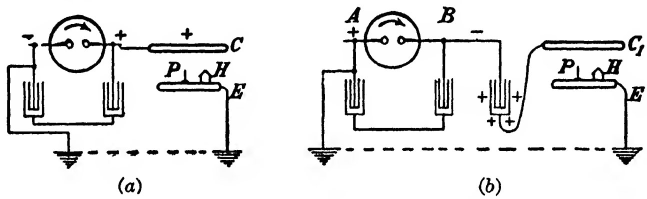


FIG. 6.—Machine charging the plate C which is discharged by the point P, preventing a spark to H, illustrating the action of a lightning rod (b) Condition under which a lightning rod does not protect.

charges the cloud so rapidly that usually no lightning takes place. If it does, it always strikes the sharp point. This experiment illustrates the flow of ions from the lightning rod but exaggerates its efficacy on account of the greater distance and the comparatively large charge of the cloud.

The other kind of lightning ("impulsive discharge") is illustrated in Fig. 6(b). The cloud C_1 now includes the lower coating of a Leyden jar which is shown to be positively charged. The inner coating of the Leyden jar forms the second cloud B and includes one electrode of the electrostatic machine. The electrode A of the machine represents the third cloud. When the machine charges the clouds, the lower coating of the Leyden jar, which is a part of the cloud C_1, becomes positively charged by induction, as represented; but the repelled electrons charge the other end of the cloud C_1 negatively. The lightning rod on E "collects" (Art. XI–10) the free negative charge from C_1. This is not the manner in which C_1 is left uncharged in nature, but the plates represent the conditions existing in actual clouds. Cloud C_1 has all its charge bound on its "upper" side, which is

the outer coat of the Leyden jar. There are no ions between the lightning rod and the cloud. If now, by bringing the knobs of the machine together, the cloud A is made to discharge cloud B, the bound charge on the lower coating of the jar, *i.e.*, the "upper part" of C_1, suddenly becomes free, and the whole cloud is instantly raised to a high potential which causes a discharge to the earth by the path of least resistance. Since there are no ions between the lightning rod and the cloud, the lightning strikes the house as readily as it does the lightning rod if the two are of the same height. In the case of an actual house with ionized gases rising from the chimney this form of lightning always strikes the chimney. The lightning rod gives no protection from this infrequent kind of lightning.

Lightning rods are not so necessary in the city as in the country on account of the many high buildings with tall stand pipes and metal framework. The lightning strikes these and the many well-protected smokestacks of factories. In addition, if a single flash of lightning is to strike a given city area in which there are 50 houses, for example, and another is to strike an equal country area in which there may be only one house, the chance of any one city house being struck is only one-fiftieth that of the lone rural dwelling.

Lightning produces serious disturbances in power-transmission lines unless these lines are protected by "lightning arrestors." Lightning may strike a line directly, but the disturbance is produced more often by an induced charge. The charged cloud gradually induces a bound charge on the line, which charge becomes free and produces a serious high-potential surge when the cloud suddenly loses its charge through lightning. This transient surge may injure both the insulation and the equipment attached to the line.

Protection against these surges is made partly by a grounded wire suspended above the transmission lines and partly by lightning arrestors. The lightning arrestor is so constructed that the suddenly increased voltage causes the excess current to pass more readily through the spark gaps of the arrestor to the earth than through the line which has a comparatively high inductance.

Questions

1. How may the earth's magnetism be explained?

2. What are the daily annual and secular variations in the earth's magnetic field? How large are they?

3. What are the period and magnitude of the secular variation?

4. How many ions of each kind are found in each cubic centimeter of air?

5. How many ions of each kind are being produced per second by each of the three sources? Why does not the number per cubic centimeter continually increase when they are being continually produced?

6. Explain how the potential at any point in the atmosphere may be measured.

7. What is the magnitude of the potential gradient of the earth? What is the direction of the gradient?

8. How does it vary with altitude?

9. What is the Kennelly-Heaviside layer? What is the sign of its charge?

10. What is the magnitude of the total current between the earth and this layer?

11. Account for the charge of the earth and that of the conducting layer.

12. Explain the production of the northern lights.

13. Describe how a falling drop of water in supersaturated air produces a separation of electric charges

14. Explain the formation of large potential differences in thunder clouds.

15. Explain how lightning is produced.

16. Explain just what takes place when lightning strikes a house

17. What is the estimated potential of the cloud when lightning takes place?

18. How can many small charges produce such high potentials?

19. Explain the conditions which determine where lightning will strike.

20. Why does lightning usually strike a chimney?

21. Explain how a lightning rod usually protects from lightning.

22. Explain how under some conditions it does not protect

23. Explain why lightning rods are not commonly used in cities.

24. Give the principle of the lightning arrestor

25. If the earth is negatively charged, why are not bodies on its surface likewise charged and therefore repelled? (Not answered in text. Student should calculate the charge and the force of repulsion on a body of some given area.)

Experiments

1. Thundershower—a nearly vertical jet of water breaking into spray forms large or small drops depending on the nearness of a charged electrophorus. Size of drops is judged by their patter on the bottom of a tin tray.

2. Ordinary lightning and the action of a lightning rod.

3. Cases where a lightning rod does not protect.

4. Slides of lightning (taken with stationary and rotating cameras and with a rotating lens).

5. Electroscope inside a metal vessel not affected by a large spark striking the vessel.

6. Principle of the lightning arrestor illustrated.

*

APPENDIX I

SYMBOLS

Energy

W, w = work or energy in ergs.
W_J = work or energy in joules.
P = power in watts.
P_{kw} = power in kilowatts.
J_J = mechanical equivalent of heat in joules per calorie = 4.18.

Magnetism

m = pole strength of a magnet.
M = magnetic moment.
H, $\mathcal{H}$ = intensity of magnetic field in oersteds. (Horizontal component of the earth's magnetic field is always represented by H.)
B = magnetic field density or magnetic induction in gauss.
Φ, ϕ = magnetic flux in maxwells. (The Greek letter is pronounced phi.)
mmf = m.m f. = magnetomotive force.
$\mathfrak{F}$ = magnetomotive force in gilberts.
μ = permeability. (The Greek letter is pronounced mu.)
μ_0 = permeability of nonmagnetic medium = 1.
$\mathcal{R}$ = reluctance.

Electricity

$\left.\begin{array}{r} Q'', Q', Q \\ q'', q', q \end{array}\right\} = \left\{\begin{array}{l}\text{number of units of quantity of electricity in electrostatic,} \\ \text{electromagnetic, and practical units, respectively.}\end{array}\right.$
F, F'' = intensity of electric field; potential gradient.
I'', I', I = number of units of current in statamperes, abamperes, and amperes, respectively.
I_0 = effective alternating current.
d.c. = direct current.
a.c. = alternating current.
$\left.\begin{array}{r} E'', E', E \\ e'', e', e \\ V'', V', V \end{array}\right\} = \left\{\begin{array}{l}\text{potential difference or electromotive force in the three} \\ \text{systems of units. } e'', e', e \text{ also represent quantities of} \\ \text{electricity on electrons, protons, and ions.}\end{array}\right.$
emf = e.m f. = electromotive force.
E_0 = effective alternating e.m.f. in volts.
p.d. = potential difference.
Ω, R, r = resistance in ohms.

ρ = specific resistance. (The Greek letter is pronounced rho.)
C'', C', C = capacitance in electrostatic, electromagnetic, and practical units.
L = self-inductance in henrys
M = mutual inductance in henrys.
μf = microfarads.
$\mu\mu$f = micromicrofarads.

Miscellaneous

B.P. = basic phenomenon.
α, β, γ = rays from radioactive substances. (The Greek letters are pronounced alpha, beta, gamma)
a, α = linear and angular acceleration, respectively; also angle of lag
v, V_0 = velocity in centimeters per second
ω = $2\pi f$ = angular velocity in radians per second. (The Greek letter is pronounced omega)
N = number of turns.
n = number of revolutions or of vibrations per second; number.
ν = radiation frequency. (The Greek letter is pronounced nu.)
H = calories of heat; intensity of magnetic field
f = force; frequency.
d = distance.
l = length, distance.
λ = wave length; mean free path. (The Greek letter is pronounced lambda)
g = acceleration due to gravity.
N.P.T. = normal pressure and temperature.
c.g.s.u. = centimeter-gram-second units
t = time in seconds, temperature in degrees centigrade.
T = torque per radian of a twisted fiber; any torque.
L = torque; length.
I_0 = moment of inertia.
M, m = mass.
c = velocity of light = 3×10^{10} cm/sec
θ = angle in radians. (The Greek letter is pronounced theta)
Σ = sum of such terms as. (The Greek letter is pronounced sigma)
$\rightarrow$ Indicates direction of motion of masses or of electrons; the direction of electric or magnetic fields, or of pulses in space; the direction of e m f , etc.
$\otimes$ Indicates electrons moving toward the paper or from the observer. The circle sometimes is omitted. This also represents the direction of electric or magnetic lines of force. (Imagine the cross to be the feathered end of an arrow.)

$\odot$ Indicates electrons moving up from the paper or toward the observer. This also represents the direction of electric and magnetic lines of force. (Imagine the dot to be the pointed end of an arrow.)

$\equiv$ Equivalent in magnitude to.

$\cong$ Approximately equal to.

$|\ |$ Terms so inclosed are treated only quantitatively.

 Upper cross in each case represents direction of electron flow, and the lower cross and dot represent directions of electron acceleration in the flow.

 Connection to earth.

 Capacitive reactance; condenser.

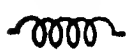
 Variable condenser.

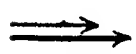
 Battery. (The longer line represents the + pole.)

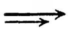
 Telephone receiver.

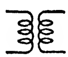
 Galvanometer.

 Ohmic resistance.

 Inductive reactance; inductive coil.

 Positive acceleration.

 Negative acceleration.

 High-frequency transformer.

 Any transformer with iron core.

*

APPENDIX II

TABLE OF NUMERICAL CONSTANTS

1 radian	57.296°
π	3.14159
1 inch (in)	2.5400 cm
1 mile (mi)	1.6093 km
1 meter (m)	39.370 in
1 kilometer (km)	0.62137 miles
1 micron (μ)	10^{-6} meters
1 Ångstrom unit (Å.U.)	10^{-10} meters
1 x-ray unit (X.U.)	10^{-13} meters
1 pound (av.) (lb)	453.592 grams
1 kilogram (kg)	2.20462 lb
1 year (tropical) (yr)	31,556,925 98 sec
1 foot-pound (ft-lb)	1.3558 joules 13.8255 kg-cm
1 joule	10.197 kg-cm 0.73755 ft-lb
1 kilowatt-hour (kw-hr)	3.6×10^6 joules 2,655,403 ft-lb
1 horsepower (hp)	745.65 watts
Acceleration due to gravity (Minneapolis)	980.596 cm/sec^2
Acceleration due to gravity (normal)	980.665 cm/sec^2
Density of mercury (N.P.T.)	13.59509 grams/cm^3
1 atmosphere (760 mm of Hg)	1,013,249 dynes/cm^2
Velocity of light (c)	2.99774×10^{10} cm/sec
Calorie (15°C) (cal)	4.1852 joules 42.68 kg-cm
Absolute zero of temperature (0°K)	−273.18°C
1 ampere (amp)	3×10^9 statamperes
1 international ampere (int amp)	0.99995 ± 5 absolute amp
1 international ohm	1.00051 ± 2 absolute ohms
1 international volt	1.00046 ± 5 absolute volts
1 faraday	96,494 ± 5 international coulombs/gram equivalent 96,489 ± 5 absolute coulombs /gram equivalent
1 elemental unit of quantity (e'')	$(4.770 \pm 5) \times 10^{-10}$ statcoulombs

1 statcoulomb	2.096×10^{9} elemental units
1 coulomb	6.288×10^{18} elemental units
1 micromicrofarad (mmf or $\mu\mu f$)	0.90000 statfarads
Number of molecules/cm³ of gas (N.P.T.)	27.06×10^{18}
Number of atoms in a gram of hydrogen	6.017×10^{23}
Atomic weight of hydrogen	1.0078
Volume per mole of gas	22,414.1 cm³
Avogadro's number	6.064×10^{23} per mole
Human population of earth (approximately)	1.75×10^{9}
Mass of electron, deflection method	$(8.99 \pm 1) \times 10^{-28}$ grams
Mass of electron, spectroscopic method	$(9\ 04 \pm 1) \times 10^{-28}$ grams
Mass of proton	$(1.6609 \pm 2) \times 10^{-24}$ grams
Mass of α-particle	$(6.598 \pm 7) \times 10^{-24}$ grams
Ratio, proton mass to electron mass (deflection method)	$1{,}847 \pm 2$
Ratio, proton mass to electron mass (spectroscopic method)	$1{,}838 \pm 1$
Radius of electron	1.88×10^{-13} cm
Radius of proton	1.03×10^{-16} cm
Diameters of nuclei of atoms	10^{-13} to 10^{-12} cm
Radius of orbit of orbital electron in normal hydrogen	$(0\ 5282 \pm 4) \times 10^{-8}$ cm
Speed of electron in normal hydrogen orbit	$(2.182 \pm 2) \times 10^{8}$ cm/sec
Planck's constant (h)	$(6\ 547 \pm 8) \times 10^{-27}$ erg-sec
Density of copper	8.93 grams/cm³
Density of tungsten	18.8 grams/cm³
Distance between centers of adjacent atoms of copper	2.28×10^{-8} cm
Number of atoms in 1 cc of copper (also number of free electrons)	$8\ 51 \times 10^{22}$
Number of atoms in 1 cc of tungsten	$6\ 26 \times 10^{22}$
Diameter of smallest microscopic particle	30×10^{-6} cm
Diameter of ultramicroscopic particles	0.4×10^{-6} cm
Diameter of large colloidal particles	10×10^{-6} cm
Wave length of sodium light (D_1)	58.96×10^{-6} cm

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APPENDIX III

ELECTRIC AND MAGNETIC UNITS

MAGNETIC UNITS

A *unit magnetic pole* is a point-pole which acts in a vacuum with a force of 1 dyne on another equal pole 1 cm distant Art. VII–1

Magnetic moment = pole strength × the distance between the poles. Art VII–5

A magnetic field has unit strength or an intensity of 1 oersted if it acts on a unit magnetic pole in the field with a force of 1 dyne. Art VII–3

A *maxwell* is one magnetic line of force. Art VII–4

The *density of magnetic flux* is unity when each square centimeter of the field contains one magnetic line of force. Art VII–4

The m m.f. is 1 *gilbert* when the magnetizing field of 1 oersted is 1 cm in length. Art XX–4

The reluctance of a magnetic circuit is 1 *unit* when an m m.f. of 1 gilbert produces a magnetic flux of 1 maxwell. Art. XX–5

UNITS OF QUANTITY OF ELECTRICITY

Elemental unit of electricity is the charge on a single electron or proton. Art. I–2

The electrostatic unit of quantity of electricity, the *statcoulomb*, is that quantity of electricity which, when concentrated at a point in a vacuum, repels an equal like charge at a distance of 1 cm with a force of 1 dyne. Art III–1

The *abcoulomb* is the quantity of electricity which passes any plane in a circuit in 1 sec when the current is 1 abampere. Art. VIII–3

The *coulomb* is the quantity of electricity which passes any plane in a circuit in 1 sec when the current is 1 amp. Art. VIII–3

The *international coulomb* is the quantity of electricity which passes any plane in a circuit in 1 sec when the current is 1 international ampere

The *faraday* is 96,494 international coulombs and is the quantity of electricity which deposits 1 gram equivalent of any element. Art. XI–3

UNITS OF CURRENT

The *statampere* is the current when electricity is passing any plane in the circuit at the rate of 1 statcoulomb/sec Art. VIII–2

The *abampere* is that current which when flowing in a loop of 1 cm radius produces a magnetic field of 1 oersted at the center of the loop for each centimeter length of the wire. Art. VIII–2

The *ampere* is 0.1 of an abampere. Art. VIII–2

The *international ampere* is the unvarying electric current which when passed through a solution of silver nitrate in water deposits silver at the rate of 0.00111800 grams/sec. Art. XI–5

UNITS OF POTENTIAL DIFFERENCE AND OF E.M.F.

The electrostatic unit of potential difference, the *statvolt,* exists between two points when the work required to move a statcoulomb of electricity between the points is 1 erg. Art. III–3

An electromagnetic unit of potential difference, the *abvolt,* exists between two points when 1 erg of work is expended in moving an abcoulomb of electricity from one of the points to the other Art. IX–4

The practical unit of potential difference, the *volt,* is 10^8 abvolts (by definition) and exists between two points when 1 joule of work is expended in moving 1 coulomb of electricity from one of the points to the other. Art IX–4

The *international volt* is the potential difference which, when steadily applied to a conductor the resistance of which is 1 international ohm, produces a current of 1 international ampere. Art. XI–5

The *practical unit of e m.f*, the volt, is identical with that of potential difference and exists when the forces displacing the electrons maintain a potential difference of 1 volt between the extremities of an open circuit or produce an *RI* drop of 1 volt in a closed circuit An e m.f. of 1 volt is induced in a circuit when the magnetic flux within the circuit is changing at the rate of 10^8 maxwells/sec and is 1/1 01830 part of the e.m f. of a standard cadmium cell. Arts. IX–4; XIII–3, 4

PRACTICAL UNITS OF RESISTANCE

The resistance of a conductor is 1 *ohm* when a potential difference of 1 volt causes a current of 1 amp to flow Art IX–8

The *international ohm* is the resistance of a column of mercury, at 0°C, 14.4521 grams in mass and 106 30 cm in length. Such a column of mercury has a cross section of 1 mm^2 and its resistance, as closely as it can be measured, is 1 practical ohm. Art. XI–5

UNITS OF CAPACITANCE

A conductor has a capacitance of 1 *statfarad* when its potential is changed 1 statvolt by 1 statcoulomb of electricity. Art. XII–1

A conductor has a capacitance of 1 *farad* when its potential is changed 1 volt by 1 coulomb of electricity, and a condenser has a capacitance of 1 farad when the potential difference between its plates is changed 1 volt by a transfer of 1 coulomb of electricity from one set of plates to the other. Art. XII–1

PRACTICAL UNIT OF INDUCTANCE

The mutual inductance of two circuits is 1 *henry* when the current changing at the rate of 1 amp/sec in the primary circuit induces an e.m.f. of 1 volt

in the secondary. The self-inductance is 1 henry when the current changing at the rate of 1 amp/sec induces a counter e.m.f. of 1 volt in its own circuit.

Arts. XVII–4, 6

UNIT OF INTENSITY OF ELECTRIC FIELD

An electric field has *unit intensity* when it acts on a statcoulomb of electricity in the field with a force of 1 dyne.

Art. III–2

APPENDIX IV

LAWS AND RELATIONSHIPS

(B.P.1, B.P.2, B.P.3, and Laws A, B, and C are given under Laws I–1; V–3, 6, 7, 8; XIII–7; XVI–1)

I–1. There are two kinds of electricity, of which charges of like kind repel one another, and charges of unlike kind attract. (This law is designated as B.P.1.) Art. 1

2. Elemental electric fields extend indefinitely, interpenetrate one another freely, exert forces on electric charges placed in them, possess inertia, and, when in motion, their lines of force move parallel to themselves. Art. 3

3. An electric field, when at rest, acts with the same force on any electric charge regardless of whether this charge is in motion or at rest. Art. 3.

4. Each of several fixed charges exerts the same force on a given charge as though it acted alone. Art 3

5. Electric lines of force tend to become as short as possible and to repel one another. Art. 3

6. An electric charge in an electric field possesses potential energy and is urged by the field toward points where its potential energy will be less. Art. 3

III–1. $f = \pm\frac{1}{K}\frac{Q_1''Q_2''}{d^2}, \quad f_0 = \pm\frac{Q_1''Q_2''}{d^2}.$ Art. 1

2. $e'' = 4.770 \times 10^{-10}$ statcoulombs. Art. 1

3. $f = Q''F''$ dynes. Art. 2

4. $F'' = \frac{Q''}{d^2}$ e.s.u. Art. 2

5. The resultant of several superposed electric fields is their vector sum. Art. 2

6. $W = Q''V'' = Q''E''$ ergs. Art. 3

7. The electric field moves positive charges from points of higher to points of lower potential and negative charges from points of lower to points of higher potential. Art. 4

8. $V'' = F''l$ statvolts. $F'' = \frac{V''}{l}$ e.s.u. Art. 6

9. $V'' = \pm\frac{Q''}{d}$ statvolts. Art. 7

10. $V'' = \pm\frac{Q''}{R}$ statvolts. Art. 8

11. Potential is a scalar quantity. $V'' = \sum\frac{q''}{r}.$ Art. 9

IV-1. A free charge on a homogeneous conductor distributes itself over the outer surface only, and in such a manner that no potential difference exists between any two points within or on the surface. Art 3

2. The density of electric charge is greatest at the pointed end of a conductor and especially at sharp points. Art 3

3. An electroscope measures the potential and not the quantity of electricity on the system of which the electroscope is a part Art 6

4. For every free charge there is induced an equal opposite bound charge on neighboring conductors or on the earth. Art 7

5. The production of a charge necessitates the production of an equal opposite charge. Art. 8

V-1. Motion endows an electric field with a property which gives it power to exert magnetic forces on electric charges because of their motion Art 1

2. An electric field, by convention, acts on moving charges because they are charges, regardless of their motion; a magnetic field (an attribute of a moving electric field) acts on the charges only because of their motion Art. 1

3. Electron flows in two parallel conductors evoke forces of attraction between the conductors when both flows have the same direction, and forces of repulsion when the flows have reverse directions; in conductors at right angles to each other, the electron flows evoke torques which tend to turn the conductors until they are parallel and their flows have the same direction. (This law is designated as B P 2) Art 1

4. The north pole of a magnetic loop (elemental magnet) is the pole which when faced by the observer appears to be due to a clockwise electron motion in the energized loop. Art 3

5. A magnetic field exerts forces on conductors carrying electron flows and thereby torques on magnetic loops until the + poles of the loops face the conventional direction of the magnetic lines of force. Art 4

6. When looking in the direction of an electron flow the direction of the magnetic lines of force about the flow is counterclockwise. Art. 4

7. An electric charge moving at right angles to the lines of force in a magnetic field is urged from the strengthened toward the weakened part of the field. (Laws 6 and 7 are designated as Law A) Art 5

8. A moving electric field and its associated magnetic field always exist together and are so related that, if they are moving from the observer with the electric lines pointing upward, the magnetic lines point from left to right. (This law is designated as Law B) Art. 8

VI-1. Unlike magnetic poles attract, and like magnetic poles repel one another. Art. 1

2. Every magnetic loop is linked by magnetic lines of force which are closed loops and which emerge from the positive-pole side of the loop. Art. 2

3. A stationary magnet is equivalent to a cylindrical electron whirl about the axis of the magnet; and a magnet of changing magnetization is equivalent to an accelerating whirl Art 7

4. Magnetic lines of force tend to become as short as possible and to repel one another. Art. 10

VII–1. The position of a magnetic point-pole is that position which makes the force between two such point-poles vary inversely as the square of the distance. Art. 1

2. $f = \pm\frac{1}{\mu}\frac{mm'}{d^2}$ dynes. Art 2

3. $f = Hm$ dynes Art 3

4. A magnetic field is a space which (1) exerts forces on moving charges because of their motion, (2) turns magnetic loops into positions in which their + poles face in the direction of the field, and (3) urges the conventional + magnetic point-pole in the direction of the field and the − point-poles in the reverse direction. Art. 3

5. $\phi = BA$ maxwells. Art. 4

6. $T = HM \sin \theta$. Art. 5

7. $t = 2\pi\sqrt{\frac{I_0}{MH}}$ sec. Art. 7

8. $\mathcal{H} \cong \frac{2M}{d^3}$ oersteds. Art. 8

VIII–1. $\mathcal{H} = \frac{I'l}{r^2}$ oersteds $\quad B = \mu\mathcal{H} = \mu\frac{I'l}{r^2}$ oersteds. Art. 2

2. $I'' = \frac{Q''}{t}, \quad Q' = I't, \quad Q = It.$ Arts. 2, 3

3. $Q = 10Q' = \frac{Q''}{3 \times 10^9}.$ Art. 4

4. $I = 10I' = \frac{I''}{3 \times 10^9}.$ Art. 4

5. $f = [BlI']$ dynes. Art. 6

6. $i' = \frac{e'v}{l}$ abamperes. Art. 7

7. $f_1 = Be'v = \frac{Be''v}{c}$ dynes. Art. 8

8. $f_1 = e''\left[F'' + \frac{[vB]}{c}\right]$ dynes. Art. 8

IX–1. Whenever a current is flowing, a quantity of electricity, $Q\ (= It)$, passes every plane perpendicular to the conductor in the time t. This quantity, for the purpose of calculating the expenditure of energy, may be considered the quantity that has been transferred through the potential difference between any two planes under consideration. Art. 1

2. $W = (E_1'' + E_0'')I''t = (E_2'' + E_0'')I''t = E''I''t$ ergs. Art. 4

3. $W = E'Q' = E'I't$ ergs. Art. 4

4. $W_J = EQ = EIt$ joules. Art. 4

5. $E = \frac{E'}{10^8} = 300E''.$ Art. 5

6. $E = \frac{J_JH}{It}$ volts. Art. 6

7. $E = RI.$ Art. 7

8. The strength of an electric current is proportional to the potential difference; hence the electron-flow velocity is proportional to the potential gradient. Art. 7

9. $W_J = EQ = EIt = RI^2t = \frac{E^2}{R}t$ joules. Art. 9

10. $P = \frac{W_J}{t} = EI = RI^2 = \frac{E^2}{R}$ watts. Art. 9

11. Cost $= P_{kw}t_h r$. Art. 9

X–1. $R = \frac{\rho l}{A}$. Art. 1

2. $R \propto \frac{l}{N\lambda A}$. Art. 2

3. $R_t = R_0(1 + \alpha t)$. Art. 2

4. $e \propto r$. Art. 4

5. $R = r_1 + r_2$ (series). Art. 5

6. $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$ (parallel). Art. 6

XI–1. $M = ZIt$ grams. Art. 3

2. $Z = K\frac{\text{atomic weight}}{\text{valence}}$. Art. 3

3. $I = \frac{M}{Zt}$ amp. Art. 4

4. $I = \frac{E - e}{R + r}$. Art. 6

XII–1. $C'' = \frac{Q''}{V''}$ statfarads, $C' = \frac{Q'}{V'}$ abfarads, $C = \frac{Q}{V}$ farads. Art. 1

2. $C = 10^9 C' = \frac{C''}{9 \times 10^{11}}$. Art. 2

3. $C'' = R$ statfarads. Art. 5

4. $C'' = \frac{KA}{4\pi d}$. $C' = \frac{KA(N-1)}{4\pi d}$. $C = \frac{KA(N-1)}{11.31 \times 10^{12} d}$. Art. 7

5. $C = c_1 + c_2$ (parallel). Art. 10

6. $\frac{1}{C} = \frac{1}{c_1} + \frac{1}{c_2}$ (series). Art. 11

7. $W_J = \frac{1}{2}CV^2$ joules. Art. 12

8. $w_1 = \frac{K(F'')^2}{8\pi}$ ergs/cm³. Art. 13

XIII–1. $P = f_0 v = f_1 v_1$ ergs/sec. Art. 2

2. $w = \phi I'$ ergs. Art. 2

3. $E' = \frac{f_1 l}{e'}$ abvolts. Art. 3

4. $E'' = V'' = F''l$ statvolts. $E = 300F''l$ volts. Art. 3

5. $E' = \frac{\phi}{t}$ abvolts Art. 4

6. $E = \frac{N\phi}{10^8 t} = -\frac{Nd\phi}{10^8 dt}$ volts. Art. 4

7. A moving magnetic field is assumed to have associated with it a nonconservative electric field which is the primary cause of electromagnetic induction; the induced e.m.f. is ascribed to the magnetic field rather than to the electric only because the presence, direction, and magnitude of the e.m.f. can be determined more conveniently in terms of its cutting flux. (This law is designated as B.P.3.) Art. 5

8. $F'' = \frac{[vB]}{c}$ e.s.u. Art. 6

9. $\mathcal{H} = \frac{[vF''_E]}{c}$ oersteds. Art. 6

10. A moving conservative electric field evokes an associated magnetic field and a moving magnetic field evokes a nonconservative electric field; in either case the intensity of the evoked field varies as the velocity and as the intensity of the primary moving field, and its direction with respect to the moving field is that given by Law B. Art. 6

11. $E = \frac{\text{rate of change of line turns}}{10^8}$. Art. 7

12. The direction of an induced e.m.f. is such as to tend to produce a flow whose magnetic flux (1) in the case of a conductor strengthens the inducing flux on the cutting side and thereby evokes a force acting in opposition to that which produces the flow and (2), in the case of a loop, also tends to oppose any change in the flux linking the loop. (This law is known as Lenz's law.) Art. 8

XIV–1. The + ions of metals tend to pass into solution, and the + ions of solutions tend to pass out of solution. Art. 3

2. A voltaic cell transforms chemical energy into electric energy. In a perfect voltaic cell, the excess of chemical energy in joules per coulomb liberated at one plate over that absorbed at the other gives the e.m.f. of the cell in volts. Art. 9

XV–1. Electrostatic machines convert mechanical into electric energy. Art. 5

2. $e_S = At + \left(\frac{B}{2}\right)t^2$ Art. 8

3. $P_S = A + Bt$ Art. 8

4. Thermocouples transform heat energy directly into electric energy. Art. 8

5. Photovoltaic cells transform light energy directly into electric energy. Art. 9

XVI–1. Any acceleration of the electron flow within a conductor or a magnetization whirl evokes an electromagnetic pulse which emanates from the conductor or magnet with the velocity of light. The direction of

the nonconservative electric component of this pulse is that of the acceleration, and the direction of the magnetic component necessarily conforms to Law B; from which it follows that when an electron flow is accelerating in any direction, all neighboring electrons are urged in the reverse direction (by the electric component) **and, after being set in motion, are also urged** (by the magnetic component) **in the direction of the pulse propagation. See simplified statement, Law XVI–2.** (This law is designated as Law C.) Art. 5

2. The direction of the induced flow is the reverse of that of the electron acceleration in the inducing flow. Art. 5

XVII–1. The magnitude of the e.m.f. an accelerating flow induces in a neighboring circuit is determined by Eq. XIII–11. This equation applies to electromagnetic pulses as it does to relative motions between conductors and magnetic fields. Art. 2

2. $N\phi = 10^8 QR.$ Art. 3

3. $e = -M\dfrac{di}{dt}$ Art. 4

4. $e = M\dfrac{I_p}{t} = \dfrac{N\phi}{10^8 t} = iR$, volts. Art. 4

5. $e = -L\dfrac{di}{dt}.$ Art. 6

6. $e_0 = Ri + L\dfrac{di}{dt}.$ Art. 6

7. $L = \dfrac{N\phi}{10^8 I}$ henrys. Art. 6

8. $L \propto N^2.$ Art. 7

9. $P = EI = RI^2 + eI$ watts. Art. 8

10. $w_J = \dfrac{1}{2}LI^2$ joules. Art. 9

XVIII–1. $e = \dfrac{\omega N\phi}{10^8} \sin \theta.$ Art. 1

2. $E_{av} = 0.637E_m.$ Art. 2

3. $E_r = 0.707E_m \qquad I_r = 0.707I_m.$ Art. 2

4. $e_L = \omega LI_m \cos \theta$ volts. $E_m = \omega LI_m$ volts. Art. 6

5. $e_0 + e_L = Ri.$ Art. 7

6. $e_0 = Ri + L\dfrac{di}{dt}.$ Art. 8

7. $I = \dfrac{E}{\sqrt{R^2 + (\omega L)^2}}$ amp. Art. 9

8. $E_c = -\dfrac{I}{\omega C}.$ Art. 10

9. $I = \dfrac{E}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}}.$ Art. 10

10. $$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}.$$ Art. 10

11. $$e_0 = Ri + L\frac{di}{dt} + \frac{q}{C}.$$ Art. 10

12. $$\tan \alpha_L = \frac{\omega L}{R}. \quad \tan \alpha_c = \frac{1}{\omega CR}.$$

$$\tan \alpha_X = \frac{\omega L}{R} - \frac{1}{\omega CR}.$$ Art. 11

13. $$P = EI \cos \alpha \text{ watts.}$$ Art. 12

XX–1. The work done in moving a unit point-pole around any closed path linking a coil of N turns is

$$w = 4\pi NI' = 0.4\pi NI \text{ ergs.}$$ Art. 2

2. $$H = \frac{w}{l} = \frac{0.4\pi NI}{l} = \frac{1.257NI}{l} \text{ oersteds.}$$ Art. 3

3. $$\mathfrak{F} = Hl = 1.257NI \text{ gilberts.}$$ Art. 4

4. $$\phi = \frac{\mathfrak{F}}{\mathfrak{R}} \text{ maxwells.}$$ Art. 5

5. $$B = \mu H \text{ gauss.}$$ Art. 7

6. $$\mathfrak{R} = \frac{l}{\mu A} + \frac{l_1}{A} \text{ units.}$$ Art. 8

XXI–1. $$I = \frac{10T}{BlND}\theta = K\theta = K'd.$$ Art. 3

2. $$i = \frac{S}{G + S}I.$$ Art. 4

3. $$\frac{r_1}{r_2} = \frac{l_3}{l_4}.$$ Art. 14

4. $$Q = Kd$$ Art. 16

5. $$Q_s = \frac{MI_p}{R_s}.$$ Art. 18

6. $$\frac{N_x\phi_x}{N_s\phi_s} = \frac{d_x}{d_s}.$$ Art. 20

XXII–1. $$\frac{E_p}{E_s} = \frac{N_p}{N_s}.$$ Art. 3

2. $$\frac{I_p}{I_s} = \frac{N_s}{N_p}.$$ Art. 3

3. $$E_sI_s \cos \alpha \cong E_pI_p \cos \alpha.$$ Art. 3

XXV–1. $$v = \sqrt{\frac{2V''e''}{m}} \text{ cm/sec.}$$ Art. 9

XXVI–1. $$F'' = \frac{V}{300d} \text{ e.s.u.}$$ Art. 1

2. $$v = c\frac{F''}{B} \text{ cm/sec.}$$ Art. 2

3. $$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}.$$ Art. 2

4. $$e'' = \frac{Mg - B}{F''} = 4.770 \times 10^{-10} \text{ statcoulombs.}$$ Art. 3

5. $$\frac{e''}{m} = \frac{cv}{Br}.$$ Art. 4

XXVIII–1. $$f = \frac{1}{2\pi\sqrt{LC}}. \qquad f = \frac{\sqrt{LC - \frac{1}{4}R^2C^2}}{2\pi LC}.$$ Art. 2

2. $$R < \sqrt{\frac{4L}{C}}.$$ Art. 2

3. $$\lambda = \frac{c}{n} = \frac{c}{f} = \frac{c}{\nu}.$$ Art. 4

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APPENDIX V

SPECIAL PROOFS

1. The Effect of a Material Dielectric on the Forces Acting between Electric Charges.—Imagine the charged body B to be in the electric field of the point-charge A within a material dielectric, as shown in Fig. 1. Each atom of the dielectric, due to the action of the electric field, is an electric doublet (Art. I–1). The space about the point-charge then may be divided into concentric spherical charged shells, as shown. The charged shells in the region X between the two charges produce externally the same field as though their charges were concentrated at the center (Art. III–2). The two sets of paired equal and opposite kinds of charges then neutralize one another's action on the charge of B. The uniformly charged shells of the region Y completely inclose the charge on B, and therefore they too exert no force upon it (Art. IV–3).

The space Z which is a spherical shell between the regions X and Y is penetrated by the charged body B. Inspection shows that the boundary doublets mm produce the electric field through B represented by the broken-line arrows. This field weakens that, F'', due to the charge A within the body B. All the other doublets of the shell Z, such as p, have the nearer charges of such sign that they too contribute to the weakening of the field through B. The charge on B, then, is acted on with a smaller force than it would be in open space. That the decrease factor K has the value of the dielectric constant is shown by experiment and from the following considerations:

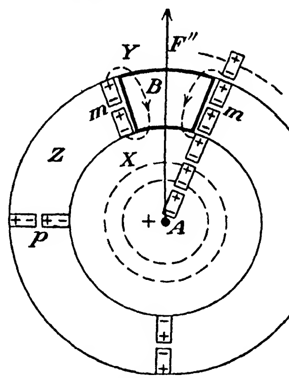

Fig. 1.

The capacitance of a condenser in a material dielectric is

$$C_K = KC_o,$$

where C_o is the capacitance in open space and K the dielectric constant of the material. For the same charge

$$Q = C_K V_K = C_o V_o,$$

from which

$$K = \frac{C_K}{C_o} = \frac{V_o}{V_K} = \frac{F_o''d}{F_K''d} = \frac{F_o''}{F_K''} = \frac{f_o}{f_K}.$$

Then

$$f_K = \frac{f_o}{K},$$

where f_K is the force acting on each of the condenser plates and therefore on any two such charged conductors when they are immersed in a dielectric, and f_o the force in open space. Each plate is in the electric field of the other and therefore in a field whose intensity is one-half that between the plates.

2. The Effect of a Magnetic Medium on the Forces Acting between Magnetic Point-poles.—It was shown (Art. VI–11) that, when a space is surrounded by a magnetic medium, that space is partly screened from the magnetic fields of external magnets. A *permanent* magnet or the material of a solenoid occupies such a screened space. The forces acting on the moving electrons in the whirls of the magnet or within the loops of the

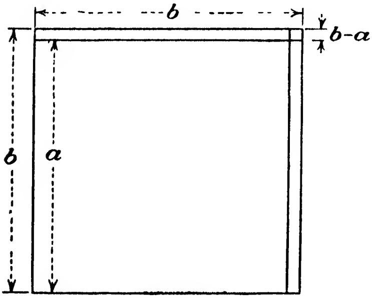

FIG. 2.

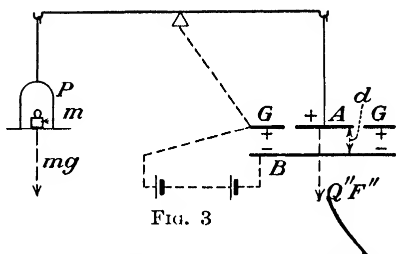

FIG. 3

solenoid are therefore diminished and consequently the forces acting on the conventional point-poles. That the diminution is by the factor $1/\mu$ is not easily shown.

3. The Average of the Squares of Two Quantities Which Differ in Magnitude by an Infinitesimal Amount.—The two superposed squares, Fig. 2, have one corner in common so that the larger square extends the distance $b - a$ beyond two of the sides only. If the sides differ by only an infinitesimal amount the small corner area, $(b - a)^2 \cong 0$.

The average of the two superposed areas is seen by inspection to be

$$\frac{a^2 + b^2}{2} = \frac{a^2 + [a^2 + 2a(b - a) + (b - a)^2]}{2} = ab + \frac{1}{2}(b - a)^2$$

From which, since $(b - a)^2 \cong 0$,

$$\frac{a^2 + b^2}{2} \cong ab.$$

4. Disk Electrometer.—The disk electrometer was described in Art. XXI–9. As there stated it is used for making absolute determinations of p.d. between two parallel plates and thereby the potential of one of them

and of any material conductor connected thereto if one of the plates is grounded.

The disk A and the pan P, Fig. 3, normally balance. When the disk, the guard ring G, and the plate B are charged as shown. The disk is attracted by B with a force $Q''F''$, where F'' is the intensity of the electric field at A due to the charge on B and Q'' is the charge on A. The mass m on the pan pulls down with a force mg dynes and the disk is attracted with a force of $Q''F''$ dynes. Then

$$f = Q''F'' = mg,$$

when the two forces are made to balance

The capacitance of the disk A whose area here is represented by A, because of the neighboring equally and oppositely charged plate B, is that of two parallel plates Then (Art XII-7)

$$C'' = \frac{A}{4\pi d}$$

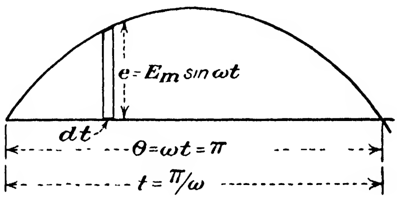

FIG 4

The intensity of the electric field (Art III-6) between the disk and plate is

$$F_{A+B} = \frac{V''}{d}.$$

One-half of the intensity of the field is due to each of the two surfaces The charge on the disk A with its electric field, then, is in the electric field due to the charge on B and therefore is in an electric field

$$F'' = \frac{F_{A+B}}{2} = \frac{V''}{2d}.$$

The force acting on the electricity of the attracted disk is

$$f = Q''F'' = C''V'' \times \frac{V''}{2d} = \frac{A}{4\pi d}\frac{(V'')^2}{2d} = \frac{A(V'')^2}{8\pi d^2} = mg.$$

Then

$$V'' = d\sqrt{\frac{8\pi mg}{A}},$$

which enables the potential of the disk to be calculated from easily measurable quantities

5. Average Value of Alternating E.M.F. and Current.—The average value of an alternating e.m.f. in all the cycles together, when the direction of flow is disregarded, is the average value in a half cycle. Figure 4 represents such a half cycle, where the average e.m f. is the area of the half cycle divided by its base; *i.e.*,

$$E_{av} = \frac{\text{Area}}{\text{base}} = \frac{1}{\pi/\omega}\int_0^{\frac{\pi}{\omega}} E_m \sin \omega t \, dt$$

$$= -\frac{1}{\pi}E_m \cos \omega t\Big|_0^{\frac{\pi}{\omega}} = \frac{2}{\pi}E_m = 0.637E_m.$$

The expression for I_{av} is found in a similar manner.

6. Effective Value of Alternating E.M.F. and Current.—The effective value of an alternating e.m f. (see Fig. 4) is

$$E_r = \sqrt{\text{average } e^2}.$$

$$\text{Average } e^2 = \frac{\Sigma e^2}{\pi/\omega} = \frac{1}{\pi\omega}\int_0^{\frac{\pi}{\omega}} E_m^2 \sin^2 \omega t dt = \frac{\omega}{\pi}E_m^2\int_0^{\frac{\pi}{\omega}} \frac{1}{2}(1 - \cos 2\omega t)dt$$

$$= \frac{\omega}{\pi}E_m^2 \frac{1}{2}\left(t - \frac{1}{2\omega}\sin 2\omega t\right)\Big|_0^{\frac{\pi}{\omega}} = \frac{1}{2}E_m^2.$$

Then

$$E_r = \sqrt{\frac{1}{2}E_m^2} = \frac{E_m}{\sqrt{2}} = 0.707E_m.$$

The expression for I_r is obtained in the same manner.

7. Power Delivered by an Alternating Current.—The power being delivered by an alternating current, at any instant, is

$$\begin{aligned} p = ei &= E_m \sin \omega t \times I_m \sin (\omega t \pm \alpha) \\ &= E_m I_m[\sin^2 \omega t \cos \alpha - \sin \omega t \cos \omega t \sin \alpha] \\ &= \frac{E_m I_m}{2}[(1 - \cos 2\omega t) \cos \alpha - \sin 2\omega t \sin \alpha] \\ &= \frac{E_m I_m}{2} \cos \alpha - \frac{E_m I_m}{2} \cos (2\omega t - \alpha). \end{aligned}$$

The power being delivered is the average of the powers of all the successive instants in a half cycle. This can now be derived by the same method as the E_{av} was derived in Art. 5 and is

$$P = E_r I_r \cos \alpha.$$

8. Electric Field Evoked by the Motion of a Magnetic Loop.—1. *Conditions about and within the conductor of a stationary magnetic loop* are shown in Fig. 5. It has been shown that the electron and proton fields neutralize each other in the regions outside the conductor and that in these regions

there exists what is called a *magnetic field*, as represented in two views in Fig. 5(*a*),(*b*). The magnetic field within the conductor of the loop, however, may be resolved into two components; one produced in each small section S by the flow in the remainder of the loop, as represented pointing toward the observer by the dots in Fig. 5(*c*), and due to which the flow

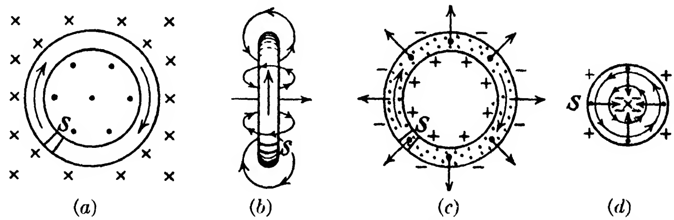

FIG. 5.—(*a*)(*b*) Two views of a stationary magnetic loop and of its external magnetic field, (*c*) the direction of that component of the magnetic field within the conductor of the loop which is produced in each small section *s* by the remainder of the loop, and the forces acting on the flow electrons due to this component, (*d*) cross-sectional view of the small section *s*, showing the circular component of the magnetic field due to the flow in the small section itself and the forces acting on the flow electrons due to this component.

electrons are urged to the outer circumference of the loop as represented by the diverging arrows; and the other, a circular component, produced by the electron flow in each small section itself, due to which the flow electrons are urged toward the center of each section as represented by the converging arrows in Fig. 5(*d*). Each of these components, therefore, contributes to a

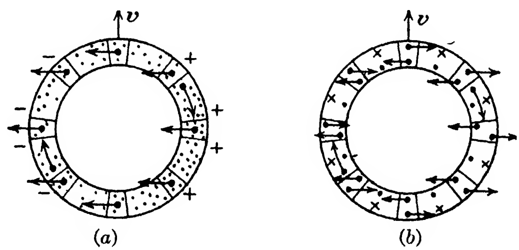

FIG. 6.—Electron displacements in a magnetic loop caused by the electromagnetic reaction between the free electrons and each of the two components of the magnetic field because of the loop's rectilinear velocity v. (*a*) The reaction with the component produced in each small section by the flow in the remainder of the loop; (*b*) the reaction with the circular component due to the flow in each small section itself.

displacement of the flow electrons within the conductor. In the one case the displacement is symmetrical with respect to the center of the loop, and in the other with respect to the center of the conductor. In each case, therefore, the external electric field due to each kind of charge is that which it would be if the charge were located at the center (Art. III-2). The two equal fields due to the two equal charges of opposite kind, therefore, neutralize each other, in each of the two cases, in the space about the loop.

2. *Conditions within the conductor of the magnetic loop when the loop is given a translational velocity* are represented in Fig. 6(*a*),(*b*) for each of the two components of the magnetic field within the loop. When the loop is given a translational velocity *v*, the flow electrons also are given that velocity. The electrons can then be considered as moving with that velocity in each of the two component fields. The reaction with the component field produced in each small section by the remainder of the loop, due to this translational motion, causes the electrons everywhere to be urged to the left. The right and left sides of the loop then appear (to the stationary observer) to be charged oppositely, as represented in Fig. 6(*a*).

In the circular or rather toroidal component which is represented by the crosses and dots in Fig. 6(*b*), the flow electrons are urged in reverse directions on the inner and outer sides of each small section, as represented by the horizontal arrows. It is seen that the forces acting on the electrons on the

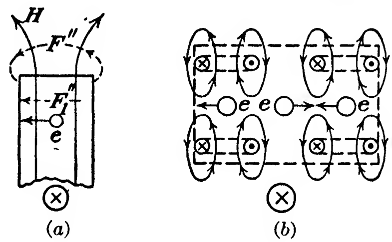

FIG. 7.—(*a*) A magnet moving from observer, (*b*) a magnet composed of four magnetons showing the magnetic field in detail and the forces acting on individual electrons when the magnet is in motion.

side of the wire more distant from the center of the loop are all in the same direction to the right. These forces have components urging the electrons along the lower and upper halves of the loop toward the extreme right. Electrons on the side nearer the center of the loop similarly are urged toward the extreme left of the loop. The two sets of forces tend to displace the electrons in reverse directions and therefore affect the displacement of them but little in the regions distant from the upper and lower sides of the loop.

3. A stationary observer viewing this moving loop reasons it to be charged by the four kinds of displacements, the latter two of which, Fig. 6(*a*),(*b*), contribute to a nonconservative electrostatic field in the space about the loop.

9. Electric Field about a Moving Magnet.—A magnetic atom and a magneton are essentially magnetic loops. The phenomena associated with the magnetic field about a magnet composed of such elemental units, therefore, are those about a magnetic loop. Law A_1, then, applies to its field and to all magnetic fields.

In a magnet, which is usually a conductor, the electron displacement within the magneton may be replaced by an equivalent displacement of the free electrons within the mass of the magnet. Electromagnetic reaction (Law A_2) between the free electrons, which are moving with the magnet, and the field of the magnet, urges the free electrons as shown in Fig. 7(*a*) and in detail, for a magnet of four magnetons, in Fig. 7(*b*).

APPENDIX VI

DIMENSIONAL EQUATIONS OF THE ELECTROMAGNETIC SYSTEM

$$v = \frac{d}{t} = [LT^{-1}]$$

$$a = \frac{v}{t} = [LT^{-2}]$$

$$f = ma = [MLT^{-2}]$$

$$v = fd = [ML^2T^{-2}].$$

The magnetic permeability μ is assumed to be dimensional. Then when $\overline{m}$ represents pole strength and since $f = \overline{m}/\mu d^2$,

$$\overline{m} = d\sqrt{\mu f} = [M^{1/2}L^{3/2}T^{-1}\mu^{1/2}]$$

$$H = \frac{f}{\overline{m}} = [M^{1/2}L^{-1/2}T^{-1}\mu^{-1/2}].$$

$$\phi = 4\pi\overline{m} = [M^{1/2}L^{3/2}T^{-1}\mu^{1/2}].$$

$$B = \frac{\phi}{A} = [M^{1/2}L^{-1/2}T^{-1}\mu^{1/2}].$$

$$\mathfrak{F} = Hl = [M^{1/2}L^{1/2}T^{-1}\mu^{-1/2}].$$

$$\mathcal{R} = \frac{\mathfrak{F}}{\phi} = [L^{-1}\mu^{-1}].$$

$$E = \frac{N\phi}{10^8 t} = [M^{1/2}L^{3/2}T^{-2}\mu^{1/2}].$$

$$I = \frac{10Hr^2}{l} = [M^{1/2}L^{1/2}T^{-1}\mu^{-1/2}].$$

$$R = \frac{E}{I} = [LT^{-1}\mu].$$

$$Q = It = [M^{1/2}L^{1/2}\mu^{-1/2}].$$

$$C = \frac{Q}{E} = [L^{-1}T^2\mu^{-1}].$$

$$L = [L\mu].$$

When μ is assumed to be nondimensional, the foregoing equations are modified by dropping the term μ.

APPENDIX VII

IMPORTANT DATES

B.C. 2639. First record of magnetic compass (China)

B.C. 600. Thales observed electrostatic attraction of light objects by charged amber.

B.C. 425. Euripides mentioned that lodestone attracts iron.

A.D. 426. St Augustine distinguished between magnetic and electric attractions.

1000. Marine compass used by the Finns.

1544. Hartman noted dip of compass.

1600. Gilbert made a magnetometer and an electroscope.

1650. Von Guericke made a friction electric machine.

1675. Isaac Newton discovered electrostatic induction.

1729. Gray distinguished between conductors and insulators.

1733. Dufay discovered that there were two kinds of electric charges.

1745–1746. Winckler, Von Kleist, and Musschenbroek independently invented the Leyden jar (condenser).

1746. Gralath made the first electric measuring instrument (an electrometer).

1752. Benjamin Franklin was the first fully to realize the importance of points on conductors and showed the relation of lightning to electricity.

1784. Coulomb proved the law of inverse squares for the attraction and repulsion of electric charges and of magnetic poles.

1786. Galvani observed the contraction of frog's legs when touched by two connected unlike metals.

1799. Volta constructed the first voltaic battery ("galvanic pile").

1800. Nicholson and Carlisle decomposed water by electric current; Wollaston electroplated; Davy made an arc light with carbon electrodes.

1812. Schilling made rubber-covered insulated wire.

1820. Oersted discovered action of a current on a magnet; Ampère discovered that a current in a loop is equivalent to a magnetic shell; Ampère discovered electromagnetic reaction (Law A).

1821. Faraday made an energized conductor rotate about a magnetic pole. Seebeck devised the thermocouple.

1825. Sturgeon made an electromagnet.

1826. Ohm discovered "Ohm's law."

1831. Faraday discovered that an e.m f. is induced in a conductor by relative motion between it and a magnetic field and con-

structed a disk dynamo. Henry constructed a motor with a commutator. Henry and Faraday (independently) discovered mutual induction.

1832. Henry discovered self-induction Gauss devised the absolute system of magnetic units

1833–1838 Gauss and Weber telegraphed using a solenoid which when moved over a pole of magnet produced an intermittent current which transmitted the signal to a moving-magnet galvanometer.

1837 Morse invented the practical telegraph

1841. Wheatstone made a dynamo with multiple-coil armature giving continuous current

1842. Henry discovered that the discharge of a condenser is oscillatory Masson and Brequet made an induction coil

1846. Weber devised the absolute system of electric units

1846. First commercial use of arc light

1850 Electric bell (Mirand)

1858 First successful Atlantic cable (Cyrus Field)

1864 Maxwell propounded the electromagnetic theory of light.

1868. Duplex telegraphy (Stearns)

1873. Rowland explained the magnetic circuit

1874. Stoney calculated roughly the magnitude of the elemental charge. Crookes considered cathode rays as "radiant matter" (see 1897–1899).

1876. Rowland proved that magnetic fields are produced by moving charges Bell invented the telephone.

1879. Edison invented the practical incandescent lamp.

1881. Commercial electric railway (Berlin). International Electrical Congress adopted "c.g s " units

1882. Electric power transmitted 37 miles.

1883. Transformer used commercially.

1888. Rotating magnetic field by two-phase currents (Farraris). Hertz demonstrated the existence of the electromagnetic waves propounded by Maxwell in 1864. Sprague constructed the first modern electric railway at Richmond. See 1881.

1893 International electric units adopted

1894. Coherer of fine filings used for detection of electromagnetic waves.

1895. Rontgen discovered x-rays.

1896. Radioactivity discovered by Becquerel.

1897. Marconi sent radio messages 9 miles.

1897–1899. Nature of the electron as a corpuscle established (J J Thomson, Townsend).

1898. Radium discovered by Madame Curie.

1899. Pupin discovered use of "loading coils" for long-distance telephony.

1901. Quantum theory of radiation propounded (Planck).

1906. Tungsten lamp and crystal detector introduced. De Forest invented the three-electrode vacuum tube used in modern radio telephony.
1907. Transatlantic radio telegraphy.
1911. Drawn-wire tungsten lamp (1.1 watts/candle).
1913. Fajans and Soddy discovered isotopes Nitrogen-filled tungsten lamp (0.5 watts/candle) first used
1915. Transcontinental telephony.
1919. Multiplex telephony.
1920. First 220,000-volt transmission line.
1923. Photon discovered (Einstein, 1905; A H Compton, 1923).
1925–1929. Beginning of television.
1926. Existence of cosmic rays demonstrated (Millikan, Kolhorster, Hess).
1932. Neutron discovered (Chadwick).
1932. Positron discovered (C. D. Anderson).

APPENDIX VIII

SUGGESTIONS FOR ABRIDGED REVIEW

(Numbers refer to chapters)

Part I

1. (*a*) B.P.1, electric field, potential difference. (*b*) Electric fields which neutralize each other's action on electric charges must be imagined still to exist superposed. (*c*) Properties of an electric field.

2. (*a*) Free electrons in conductors (*b*) The "ether."

3. (*a*) E s u. of electricity (statcoulomb), unit field, law of inverse squares for electric charges. (*b*) Magnitude of elemental charge in statcoulombs, number of elemental units in a statcoulomb. (*c*) Distinguish between + and − potential. (*d*) Give four points of view in describing why an electric charge feels a force urging it in any given direction. (*e*) Derive expression for the potential at any point due to a point charge. (*f*) Show that potential is a scalar quantity and that $E'' = \Sigma q''/r$.

4. (*a*) Explain why a static charge is distributed over the surface of a conductor and why its density is greatest at the pointed end of the conductor. (*b*) Show how the total induced charge is proved to be equal to the inducing charge and that two equal opposite charges are always produced at the same time. (*c*) Show that an electroscope measures the potential of the conductors to which it is connected.

5. (*a*) Distinguish between electric and magnetic forces illustrating by means of a basic experiment. (*b*) Describe B.P.2 in terms of forces acting between neighboring energized conductors. (*c*) Illustrate the relativity nature of electric and magnetic forces. (*d*) Concept of magnetic line and magnetic field derived from B.P.2. (*e*) "Stationary" magnetic field about an electric current. (*f*) Laws A and *B*.

6. (*a*) Why has the magnetic field about a loop, a solenoid, or a toroid the observed form? (*b*) Molecular theory of magnetism. (*c*) The external equivalence of a magnet to a cylindrical electron whirl. (*d*) Changing magnetization described in terms of electron acceleration of the magnetization whirl. (*e*) Forces acting on a magnet in a magnetic field in terms of the molecular magnets. How location of the point-poles is determined.

7. (*a*) Unit magnetic pole, unit magnetic field, law of inverse squares for magnetic poles, oersted, maxwell, gauss. (*b*) Electric and magnetic doublets. (*c*) Measurement of the horizontal component of the earth's magnetic field.

8. (*a*) Define the units of current and of quantity in each of the three systems of units. (*b*) Give the relation of the magnitudes of the units, and

of their number in any given quantity or current. (c) Derive the expression for the current, in amperes, in a tangent galvanometer. (d) Show that the current strength is proportional to the number of moving electrons and to their velocity. (e) Show how it follows from the definition of an abampere that the force acting on a wire in a magnetic field is $f = BlI'$ dynes. (f) Show that the contribution by each electron to the current in any part of a conductor of length l is $i' = e'v/l$. (g) Show that in a magnetic field an electron feels a force $f = He'v$ dynes.

9. (a) Define abvolt, volt, statvolt. (b) Give the relation between the magnitudes of the units and of the numbers of the units in any given potential difference. (c) Derive the expression for potential difference as measured by the calorimeter method. (d) State Ohm's law, and show, by means of it, that the velocity of the free electrons is proportional to the potential gradient. (e) Define ohm, watt, kilowatt-hour. (f) Show that

$$P = EI = RI^2 = E^2/R \text{ watts.}$$

10. (a) Show that $R = \rho l/A$. (b) Show that along a simple circuit, potential difference $\propto R$. (c) Derive the expression for the resistance of wires in series and in parallel.

11. (a) Show how Faraday's two laws of electrolysis follow from our knowledge of the nature of conduction in solids and electrolytes. (b) How are the natural ions in the atmosphere produced, and what is their number? (c) Explain the corona. (d) Show why sharp points discharge a conductor.

12. (a) Give the defining equation for capacitance, and define farad and statfarad (b) Show that $C = C''/9 \times 10^{11}$. (c) Show, from four points of view, why the potential of a plate is decreased by a neighboring grounded plate, and that, thereby, its capacitance is increased. (d) Explain how a material dielectric increases the capacitance of a condenser. (e) Explain why the two plates of a condenser become charged oppositely when connected to any two points in a closed electric circuit. (f) Prove $w = \frac{1}{2}CV^2$ joules. (g) Derive the expression for the capacitance of condensers in parallel; in series.

13. (a) Explain, in terms of Law A, why the electrons in the part of an electric circuit which is cutting through a magnetic field are urged along the wire and why the wire feels a force opposing its motion. State Lenz's law. (b) Show that $E' = fl/e'$. (c) Distinguish between potential difference and e.m.f. Why is e.m.f. measured in terms of potential difference? (d) Prove $w = \phi I'$ ergs. (e) Prove that $E = N\phi/10^8t = Hlv/10^8$ volts. (f) Show that E = rate of change of flux turns/10^8 volts. (g) Show why the induced e.m.f. is ascribed to an electric field when it appears to be produced by a moving magnetic field cutting through the wire. State B.P.3. (h) Derive the expression for the intensity of this nonconservative electric field in terms of v and B.

14. (a) Explain how, in dilute sulphuric acid, zinc becomes charged negatively and copper positively. (b) Explain the action of and draw the

potential diagram for the simple voltaic cell and for the Daniell cell. How is the energy required for the production of the current obtained? (*c*) Give the principle of action for the storage cell. (*d*) Electromotive force of cells in series and in parallel.

15. (*a*) Explain the action of the simple electrostatic machine, and show that the charges are produced at the expense of external work. (*b*) Give the function of the Leyden jars employed with the machines. (*c*) Explain the action of a Van de Graaff electrostatic generator. (*d*) Explain the Peltier and the Thomson effects and the action of the thermocouple. (*e*) Show how the heat energy is being converted into the electric energy. (*f*) Explain the action of a photovoltaic cell.

16. (*a*) Show how the cylindrical electromagnetic pulse is produced about a wire by the acceleration of its free electrons. (*b*) State Law C. (*c*) Explain how the nonconservative electric field of the pulse differs from an electrostatic field. (*d*) Show how the pulse establishes a magnetic field about a conductor. How it removes an established magnetic field. (*e*) Show that in the electromagnetic pulse about an electric circuit the electric lines of force form closed loops.

17. (*a*) Explain, with reference to the accelerating electrons, the direction in which neighboring electrons are urged by the electromagnetic pulse. Show how in a neighboring circuit the induced e.m.f. is again expressed by the equation E = rate of change of line turns/10^8. (*b*) Write the defining equations for mutual inductance and self-inductance. Define henry (*c*) Show that $e = M\frac{I_p}{t} = \frac{N\phi}{10^8 t} = Ri$, and show what relationships are obtained from this. (*d*) Derive, for the case of an increasing current, the expression showing the portion of the total power being expended at any instant in heating the wire and that which is being transferred into the energy of the magnetic field. (*e*) What is the relation of the energy of the magnetic field to the kinetic energy of the moving electrons? Explain electric inertia. (*f*) Show that the energy of the magnetic field about a circuit is $w = \frac{1}{2}LI^2$.

18. (*a*) Show that in a rectangular loop rotating in a magnetic field $E = (\omega N\phi/10^8) \sin \theta$, and describe how alternating and how pulsating currents are transferred from the loop to the external part of a circuit. (*b*) Show that alternating electromagnetic pulses emanate from alternating currents. (*c*) Show by rectangular coordinates and by a vector diagram that in a circuit containing L and R the current lags in phase behind the impressed e.m.f. (*d*) Show that in alternating-current circuits containing L and R, $I = E/\sqrt{R^2 + (L\omega)^2}$; and derive a similar equation for circuits containing C and R; and for circuits containing L, C, and R. (*e*) Derive the expression for phase difference in each of the three cases. (*f*) What is the power factor?

19. Explain the direction of the eddy currents (1) in a mass entering a magnetic field; (2) in a rotating cylinder; (3) in a disk rotating under a magnet; (4) in a disk above a rotating horseshoe magnet; and (5) in a magnet when the intensity of its magnetization is changing.

20. (*a*) Show that the work required to move a unit magnetic point-pole completely around a loop path linking a coil of wire is $w = 4\pi NI'$ ergs. (*b*) Intensity of the magnetic field within a toroid and a solenoid. (*c*) Define magnetic potential difference and m.m.f. Definite gilbert. Show that $\mathfrak{F} = 4\pi NI' = 0.4\pi NI$ gilberts. (*d*) Give the equation for the magnetic flux in a magnetic circuit, and show that $\mathfrak{R} = 1/\mu A$. (*e*) Show that the reluctance of an air gap is relatively large.

Part II

21. (*a*) Give the principal current- and voltage-measuring instruments for both direct and alternating currents. (*b*) Give reason for the difference in the construction of the moving-coil ammeter and the voltmeter. (*c*) How is a uniform radial magnetic field produced? (*d*) Derive equation and show how resistance is measured by the Wheatstone bridge; how potential difference is measured by the potentiometer. (*e*) Give the principle of the ballistic galvanometer, and show how capacitances are compared and how the intensity of the magnetic field is measured by its means.

22. (*a*) Explain in a transformer (1) the direction of eddy currents in the core and their almost complete elimination; (2) the choking of the primary current; (3) the decrease of this choking by the action of the current induced in the secondary; (4) that at any instant the counter-e.m f. per turn in the primary and the e.m.f. per turn in the secondary are each equal to the rate with which the magnetic flux is then changing within the core; (5) that $E_p/E_s = N_p/N_s$, and $I_p/I_s = N_s/N_p$. (6) Explain question (2) by referring directly to the electromagnetic pulses. (7) Show why, if the core is an iron ring, no magnetic field is detectable outside the metal of the ring although electromagnetic pulses cut both the primary and the secondary coils and an electric field is impressing an e.m.f. on both coils. (*b*) Explain the action of the induction coil and give the reason for one of the electrodes of the secondary being "positive" and the other "negative." (*c*) Explain why, when one electrode of the coil is a sharp point and the other a circular disk, the spark takes place between the circumference of the disk and the point when the disk is the positive electrode, and between the center of the disk and the point when the disk is the negative electrode.

23. (*a*) Explain why the core of the armature in a generator or motor is made of laminated iron and why the windings are placed in slots cut in the core. (*b*) Explain why a starting resistance is used with an electric motor. (*c*) Explain the production of three-phase currents and the action of the induction motor. (*d*) Show how single- and three-phase e.m.fs. are obtained from a three-phase transmission line. (*e*) Explain why high potentials and alternating currents are used in the transmission of electric power.

24. (*a*) Laws of heating and lighting by electricity. (*b*) Explain the thermoelectric diagram. (*c*) Measurement of temperature by means of the thermocouple. (*d*) The resistance thermometer.

25. (*a*) Describe the production of cathode rays at the negative electrode of a vacuum tube; the stream of positive ions flowing through any orifice

in the negative electrode. (*b*) Explain the action of the two- and three-electrode vacuum tubes. (*c*) Show how, in a three-electrode tube, the plate potential varies the position of the characteristic plate-current curve. (*d*) Show how this variation can be made by a C battery energizing the grid. (*e*) Show how an alternating potential applied to the grid affects the plate current in each of the three standard positions. (*f*) Explain the action of an electron gun and derive the expression for the velocity of the projected particles.

26. (*a*) Draw diagram, explain, and derive the equation for (1) the velocity of the electrons in a cathode stream in terms of the intensities of the crossed electric and magnetic fields, (2) the charge on an electron or proton, and (3) the mass of an electron or ion. (*b*) What are isotopes and how were they discovered?

27. (*a*) What are the energy levels of atoms? (*b*) Describe the process by which energy is emitted from atoms. (*c*) Define photon. (*d*) Describe the production of cathode rays in the two types of x-ray tubes; of the x-rays. (*e*) How are the intensity and the hardness of x-rays controlled in the Coolidge tube? (*f*) Give the nature and the properties of general and of characteristic x-rays. In what respects do x-rays resemble both waves and particles? (*g*) Give the nature of the three kinds of "rays" emitted by radioactive substances. (*h*) Give the experimental evidence for the existence of the nucleus of an atom; for the atomic number of an atom. (*i*) Explain the inertia masses of elemental charges. Why do the nuclear electrons affect the mass of the nucleus? (*j*) The artificial disintegration of atomic nuclei. (*k*) The transformation of mass into energy and of energy into mass. (*l*) What are neutron and positron? Artificial radioactivity?

28. (*a*) Explain why, under proper conditions, an electric charge in a circuit oscillates and why the frequency is decreased by an increase in L or in C. (*b*) Show that $f = (2\pi\sqrt{LC})^{-1}$. (*c*) Show the relation of the electromagnetic pulse and of the changing electrostatic field in the neighborhood of a Hertz oscillator with respect to their action on the free electrons in neighboring conductors. Why does the effect produced by the changing electrostatic field diminish more rapidly with distance than that due to the electromagnetic pulse? (*d*) Show why electromagnetic waves radiate efficiently only from open-circuit oscillators. (*e*) Draw diagram of apparatus for producing damped open-circuit oscillations (Tesla coil). (*f*) What are high-frequency currents? (*g*) Draw diagram showing how undamped open-circuit oscillations are produced by the vacuum-tube oscillator. (*h*) Draw diagram and explain how the presence of electromagnetic waves is detected by means of the crystal detector and by the three-electrode vacuum tube.

29. (*a*) Explain the action of a simple telephone, of the loading coils, and of the thermionic relay. (*b*) Show how the carrier waves are modulated to transmit speech messages; how the detector tube causes the telephone receiver to reproduce the speech. (*c*) Draw diagram and explain the action of a simplified receiving set consisting of one radio- and one audio-frequency amplification. (*d*) Explain how the photoelectric cell responds to varying intensities of light and how it is employed in the transmission of pictures

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Ltd., Bournemouth

how the transmitted electric values of a scene are ...es and how these light values are made to reproduce the scene.

30. (*a*) How is the earth's magnetism explained? (*b*) Explain the charge of the earth and that of the upper layers of the atmosphere. (*c*) What is the potential gradient of the atmosphere near the surface of the earth? (*d*) How do clouds obtain their high potentials? (*e*) Explain the production of the two main forms of lightning. (*f*) Explain how lightning rods protect and why under certain conditions they do not.

APPENDIX IX

SQUARES, CUBES, AND RECIPROCALS

Nos	Squares	Cubes	Reciprocals	Nos.	Squares	Cubes	Reciprocals
1	1	1	1.000000000	51	26 01	132 651	.019607843
2	4	8	.500000000	52	27 04	140 608	.019230769
3	9	27	.333333333	53	28 09	148 877	.018867925
4	16	64	.250000000	54	29 16	157 464	.018518519
5	25	125	.200000000	55	30 25	166 375	.018181818
6	36	216	.166666667	56	31 36	175 616	.017857143
7	49	343	.142857143	57.	32 49	185 193	.017543860
8	64	512	.125000000	58	33 64	195 112	.017241379
9	81	729	.111111111	59	34 81	205 379	.016949153
10	1 00	1 000	.100000000	60	36 00	216 000	.016666667
11	1 21	1 331	.090909091	61	37 21	226 981	.016393443
12	1 44	1 728	.083333333	62	38 44	238 328	.016129032
13	1 69	2 197	.076923077	63	39 69	250 047	.015873016
14	1 96	2 744	.071428571	64	40 96	262 144	.015625000
15	2 25	3 375	.066666667	65	42 25	274 625	.015384615
16	2 56	4 096	.062500000	66	43 56	287 496	.015151515
17	2 89	4 913	.058823529	67	44 89	300 763	.014925373
18	3 24	5 832	.055555556	68	46 24	314 432	.014705882
19	3 61	6 859	.052631579	69	47 61	328 509	.014492754
20	4 00	8 000	.050000000	70	49 00	343 000	.014285714
21	4 41	9 261	.047619048	71	50 41	357 911	.014084507
22	4 84	10 648	.045454545	72	51 84	373 248	.013888889
23	5 29	12 167	.043478260	73	53 29	389 017	.013698630
24	5 76	13 824	.041666667	74	54 76	405 224	.013513514
25	6 25	15 625	.040000000	75	56 25	421 875	.013333333
26	6 76	17 576	.038461538	76	57 76	438 976	.013157895
27	7 29	19 683	.037037037	77	59 29	456 533	.012987013
28	7 84	21 952	.035714286	78	60 84	474 552	.012820513
29	8 41	24 389	.034482759	79	62 41	493 039	.012658228
30	9 00	27 000	.033333333	80	64 00	512 000	.012500000
31	9 61	29 791	.032258065	81	65 61	531 441	.012345679
32	10 24	32 768	.031250000	82	67 24	551 368	.012195122
33	10 89	35 937	.030303030	83	68 89	571 787	.012048193
34	11 56	39 304	.029411765	84	70 56	592 704	.011904762
35	12 25	42 875	.028571429	85	72 25	614 125	.011764706
36	12 96	46 656	.027777778	86	73 96	636 056	011627907
37	13 69	50 653	.027027027	87	75 69	658 503	.011494253
38	14 44	54 872	.026315789	88	77 44	681 472	.011363636
39	15 21	59 319	.025641026	89	79 21	704 969	.011235955
40	16 00	64 000	.025000000	90	81 00	729 000	.011111111
41	16 81	68 921	.024390244	91	82 81	753 571	.010989011
42	17 64	74 088	.023809524	92	84 64	778 688	.010869565
43	18 49	79 507	.023255814	93	86 49	804 357	.010752688
44	19 36	85 184	.022727273	94	88 36	830 584	.010638298
45	20 25	91 125	.022222222	95	90 25	857 375	.010526316
46	21 16	97 336	.021739130	96	92 16	884 736	.010416667
47	22 09	103 823	.021276600	97	94 09	912 673	.010309278
48	23 04	110 592	.020833333	98	96 04	941 192	.010204082
49	24 01	117 649	.020408163	99	98 01	970 299	.010101010
50	25 00	125 000	.020000000	100	100 00	1 000 000	.010000000

APPENDIX X

TRIGONOMETRIC FUNCTIONS

Arc	Angle	sin	tan	sec	cosec	cot	cos		
0.0000	**0**	0.0000	0.0000	1.0000	∞	∞	1.0000	**90**	1.5708
0.0175	**1**	0.0175	0.0175	1.0002	57.2987	57.2900	0.9998	**89**	1.5533
0.0349	**2**	0.0349	0.0349	1.0006	28.6537	28.6363	0.9994	**88**	1.5359
0.0524	**3**	0.0523	0.0524	1.0014	19.1073	19.0811	0.9986	**87**	1.5184
0.0698	**4**	0.0698	0.0699	1.0024	14.3356	14.3007	0.9976	**86**	1.5010
0.0873	**5**	0.0872	0.0875	1.0038	11.4737	11.4301	0.9962	**85**	1.4835
0.1047	**6**	0.1045	0.1051	1.0055	9.5668	9.5144	0.9945	**84**	1.4661
0.1222	**7**	0.1219	0.1228	1.0075	8.2055	8.1443	0.9925	**83**	1.4486
0.1396	**8**	0.1392	0.1405	1.0098	7.1853	7.1154	0.9903	**82**	1.4312
0.1571	**9**	0.1564	0.1584	1.0125	6.3925	6.3138	0.9877	**81**	1.4137
0.1745	**10**	0.1736	0.1763	1.0154	5.7588	5.6713	0.9848	**80**	1.3963
0.1920	**11**	0.1908	0.1944	1.0187	5.2408	5.1446	0.9816	**79**	1.3788
0.2094	**12**	0.2079	0.2126	1.0223	4.8097	4.7046	0.9781	**78**	1.3614
0.2269	**13**	0.2250	0.2309	1.0263	4.4454	4.3315	0.9744	**77**	1.3439
0.2443	**14**	0.2419	0.2493	1.0306	4.1336	4.0108	0.9703	**76**	1.3264
0.2618	**15**	0.2588	0.2679	1.0353	3.8637	3.7321	0.9659	**75**	1.3090
0.2793	**16**	0.2756	0.2867	1.0403	3.6280	3.4874	0.9613	**74**	1.2915
0.2967	**17**	0.2924	0.3057	1.0457	3.4203	3.2709	0.9563	**73**	1.2741
0.3142	**18**	0.3090	0.3249	1.0515	3.2361	3.0777	0.9511	**72**	1.2566
0.3316	**19**	0.3256	0.3443	1.0576	3.0716	2.9042	0.9455	**71**	1.2392
0.3491	**20**	0.3420	0.3640	1.0642	2.9238	2.7475	0.9397	**70**	1.2217
0.3665	**21**	0.3584	0.3839	1.0711	2.7904	2.6051	0.9336	**69**	1.2043
0.3840	**22**	0.3746	0.4040	1.0785	2.6695	2.4751	0.9272	**68**	1.1868
0.4014	**23**	0.3907	0.4245	1.0864	2.5593	2.3559	0.9205	**67**	1.1694
0.4189	**24**	0.4067	0.4452	1.0946	2.4586	2.2460	0.9135	**66**	1.1519
0.4363	**25**	0.4226	0.4663	1.1034	2.3662	2.1445	0.9063	**65**	1.1345
0.4538	**26**	0.4384	0.4877	1.1126	2.2812	2.0503	0.8988	**64**	1.1170
0.4712	**27**	0.4540	0.5095	1.1223	2.2027	1.9626	0.8910	**63**	1.0996
0.4887	**28**	0.4695	0.5317	1.1326	2.1301	1.8807	0.8829	**62**	1.0821
0.5061	**29**	0.4848	0.5543	1.1434	2.0627	1.8040	0.8746	**61**	1.0647
0.5236	**30**	0.5000	0.5774	1.1547	2.0000	1.7321	0.8660	**60**	1.0472
0.5411	**31**	0.5150	0.6009	1.1666	1.9416	1.6643	0.8572	**59**	1.0297
0.5585	**32**	0.5299	0.6249	1.1792	1.8871	1.6003	0.8480	**58**	1.0123
0.5760	**33**	0.5446	0.6494	1.1924	1.8361	1.5399	0.8387	**57**	0.9948
0.5934	**34**	0.5592	0.6745	1.2062	1.7883	1.4826	0.8290	**56**	0.9774
0.6109	**35**	0.5736	0.7002	1.2208	1.7434	1.4281	0.8192	**55**	0.9599
0.6283	**36**	0.5878	0.7265	1.2361	1.7013	1.3764	0.8090	**54**	0.9425
0.6458	**37**	0.6018	0.7536	1.2521	1.6616	1.3270	0.7986	**53**	0.9250
0.6632	**38**	0.6157	0.7813	1.2690	1.6243	1.2799	0.7880	**52**	0.9076
0.6807	**39**	0.6293	0.8098	1.2868	1.5890	1.2349	0.7771	**51**	0.8901
0.6981	**40**	0.6428	0.8391	1.3054	1.5557	1.1918	0.7660	**50**	0.8727
0.7156	**41**	0.6561	0.8693	1.3250	1.5243	1.1504	0.7547	**49**	0.8552
0.7330	**42**	0.6691	0.9004	1.3456	1.4945	1.1106	0.7431	**48**	0.8378
0.7505	**43**	0.6820	0.9325	1.3673	1.4663	1.0724	0.7314	**47**	0.8203
0.7679	**44**	0.6947	0.9657	1.3902	1.4396	1.0355	0.7193	**46**	0.8029
0.7854	**45**	0.7071	1.0000	1.4142	1.4142	1.0000	0.7071	**45**	0.7854
		cos	cot	cosec	sec	tan	sin	Angle	Arc

APPENDIX XI

LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

Angle	log sin	log tan	log sec	log csc	log cot	log cos	
0	$-\infty$	$-\infty$	0.0000	∞	∞	0.0000	**90**
1	8.2419	8.2419	0.0001	1.7581	1.7581	9.9999	**89**
2	8.5428	8.5431	0.0003	1.4572	1.4569	9.9997	**88**
3	8.7188	8.7194	0.0006	1.2812	1.2806	9.9994	**87**
4	8.8436	8.8446	0.0011	1.1564	1.1554	9.9989	**86**
5	8.9403	8.9420	0.0017	1.0597	1.0580	9.9983	**85**
6	9.0192	9.0216	0.0024	0.9808	0.9784	9.9976	**84**
7	9.0859	9.0891	0.0032	0.9141	0.9109	9.9968	**83**
8	9.1436	9.1478	0.0042	0.8564	0.8522	9.9958	**82**
9	9.1943	9.1997	0.0054	0.8057	0.8003	9.9946	**81**
10	9.2397	9.2463	0.0066	0.7603	0.7537	9.9934	**80**
11	9.2806	9.2887	0.0081	0.7194	0.7113	9.9919	**79**
12	9.3179	9.3275	0.0096	0.6821	0.6725	9.9904	**78**
13	9.3521	9.3634	0.0113	0.6479	0.6366	9.9887	**77**
14	9.3837	9.3968	0.0131	0.6163	0.6032	9.9869	**76**
15	9.4130	9.4281	0.0151	0.5870	0.5719	9.9849	**75**
16	9.4403	9.4575	0.0172	0.5597	0.5425	9.9828	**74**
17	9.4659	9.4853	0.0194	0.5341	0.5147	9.9806	**73**
18	9.4900	9.5118	0.0218	0.5100	0.4882	9.9782	**72**
19	9.5126	9.5370	0.0243	0.4874	0.4630	9.9757	**71**
20	9.5341	9.5611	0.0270	0.4659	0.4389	9.9730	**70**
21	9.5543	9.5842	0.0298	0.4457	0.4158	9.9702	**69**
22	9.5736	9.6064	0.0328	0.4264	0.3936	9.9672	**68**
23	9.5919	9.6279	0.0360	0.4081	0.3721	9.9640	**67**
24	9.6093	9.6486	0.0393	0.3907	0.3514	9.9607	**66**
25	9.6259	9.6687	0.0427	0.3741	0.3313	9.9573	**65**
26	9.6418	9.6882	0.0463	0.3582	0.3118	9.9537	**64**
27	9.6570	9.7072	0.0501	0.3430	0.2928	9.9499	**63**
28	9.6716	9.7257	0.0541	0.3284	0.2743	9.9459	**62**
29	9.6856	9.7438	0.0582	0.3144	0.2562	9.9418	**61**
30	9.6990	9.7614	0.0625	0.3010	0.2386	9.9375	**60**
31	9.7118	9.7788	0.0669	0.2882	0.2212	9.9331	**59**
32	9.7242	9.7958	0.0716	0.2758	0.2042	9.9284	**58**
33	9.7361	9.8125	0.0764	0.2639	0.1875	9.9236	**57**
34	9.7476	9.8290	0.0814	0.2524	0.1710	9.9186	**56**
35	9.7586	9.8452	0.0866	0.2414	0.1548	9.9134	**55**
36	9.7692	9.8613	0.0920	0.2308	0.1387	9.9080	**54**
37	9.7795	9.8771	0.0977	0.2205	0.1229	9.9023	**53**
38	9.7893	9.8928	0.1035	0.2107	0.1072	9.8965	**52**
39	9.7989	9.9084	0.1095	0.2011	0.0916	9.8905	**51**
40	9.8081	9.9238	0.1157	0.1919	0.0762	9.8843	**50**
41	9.8169	9.9392	0.1222	0.1831	0.0608	9.8778	**49**
42	9.8255	9.9544	0.1289	0.1745	0.0456	9.8711	**48**
43	9.8338	9.9697	0.1359	0.1662	0.0303	9.8641	**47**
44	9.8418	9.9848	0.1431	0.1582	0.0152	9.8569	**46**
45	9.8495	0.0000	0.1505	0.1505	0.0000	9.8495	**45**
	log cos	log cot	log csc	log sec	log tan	log sin	Angle

APPENDIX XII

LOGARITHMS OF NUMBERS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS OF NUMBERS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

APPENDIX XIII

EXTRA PROBLEMS

1. Spheres A and B, 100 cm apart, carry, respectively, charges of $+100$ and -800 statcoulombs. The zero potential point is at what distance from the center of sphere A on the line joining the centers of the spheres?

2. Two equal spheres, one with a charge of $+200$ and the other with a charge of $+40$ statcoulombs, are 60 cm apart. They are brought into contact and then again carried to their original positions. What are the forces of repulsion before and after contact?

3. Two concentric spheres have radii of 10 and 30 cm, respectively. The inner one is charged with $+2{,}000$ statcoulombs of electricity and the outer one is grounded. (*a*) What is the potential of the inner sphere before and after the removal of the outer sphere? (*b*) What is the energy of the charge before and after the removal? (*c*) Account for the change in potential and in energy.

4. Two uniform magnetic fields of 500 and 800 oersteds, respectively, cross at right angles. (*a*) What are the intensity and the direction of the resultant field? (*b*) What is the flux density?

5. What is the current in amperes when 12×10^{18} electrons are moving by every plane of the circuit per second?

6. An isolated sphere whose diameter is 2 meters is charged to a potential of 1 million volts. (*a*) What is its charge in statcoulombs? (*b*) In coulombs? (*c*) If that charge could discharge uniformly and completely in 2 sec, what would be the intensity of the current in amperes?

7. An e m.f. of 10 volts is impressed on 20 cm of a uniform circuit whose total length is 120 cm and whose total resistance is 2 ohms. (*a*) What is the p.d. in the external part of the circuit and (*b*) the RI drop in the part containing the e.m.f.? (*c*) What is the total power being supplied to the circuit and (*d*) what part of this is expended in the external portion of the circuit?

8. When electricity sells at 3 cts/kw-hr, how much does a joule of energy cost?

9. How much does it cost (*a*) to melt 10 kg of ice when electricity sells at 3 cts/kw-hr, (*b*) to heat the ice water to 100°C, and (*c*) to vaporize the water? The heats of fusion and of vaporization are 80 and 538 calories/gram.

10. How long a piece of manganin wire whose cross-sectional area is 0.5 mm^2 is required for a resistance of 5 ohms?

11. What is the resistance of 100 meters of copper wire of 0.4 mm diameter (*a*) at 70°F and (*b*) at 100°F?

12. When electricity sells at 3 cts/kw-hr, what is the money loss per hour in mains whose resistance is 5 ohms (*a*) when a current of 2 amp is energizing the circuit and (*b*) when the current strength is 4 amp?

13. Derive the expressions for the velocity v and the time t required for the flow electrons to move D cm along a conductor of area A when the direct current is I amp. (Let N = number of flow electrons in a centimeter cube and n that in a coulomb.)

14. When the direct current is 1 amp in a copper wire of 1 mm² cross-sectional area, (*a*) what is the electron-flow velocity and (*b*) how long does it take an electron to move 1 km?

15. Assuming the number of flow electrons in tungsten to be 6.26×10^{22} per cubic centimeter and the diameter of the tungsten filament of a 40-watt lamp to be 0.043 mm, (*a*) what is the velocity of the flow electrons and (*b*) how many electrons per second pass through every plane of the filament when the direct-current impressed e.m.f. is 110 volts?

16. What is the capacitance, in microfarads, of a sphere whose diameter is 1 meter? Of the earth whose diameter is 6,377 km?

17. What is the potential gradient just outside a sphere of 20 cm radius when its charge is 80,000 statcoulombs?

18. What is the maximum potential to which a sphere of 2 meters diameter can be charged in air, the ionizing potential gradient being 30,000 volts/cm?

19. Calculate, from the electrochemical equivalent, (*a*) the number of atoms per cubic centimeter and (*b*) the distance between their centers in silver. The density of silver is 10.5 gram/cm³.

20. What time is required for a current of 3 amp to decompose electrolytically 40 grams of water?

21. A magnetic field is cutting a wire across 10 cm of its length and thereby is exerting an electric force of 4.77×10^{-14} dynes on each free electron. (*a*) What is the induced e.m.f. and (*b*) the intensity of the electric field associated with the magnetic?

22. A magnetic field 10 cm across and of 2,000 gauss density is being cut by a conductor moving with a velocity of 30 meters/sec. (*a*) What is the magnetic force acting on each electron? (*b*) What is the induced e.m.f. in the conductor?

23. (*a*) What is the intensity of the moving electron field at the center of a loop of 10 cm radius when a current of 5 amp is energizing the loop? Calculate the intensity of the magnetic field at the center of the loop from (*b*) the action of the moving electric field and (*c*) the definition of a unit current.

24. (*a*) What is the line linkage in a coil whose self-inductance is 15 millihenrys when the energizing current is 10 amp? (*b*) What energy of motion do the flow electrons possess?

25. (*a*) What is the inductive reactance in the 0.2-ohm primary of a transformer in which the magnetization current is 0.1 amp when the 60-cycle impressed e.m.f. is 110 volts? (*b*) What is the self-inductance of the primary coil? (*c*) The phase lag?

26. (*a*) What is the capacitive reactance in a condenser branch of 2 ohms resistance when a 60-cycle e.m.f. impressed upon it produces a current of 4 amp? (*b*) What is the phase lead? (*c*) The power being expended?

27. A 60-cycle e.m.f. is impressed on a part of a circuit containing a 5-microfarad condenser and an in-series resistance of 5 ohms. (*a*) What is the current? (*b*) The phase lead? (*c*) What self-inductance of negligible resistance must be placed in series to bring the current into phase? (*d*) What would then be the intensity of the current?

28. (*a*) What series capacitance must be placed in the circuit of Prob. XVIII–3 to bring the current into phase? (*b*) How much does this change the current and the power delivered?

29. (*a*) How much inductive or capacitive reactance must be placed in series into each of the circuits of Probs. XVIII–4, 5 to bring the current into phase? (*b*) What changes do these produce in the current and the power delivered?

APPENDIX XIV

ANSWERS TO PROBLEMS

CHAPTER III

1. 10 dynes; repulsion; attraction. **2.** 22.75×10^{-20} dynes. **3.** 9 994 dynes; 0.123 dynes. **4.** 2 e.s.u., away from charge; 2 e s.u., toward charge. **5.** 200 dynes. **6.** 800 dynes. **7.** 3 statvolts. **8.** 1,000 ergs; 1,000 ergs. **9.** 3 statvolts. **10.** 1,200 ergs **11.** 0.05 c g s. units, 60° downward from the horizontal; 0 statvolts. **12.** -1.5 statvolts.

CHAPTER VII

1. 200 dynes. **2.** 1,000 dynes. **3.** 15 dyne-cm. **4.** 0.62 oersteds. **5.** (*a*) 0.0645 oersteds; (*b*) 0.0645 dynes. **6.** 47.37 c.g.s. units. **7.** 2,162 c g.s. units. **8.** 0.1480 oersteds; 320 c.g s. units; 32 units. **9.** 0.1024 oersteds. **10.** 0.11 oersteds. **11.** 0.16 oersteds. **12.** (*a*) 10,000 gauss; (*b*) 10,000 oersteds. **13.** (*a*) 107 oersteds; (*b*) 2,140 maxwells.

CHAPTER VIII

1. (*a*) 17.14 times; (*b*) 1.714 times. **2.** (*a*) 66.3; (*b*) 198.9×10^9. **3.** (*a*) 1,200; (*b*) 120; (*c*) 3.6×10^{12}. **4.** (*a*) 5; (*b*) 0.5; (*c*) 15×10^9. **5.** 1.57 oersteds. **6.** 15.708 cm. **7.** 0.955 c g.s. units. **8.** 0.551 amp. **9.** 326.4 grams. **10.** 1.590×10^{-21} amp. **11.** 4.770×10^{-8} dynes.

CHAPTER IX

1. 18,000 coulombs. **2.** 5,400 **3.** (*a*) 140 ergs; (*b*) 140 ergs; (*c*) 140 joules. **4.** 24,000 joules. **5.** (*a*) 87.1; (*b*) 87.1×10^8; (*c*) 0.2903. **6.** 2,000 watts. **7.** (*a*) 0.545 amp; (*b*) 201.6 ohms. **8.** 23.76 cts. **9.** (*a*) 0.767 hr; (*b*) 2 78 cts. **10.** 3.259 miles.

CHAPTER X

1. 0.356 ohms. **2.** 9 ohms. **3.** 20 amp. **4.** 15 ohms; 3.333 ohms. **5.** (*a*) 22 amp; 11 amp; (*b*) 7.333 amp; (*c*) 2,420 watts, 1,210 watts; (*d*) 268.9 watts, 537.8 watts. **6.** (*a*) 21.71 ohms; (*b*) 18.29 ohms. **7.** 560.55 ohms. **8.** 10.86 ohms.

CHAPTER XI

1. 8.05 grams. **2.** 2.37 grams. **3.** 0.2485 amp. **4.** 5.906×10^{22}. **5.** (*a*) 600 volts/cm; (*b*) 2 e.s.u.

CHAPTER XII

1. (*a*) 5 statfarads; (*b*) 5.55×10^{-21} abfarads; (*c*) 5.55×10^{-12} farads; (*d*) 5.55×10^{-6} microfarads; (*e*) 5.55 $\mu\mu$f. **2.** 505 statfarads. **3.** (*a*) 4,500 volts; (*b*) 450×10^{9} volts. **4.** 429.9 statfarads. **5.** 15,040 statfarads. **6.** (*a*) 7 microfarads; (*b*) 0.571 microfarads. **7.** (*a*) 900 microfarads; (*b*) 0.001111 microfarads. **8.** 0.02 joules. **9.** 0.01 joules. **10.** (*a*) 120 statvolts; (*b*) 1,800 statcoulombs; (*c*) 0.045 joules; (*d*) 0.018 joules; (*e*) 0.027 joules. **11.** 0.637 ergs. **12.** 11,141 ergs.

CHAPTER XIII

1. 0.04 joules. **2.** 0.4 volts. **3.** 2 volts. **4.** 0.0004 volts. **5.** 0.0016 volts. **6.** (*a*) 3.33×10^{-3} e.s.u.; (*b*) 10 volts. **7.** 3.33×10^{-5} oersteds. **8.** 3 volts, to left. **9.** 125 volts.

CHAPTER XIV

1. (*a*) 0.1375 amp; (*b*) 1.03 volts; (*c*) 0.069 volts; (*d*) 0.1418 watts; (*e*) 0.0094 watts. **2.** (*a*) 6.6 volts, 3 ohms; 1.10 volts, 0.083 ohms; (*b*) 2.13 amp, 6.00 amp; (*c*) 0.508 amp, 0.109 amp.

CHAPTER XV

1. (*a*) 0.0001 coulombs; (*b*) 10 joules. **2.** 200 cm. **3.** (*a*) 0.0002 amp; (*b*) 8 ergs/sec.

CHAPTER XVII

1. (*a*) 0.2 volts; (*b*) 1,000 volts. **2.** 0.0196 coulombs. **3.** 10 millihenrys. **4.** 28 millihenrys. **5.** 0.12 henrys. **6.** 1.224 joules.

CHAPTER XVIII

1. 44.42 volts. **2.** 12.56 volts. **3.** (*a*) 188.49 ohms; (*b*) 188.50 ohms; (*c*) 0.5836 amp; (*d*) 0.5835 amp; (*e*) 90°, 89°23.5′; (*f*) 0 watts, 0.681 watts. **4.** (*a*) 26.52 ohms; (*b*) 33.22 ohms; (*c*) 3.31 amp; (*d*) 52°1.3″. **5.** (*a*) 188.5 ohms; (*b*) 66.3 ohms; (*c*) 123.9; (*d*) 0.888 amp; (*e*) 83°21.4′; (*f*) 11.3 watts.

CHAPTER XX

1. 628.5 gilberts. **2.** (*a*) 25; (*b*) 0.524 oersteds; (*c*) 31.4 gilberts; (*d*) 31.4 gilberts; (*e*) 0.003 units; (*f*) 10,475 maxwells; (*g*) 1,309 gauss. **3.** (*a*) 0.25 units; (*b*) 0.253 units; (*c*) 124.2 maxwells; (*d*) 15.5 gauss; (*e*) 84.3; (*f*) 0.503 units, 62.5 maxwells.

CHAPTER XXI

1. (*a*) 0.1123 dynes/cm; (*b*) 5.36°. **2.** 0.005 ohms. **3.** (*a*) 17.83 volts; (*b*) 3.567 ohms. **4.** 19,980 ohms. **5.** 1.261 volts. **6.** 1.629×10^{-6} coulombs. **7.** 2.006 microfarads. **8.** 0.00005 coulombs. **9.** 2,666 oersteds.

CHAPTER XXII

1. (*a*) 2,200 volts; (*b*) 40 amp. **2.** 5.15 henrys.

CHAPTER XXIII

1. (*a*) 200 watts; (*b*) 2,100 watts; (*c*) 50.4 cts. **2.** (*a*) 5.5 amp; (*b*) 11 per cent. **3.** (*a*) 11,500 watts; (*b*) 112.15 volts; (*c*) 3,838 amp. **4.** (*a*) 1,150 volts; (*b*) 20 amp. **5.** (*a*) 6,000 kw; (*b*) 3.33 per cent; (*c*) 5,800 kw; (*d*) 58,000 volts; (*e*) 5,196 kw. **6.** (*a*) 0.248 per cent; (*b*) 0.992 per cent; (*c*) 83.3 per cent.

CHAPTER XXIV

1. 158 sec. **2.** 0.00095 volts.

CHAPTER XXV

1. (*a*) 6,072 volts; (*b*) 4.293 sec. **2.** 3.257×10^8 cm/sec.

CHAPTER XXVI

1. (*a*) 0.2 c.g.s. units; (*b*) 9.54×10^{-11} dynes. **2.** (*a*) $3\ 54 \times 10^{18}$ cm/sec^2; (*b*) 1.03×10^{10} cm/sec; (*c*) 4.77×10^{-8} ergs. **3.** (*a*) 9.54×10^{-10} statcoulombs; (*b*) 2. **4.** 1.286×10^{10} cm/sec. **5.** 80×10^{-24} grams.

CHAPTER XXVIII

1. (*a*) 0.00628 sec; (*b*) 159.2; (*c*) 1.884×10^6 meters. **2.** (*a*) 7,118; (*b*) 42,150 meters; (*c*) Yes, $R < 2{,}236$.

APPENDIX XV

SUGGESTED LESSON ASSIGNMENTS FOR PART I

(Solutions of the problems to be handed in on standard sheets and returned with grade. The assignments of articles and problems are inclusive.)

1. A note to the student; Chaps. I and II.
2. Chap. III.
3. Probs. III–1 to 12; Chap. IV.
4. Chap. V.
5. Chap. VI.
6. Chap. VII.
7. Probs. VII–1 to 13; review Chaps. I to VII
8. Chap. VIII.
9. Probs. VIII–1 to 11; Chap. IX.
10. Probs. IX–1 to 10; Chap. X.
11. Probs. X–1 to 8; Chap. XI.
12. Probs. XI–1 to 5; Chap. XII (1 to 8).
13. Chap. XII (9 to 13); Probs. XII–1 to 12.
14. Chap. XIII (1 to 4).
15. Chap. XIII (5 to 8).
16. Probs. XIII–1 to 9, XIV–1, 2; Chap. XIV.
17. Chap. XV; Probs. XV–1 to 3.
18. Chap. XVI.
19. Chap. XVII; Probs. XVII–1 to 6.
20. Chap. XVIII (1 to 8).
21. Chap. XVIII (9 to 16).
22. Probs. XVIII–1 to 5; Chap. XIX.
23. Chap. XX; Probs. XX–1 to 3.

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